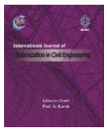
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERIN
Int. J. Optim. Civil Eng., 2024; 14(3):319-336



OPTIMIZATION OF CASCADE STILLING BASINS USING GRADIENT-BASED METAHEURISTICS

M. Golkar and R. Sheikholeslami*,†
Department of Civil Engineering, Sharif University of Technology, Tehran, Iran

ABSTRACT

Spillway design poses a significant challenge in effectively managing the energy within water flow to prevent erosion and destabilization of dam structures. Traditional approaches typically advocate for standard hydraulic jump stilling basins or other energy dissipators at spillway bases yet constructing such basins can be prohibitively large and costly, particularly when extensive excavation is necessary. Consequently, growing interest in cascade hydraulic structures has emerged over recent decades as an alternative for energy dissipation. These structures utilize a series of arranged steps to facilitate water flow, effectively dissipating energy as it traverses the cascade. Commonly deployed in scenarios involving high dams or steep gradients, the stepped configuration ensures efficient aeration and substantial energy dissipation along the structure, thereby reducing the size and cost of required stilling basins. Despite extensive research on hydraulic characteristics using physical and numerical models and established design procedures, construction cost optimization of step cascades remains limited but promising. This paper aims to address this gap by employing two novel gradient-based meta-heuristic optimization techniques to enhance the efficiency and cost-effectiveness of cascade stilling basin designs. Through comparative analyses and evaluations, this study demonstrates the efficacy of these techniques and offers insights for future research and applications in hydraulic structures design optimization.

Keywords: stilling basin; spillway; step cascades; optimization; meta-heuristics.

Received: 5 March 2024; Accepted: 26 May 2024

1. INTRODUCTION

The dam structure comprises various integral components, including the reservoir, body, spillway, energy dissipation system, tunnel, and power plant. An essential component within

---

* Corresponding author: Department of Civil Engineering, Sharif University of Technology, Tehran, Iran
† E-mail address: razi.sheikholeslami@sharif.edu (R. Sheikholeslami)

[DOI: 10.22068/ijoce.2024.14.3.599]

this structure is the energy dissipation system, necessary to mitigate the effects of hydraulic forces. As water flows from the reservoir to the downstream, the conversion of static head to kinetic energy occurs, potentially resulting in high velocities and significant pressures. Without proper dissipation, these forces can pose a threat to downstream structures. Commonly employed energy dissipation systems include stilling basins, roller buckets, and sill block aprons [1-4].

In the case of high-head dams, traditional combinations of spillways, such as siphon spillways, lateral spillways, and ogee spillways, may prove impractical due to insufficient tailwater depth or unsuitable site conditions [3-5]. These conventional designs, coupled with energy dissipation systems, often necessitate large dimensions. To mitigate this issue, cascade stilling basins offer a viable alternative. A cascade stilling basin comprises a series of successive free-fall spillways, each followed by a stilling basin. This arrangement effectively reduces flow velocity and Froude number, eliminating the risks associated with cavitation, abrasion, and vibration [1]. Moreover, by minimizing dimensions, cascade stilling basins also contribute to cost savings in spillway construction [2].

In hydraulic engineering, cascades often replace smooth chutes and are used in structures such as flood relief systems or sewage channels. According to Chanson [6], two distinct flow states must be distinguished in cascades – nappe flow and skimming flow. In nappe flow, energy conversion occurs with or without a complete hydraulic jump. In skimming flow, the water shoots over the steps, forming standing waves that extract some energy from the main flow. The type of flow is determined by the slope of the cascade, defined by the ratio of step height to step depth. Typically, flatter cascades exhibit nappe flow, while steeper ones show skimming flow. The step cascades allow water to plunge from step to step at low flows, dissipating much of its energy. At higher flows, the water skims over the steps, creating turbulent eddies in the step pockets. These eddies reduce flow velocity, thus decreasing the required size of the stilling basin for the hydraulic jump. Additionally, the turbulent eddies trap air, causing the flow to become highly aerated and increasing its bulk depth, further reducing the average flow velocity.

Optimizing the design of cascade stilling basins is essential to ensure the efficiency and cost-effectiveness of the dam structure. In recent decades, numerous optimization methods have been developed to solve engineering design problems, ranging from classical Linear Programming (LP) and Non-Linear Programming (NLP) to more advanced techniques like Dynamic Programming (DP) and meta-heuristic algorithms (see, e.g., [7-13]). Among these approaches, meta-heuristic algorithms [14] offer several significant advantages over traditional optimization techniques.

Firstly, meta-heuristic algorithms are highly flexible and can be easily adapted to a wide range of optimization problems without requiring specific mathematical formulations. This flexibility allows them to effectively handle complex, non-linear, and multi-modal problems that often arise in the design of hydraulic structures. Traditional methods like LP and NLP may struggle with such complexities due to their reliance on gradient information and other specific problem structures. Secondly, meta-heuristic algorithms can escape local optima, which is a common limitation of many classical optimization techniques. These methods often employ stochastic search strategies and population-based approaches that enhance their ability to explore the global search space thoroughly. This characteristic is particularly beneficial in optimizing the design of cascade stilling basins, where the solution landscape can be highly

[DOI: 10.22068/ijoce.2024.14.3.599]

irregular and fraught with multiple local optima.

Additionally, meta-heuristic algorithms often require fewer assumptions about the problem domain, making them more robust and versatile in practical applications. They can incorporate a variety of constraints and objective functions, which is essential for balancing the multiple performance and cost criteria involved in dam construction projects. This robustness contrasts with traditional methods that might need extensive modifications to accommodate different constraints and objectives. Furthermore, the iterative nature of meta-heuristic algorithms allows for continuous improvement and refinement of solutions. This iterative process is advantageous for complex engineering problems where initial solutions can be progressively enhanced through successive iterations.

In this paper, meta-heuristic algorithms are employed to optimize the design of cascade stilling basins. The effectiveness of two new meta-heuristics inspired by the gradient-based Newton’s method is explored. Gradient-based meta-heuristics integrate gradient and population-based techniques to determine the search direction, with Newton's method guiding the exploration of the search space. Unlike gradient methods and conventional optimization approaches, which typically adhere to a predetermined search direction towards the optimal solution, these algorithms adjust their search direction as they navigate the search space. In this study, the Gradient-based Optimizer (GBO), a novel population-based metaheuristic, is utilized, which has demonstrated its efficacy in solving various engineering problems [15], image processing [16], and task scheduling [17]. Previous studies have shown that GBO outperforms many other meta-heuristic methods in terms of solution quality and convergence [15-17]. Additionally, an improved version of GBO called the dynamic Fitness-Distance Balance (dFDB) algorithm is applied [18]. By leveraging the strengths of GBO and incorporating the dynamic fitness-distance balancing technique, the dFDB offers a promising avenue for solving optimization problems [19]. Through the application of these meta-heuristic algorithms, this paper aims to achieve an optimized design for cascade stilling basins while balancing hydraulic performance requirements with cost considerations.

[DOI: 10.22068/ijoce.2024.14.3.599]

2. LITERATURE REVIEW

In contrast to the belief that cascades for energy dissipation are a modern innovation introduced with new construction methods such as RCC and gabions, these channels have been used since ancient times. Historically, stepped channels were designed to enhance the stability of structures like overflow weirs and to dissipate flow energy. This technique was independently developed by several ancient civilizations. In fact, around 16 dams featuring stepped spillways were constructed in antiquity. These dams varied in height from 1.4 m to 50 m, in width from 3.7 m to 150 m, and handled maximum discharges of up to about 9000 m3/sec. The steps on these spillways ranged from 0.6 m to 5 m in height, with the number of steps varying between 2 and 14 [20].

The benefits of cascade structures lie in their gradual reduction of water kinetic energy, leading to heat transformation, thereby allowing for a reduction in the size of the stilling basin. These structures are particularly suited for dam spillways or combined sewer systems, ensuring the safe discharge of water over significant drops. Stepped chutes are frequently integrated into dam spillways and are also utilized in park settings, enhancing natural landscapes. In a cascade stilling basin, each step corresponds to an isolated stilling basin for hydraulic jump. Water descends from level to level on stepped chutes, with optimal dissipation expected when a hydraulic jump occurs. For steeper stepped chutes and high discharge rates, skimming flow may occur, where the step acts as a roughness element, promoting vigorous mixing of water with air [21].

The impact of step geometry, number of steps, and relative energy loss has been extensively presented and discussed in several studies. Experiments on physical models conducted over the past decades have provided deeper insights into flow characteristics and energy dissipation performance. For instance, Thorwarth [22] investigated flow instabilities on pooled stepped chutes with gentle slopes of 8.9˚ and 14.6˚, which could pose safety risks for dams. Chinnarasri and Wongwises [23] observed that energy dissipation was comparatively lower on flat stepped chutes than on steps with end sills for a 45˚ chute slope. Barani et al. [24] studied energy dissipation on a stepped spillway model with a 41.41˚ slope and 21 steps, finding that pooled stepped spillways dissipated more energy than flat stepped spillways.

Finding the optimal combination of dimensions and configurations for cascade stilling basins remains a challenging topic. Optimization of these structures falls into two main categories. The first category focuses on maximizing hydraulic performance through experimental investigations and hydraulic simulations. For instance, Tabari and Tavakoli [25] explored the relationship between energy reduction, the number of steps, step heights, and flow discharge. Frizell et al. [26] demonstrated how the energy dissipation system and the incline of steps affect the probability of cavitation. Roushangar et al. [27] simulated the energy dissipation of these systems using empirical data, while Shahheydari et al. [28] studied the correlation between flow coefficient and energy dissipation. Mero and Mitchell [29] compared the effectiveness of horizontal steps versus inclined or flat curved steps in dissipating energy. Additionally, Aal et al. [30] examined the impact of over-flow, through-flow, and under-flow breakers on energy dissipation, and Afshoon et al. [31] investigated the influence of step roughness.

The second category concentrates on optimizing the design of cascade stilling basins, with a particular focus on minimizing construction costs. There has been limited research in this area. For the first time, Vittal and Porey [32] (hereafter referred to as VP) introduced a systematic method for designing cascade stilling basins primarily focused on meeting hydraulic criteria. The VP method examines only a restricted set of alternatives before selecting the most favorable one. It determines the number and height of falls, along with the length of stilling basins, and then optimizes graphically with the objective of minimizing excavation volumes. The review of existing studies in this category indicates that various methods have been employed to enhance design efficiency. In all these studies, including the present one, the height of each fall and the length of the stilling basin beneath it are considered as decision variables. One of the first attempts to minimize the cost of cascade stilling basins was made by Bakhtyar et al. [1] using the Dynamic Programming (DP) method. Their results indicated a cost reduction of 34.4% and 31% compared to the traditional VP method for four cascades, both with and without considering concrete works, respectively. Afshar and Daraeikhah [5] later applied the Continuous Ant Algorithm (CAA), achieving improvements of approximately 18% and 16% for three and four steps, respectively. Daraeikhah et al. [33] tackled the problem using the Particle Swarm Optimization (PSO) algorithm, which

[DOI: 10.22068/ijoce.2024.14.3.599]

outperformed the VP results. Jazayeri and Moeini [3] explored four meta-heuristic algorithms, namely Artificial Bee Colony (ABC), PSO, and their improved versions (IABC and IPSO), and compared the outcomes with the VP method and Genetic Algorithm (GA). They concluded that VP failed to find the optimal solution, with IABC and IPSO showing better performance than GA for both three- and four-step cases. Finally, Jazayeri and Moeini [4] employed four different meta-heuristic algorithms: GA, Gravitational Search Algorithm (GSA), PSO, and ABC. The GA results surpassed those of VP, while the other three methods marginally outperformed GA for both three- and four-stepped cascades. PSO yielded the best results, reducing costs by 17.7% and 16.45% for three and four cascades, respectively.

Overall, the review of the literature highlights the extensive research conducted on the hydraulic performance of cascade stilling basins. These studies have primarily explored various aspects such as the impact of step geometry, number of steps, flow characteristics, and energy dissipation efficiencies. However, in terms of cost optimization, there is a noticeable lack of literature utilizing diverse methodologies to reduce the construction costs associated with cascade stilling basins. Despite the existing advancements, there remains a need for continued exploration of innovative optimization techniques to enhance the design efficiency and cost-effectiveness of cascade stilling basins. This paper addresses this gap by applying two new meta-heuristic algorithms for the optimum cost design of step cascades, thereby contributing to the ongoing development of more effective and economical cascade stilling basins.

[DOI: 10.22068/ijoce.2024.14.3.599]

3. THE UTILIZED OPTIMIZATION METHODS

3.1. Gradient-based Optimizer

The first method employed to optimize the design is the Gradient-based Optimizer (GBO), a novel meta-heuristic optimization algorithm [34]. Inspired by the gradient-based Newton's method, GBO utilizes two main operators. The first operator, the Gradient Search Rule (GSR), accelerates the convergence rate, while the second operator, the Local Escaping Operator (LEO), helps escape local optima. Together, these operators, along with a set of vectors, effectively explore the search space.

First, three parameters called *pr* (probability), *M* (total number of iterations) and ε (small number in range of [0,0.1]) must be assigned. The algorithm needs initialization, so an initial population $X_0 = [x_{0,1}, x_{0,2}, \ldots, x_{0,D}]$ is generated, in which *D* is the number of variables. Then, the objective function value $f(X_0)$ must be evaluated for each member of population and the best and worst solutions will be specified.

In each iteration (*m*), the next position of each member of population (*n*) is a vector and can be calculated using equation (1).

$$X_{n,i}^{m+1} = r_a(r_b X1_n^m + (1 - r_b)X2_n^m) + (1 - r_a)X3_n^m \tag{1}$$

where $X1_n^m$, $X2_n^m$ and $X3_n^m$ are vectors and can be calculated using equations (2-4); r_a and r_b are random numbers selected from [0,1].

$$X1_n^m = x_n^m - (randn)\rho_1 \frac{2\Delta x(x_n^m)}{yp_n^m - yq_n^m + \varepsilon} + (rand)\rho_2(x_{best} - x_n^m) \tag{2}$$

$$X2_n^m = x_{best} - (randn)\rho_1 \frac{2\Delta x(x_n^m)}{yp_n^m - yq_n^m + \varepsilon} + (rand)\rho_2(x_{r1}^m - x_{r2}^m) \tag{3}$$

$$X3_n^m = x_n^m - \rho_1(X2_n^m - X1_n^m) \tag{4}$$

where *randn* is a normally distributed random number; *rand* is a random number in [0,1]; ρ_1 and ρ_2 are defined by equation (5); x_{best} is the best solution obtained during the optimization process; and x_n^m is the current vector.

$$\rho_1 = (x.rand)\alpha - \alpha\ , \rho_2 = (2.rand)\alpha - \alpha \tag{5}$$

These parameters depend on α and β. Both can be obtained from equations (6) and (7), respectively.

$$\alpha = \left|\beta \sin\left(\frac{3\pi}{2} + \sin\left(\beta \frac{3\pi}{2}\right)\right)\right| \tag{6}$$

$$\beta = \beta_{min} + (\beta_{max} - \beta_{min})(1 - (\frac{m}{M})^3)^2 \tag{7}$$

where β_{min} and β_{max} are set to 0.2 and 1.2, respectively. Δx, which is the other parameter used in equations (2) and (3) is defined as follows

$$\Delta x = rand(1{:}N)|step| \tag{8}$$

where *rand(1:N)* is a random number with *N* dimensions and *step* is calculated using equation (9).

$$step = \frac{(x_{best} - x_{r1}^m) + \delta}{2} \tag{9}$$

δ is another parameter which can be calculated by equation (10).

$$\delta = 2rand\left|\frac{x_{r1}^m + x_{r2}^m + x_{r3}^m + x_{r4}^m}{4} - x_n^m\right| \tag{10}$$

where r_1, r_2, r_3 and r_4 are different random integers from [1,N] and not equal to *n*, and *N* is the population size. The last parameters in equations (2) and (3) are yp_n^m and yq_n^m, defined by equations (11) and (12).

$$yp_n^m = (rand)(\frac{z_{n+1}^m + x_n^m}{2} + (\Delta x)(rand)) \tag{11}$$

[DOI: 10.22068/ijoce.2024.14.3.599]

$$yq_n^m = (rand)(\frac{z_{n+1}^m + x_n^m}{2} - (\Delta x)(rand)) \quad (12)$$

Note that z_{n+1}^m is calculated using the following equation where x_{worst} is the worst solution obtained during the optimization process.

$$z_{n+1}^m = x_n^m - randn \frac{(2\Delta x)x_n^m}{x_{worst} - x_{best} + \varepsilon} \quad (13)$$

Also, the LEO produces a new position if a random number is less than *pr*. This new position is calculated from equation (14) if a random number from [0,1] is less than 0.5. Otherwise, it is calculated from equation (15).

$$X_n^{m+1} = X_{LEO}^m = X_n^{m+1} + f_1(u_1 x_{best} - u_2 x_k^m) + f_2 \rho_1 (u_3(X2_n^m - X1_n^m) + u_2(x_{r1}^m - x_{r2}^m))/2 \quad (14)$$

$$X_n^{m+1} = X_{LEO}^m = x_{best} + f_1(u_1 x_{best} - u_2 x_k^m) + f_2 \rho_1 (u_3(X2_n^m - X1_n^m) + u_2(x_{r1}^m - x_{r2}^m))/2 \quad (15)$$

where f_1 is a uniform random number in the range of [-1,1]; f_2 is a random number from a normal distribution with mean of 0 and standard deviation of 1; and u_1, u_2 and u_3 can be obtained from equations (16) to (18), respectively.

$$u_1 = 2L_1 rand + (1 - L_1) \quad (16)$$

$$u_2 = L_1 rand + (1 - L_1) \quad (17)$$

$$u_3 = L_1 rand + (1 - L_1) \quad (18)$$

where x_k^m is presented by the following equation

$$x_k^m = L_2 x_p^m + (1 - L_2) x_{rand} \quad (19)$$

where L_1 is a binary parameter depending on μ_1. If μ_1 is less than 0.5, L_1 is 1, otherwise, it is 0. L_2 is similar to L_1 but depending on μ_2. μ_1 and μ_2 are random numbers in the range of [0,1]. x_p^m is a randomly selected solution of the population and x_{rand} is a new solution. Then, the positions x_{best}^m and x_{worst}^m will be updated. This loop will continue until the last iteration (m=M) and the best position (x_{best}^m) will be the outcome of the method.

3.2. Dynamic Fitness-distance Balance

The second method, dynamic Fitness-distance Balance (dFDB) is based on GBO method. The GBO has a premature convergence problem. Therefore, its guide selection process in LEO must be redesigned to improve overall search performance using dFDB method [18]. There are three cases of this method presented by [18], but only the second case is used in this paper. In this case, 20% of the search process lifecycle uses LEO equations from GBO. The other 80% of this process is as follows.

[DOI: 10.22068/ijoce.2024.14.3.599]

If a random number is less than 0.5, x_{best}^{m} is calculated from equation (20); otherwise, it is calculated from equation (21).

$$X_{LEO}^{m} = X_{n}^{m+1} + f_1(u_1 x_{best} - u_2 x_k^m) + f_2 \rho_1 (u_3 (X2_n^m - X1_n^m) + u_2 (x_{dfdb} - x_{r2}^m))/2 \tag{20}$$

$$X_{LEO}^{m} = X_{dfdb} + f_1(u_1 x_{best} - u_2 x_k^m) + f_2 \rho_1 (u_3 (X2_n^m - X1_n^m) + u_2 (x_{r1}^m - x_{r2}^m))/2 \tag{21}$$

To calculate X_{dfdb}, first, a vector of normalized distance values of the vectors from best answer ($normDP_x$) is calculated for each of the existing answer vectors using equation (22). In this equation, k is the dimension of each vector, which is the number of variables.

$$DP_x = \sqrt{(P_x^1 - P_{best}^1)^2 + \cdots + (P_x^k - P_{best}^k)^2} \tag{22}$$

Then, the normal fitness values of the answer vectors ($normF_x$) is calculated. A weighting coefficient, called w_{dFDB} is calculated by equation (23).

$$w_{dFDB} = \frac{1}{\max FEs}(1 - lb) + lb \tag{23}$$

where lb is the minimum value of w_{dFDB} and maxFEs is the maximum number of evaluations of the objective function. The score of the x-th solution (SP_x) is:

$$SP_x = w_{dFDB} normF_x + (1 - w_{dFDB}) normDP_x \tag{24}$$

Finally, the vector with the largest score is chosen as X_{dfdb}.

4. STATEMENT OF THE OPTIMIZATION PROBLEM

The objective of spillway design, a process comprising two primary steps, is to ensure a safe and cost-effective structure that minimizes the combined cost of the spillway and the dam. The initial step involves selecting the type and general dimensions of the spillway to meet anticipated requirements and site conditions, followed by a detailed hydraulic and structural design. The first step in preparing the design is to evaluate fundamental data, including topography, geology, flood hydrography, storage and release requirements. Preliminary decisions can then be made regarding the type, size, and elevation of the crest, as well as whether it will be controlled. Various alternative arrangements should be considered, with the final layout determined based on economic analysis. Additionally, analyzing existing spillways can provide valuable insights into trends and preferences for spillway types under specific conditions. This section focuses on the general procedure for overall cascade stilling basins design.

[DOI: 10.22068/ijoce.2024.14.3.599]

[DOI: 10.22068/ijoce.2024.14.3.599]

4.1. Design Procedure

Only three- and four-stepped cascades have been considered because other conditions may lead to infeasible solutions [4]. No uncertainty is considered for design parameters. Here, the decision variables are height of falls (P_i) and length of stilling basins (L_i) as shown in Figure 1 and there are hydraulic and topographic criteria, which will be considered as constraints of the optimization problem and must be fulfilled.

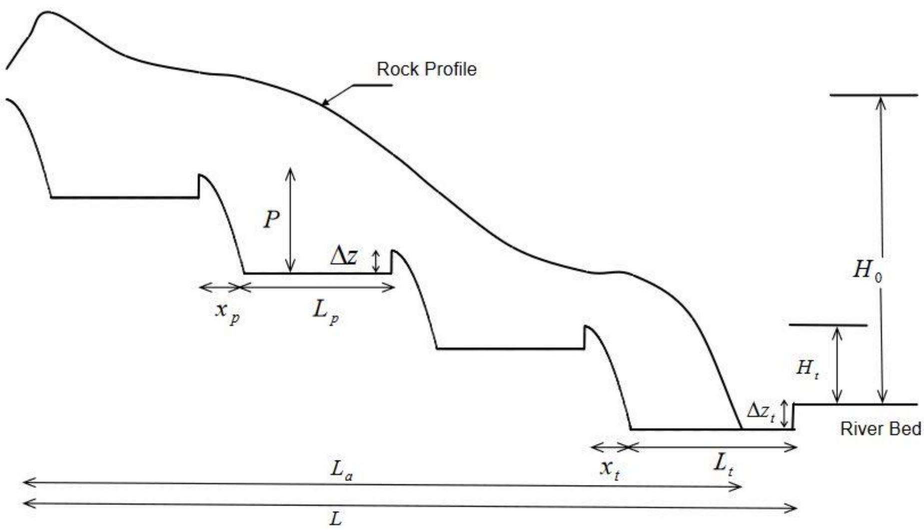


Figure 1. Longitudinal section of a typical cascade stilling basin [5]

Now the traditional VP method is presented step by step [32]:
1) The height of the last cascade (H_t) must be determined by the following equation:

$$H_t = \frac{g y_{td}^4}{7.8 q_d^2} \tag{25}$$

where g is the gravitational acceleration; y_{td} is the water depth of design discharge at tailwater; and q_d is the design discharge per unit width.

If water depth after hydraulic jump (y_2) is greater than y_{td}, the floor should be lowered by Δz_t, which is the maximum difference between Free Jump Hydraulic Curve (FJHC) and Tail Water Rating Curve (TWRC). A typical configuration of these two curves is shown in Figure 2. Therefore, the height of the last cascade (P_t) and y_{td} will be modified as follows:

$$P_t = H_t + \Delta z_t \tag{26}$$

$$y_{2d} = y_{td} = \frac{1.67 q_d^{0.5} P_t^{0.25}}{g^{0.25}} \tag{27}$$

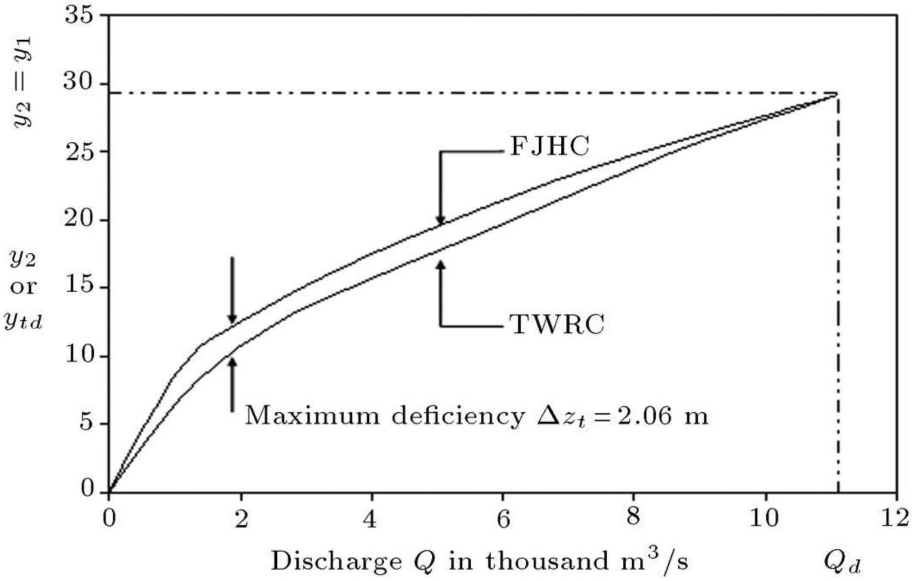


Figure 2. Free Jump Hydraulic Curve and Tail Water Rating Curve for Tehri dam [33]

2) The longitudinal length of the last cascade (x_t) is determined using equation (28), where h_{0D} is obtained from equation (29), and c is the discharge coefficient.

$$x_t = 1.455 h_{0D} \left(\frac{P_t}{h_{0D}}\right)^{\frac{1}{1.85}} \tag{28}$$

$$h_{0D} = \left(\frac{q_d}{c\sqrt{2g}}\right)^{\frac{2}{3}} \tag{29}$$

Length of the last stilling basin (L_t) can be determined from equation (30) if Froude number before hydraulic jump is equal to or greater than 4.5, otherwise, it is calculated from

$$L_t = 4.25 y_{2d} \tag{30}$$

$$L_t = 2.80 y_{2d} \tag{31}$$

3) With assuming the number of cascades (N), the height of the other cascades (P_p) will be determined from equation (32) by trial and error. Here, this height is assumed equal for all cascades, except the last one. In this equation, H_0 is the dam height above tailwater.

$$P_p = \frac{H_0 - H_t}{N - 1} + 1.671 \frac{q_d^{0.5} P_p^{0.25}}{g^{0.25}} - \left(\frac{q_d}{c\sqrt{2g}}\right)^{\frac{2}{3}} + 0.179 \frac{q_d}{g^{0.5} P_p^{0.5}} \tag{32}$$

4) To form hydraulic jump in stilling basins, the crest of each fall (preferably of ogee profile)

[DOI: 10.22068/ijoce.2024.14.3.599]

should be raised as Δz_p which can be obtained from

$$(N-1)\left(P_p-\Delta z_p\right)=H_0-H_t \tag{33}$$

The Froude number at beginning of stilling basins (before hydraulic jump) is shown by Fr_1 and is calculated from equation (34). Consequently, the water depths before (y_1) and after (y_2) hydraulic jump are determined from equations (35) and (36), respectively.

$$\frac{g^{0.5}P_p^{1.5}}{q_d}=(0.5Fr_1^{\frac{4}{3}}+Fr_1^{-\frac{2}{3}}-\frac{1}{2^{\frac{1}{3}}c^{\frac{2}{3}}})^{\frac{3}{2}} \tag{34}$$

$$y_1=(\frac{q_d}{\sqrt{g}Fr_1})^{\frac{2}{3}} \tag{35}$$

$$\frac{y_2}{y_1}=0.5(\sqrt{1+8Fr_1^2}-1) \tag{36}$$

The length of stilling basins in preceding falls (L_p) will be calculated from equation (37) where m is a constant and is considered to be 1 in this study.

$$L_p=6(my_2-y_1) \tag{37}$$

5) Horizontal length of preceding falls (x_p) is similar to last cascade and is equal to:

$$x_p=1.455h_{0D}(\frac{P_p}{h_{0D}})^{\frac{1}{1.85}} \tag{38}$$

Finally, the total length of all cascades and stilling basins (L) is defined as:

$$L=(N-1)\left(x_p+L_p\right)+(x_t+L_t) \tag{39}$$

4.2 Objective Function and Constraints

As mentioned above, the goal is to design the structure optimally. Like any other optimization problem, this problem needs an objective function and some constraints which are presented here:

$$f=\sum_{i=1}^{N}(f_1(P_i,l_i)+f_2(P_i,l_i)) \tag{40}$$

where f_1 and f_2 are excavation and concrete costs; N is the total number of cascades; P_i is the height of i-th fall; and l_i is the length of i-th stilling basin. As mentioned above, the last two parameters are decision variables.

Regarding the problem constraints, the first constraint, g_1, describes the total available

[DOI: 10.22068/ijoce.2024.14.3.599]

height where Δz_N is assumed to be zero [1]. Δz_i is presented in equation (42) and is dependent on P_i.

$$g_1 = \sum_{i=1}^{N}(P_i - \Delta z_i) + \Delta z_t \leq H_0 \quad (41)$$

$$\Delta z_i = 1.671\frac{q_d^{0.5}P_i^{0.25}}{g^{0.25}} - (\frac{q_d}{c\sqrt{2g}})^{\frac{2}{3}} + 0.179\frac{q_d}{g^{0.5}P_i^{0.5}} \quad (42)$$

The second constraint (g_2), is the total available length (L_a):

$$g_2 = \sum_{i=1}^{N}(L_i + x_i) \leq L_a \quad (43)$$

Equations (41-43) are topographical constraints. Other constraints are all hydraulic constraints which describe the maximum and minimum allowable height of each fall:

$$g_3 = P_i \leq P_{max} \quad (44)$$

$$g_4 = P_i \geq P_{min} \quad (45)$$

The upper- and lower bounds in the above equations are determined by equations (46) and (47). Note that the Froude number has maximum and minimum values of 9 and 4.5, respectively.

$$P_{max} = \frac{q_d^{\frac{2}{3}}}{g^{\frac{1}{3}}}(0.5Fr_{1,max}^{\frac{4}{3}} + Fr_{1,max}^{-\frac{2}{3}} - \frac{1}{2^{\frac{1}{3}}c^{\frac{2}{3}}}) \quad (46)$$

$$P_{min} = \frac{q_d^{\frac{2}{3}}}{g^{\frac{1}{3}}}(0.5Fr_{1,min}^{\frac{4}{3}} + Fr_{1,min}^{-\frac{2}{3}} - \frac{1}{2^{\frac{1}{3}}c^{\frac{2}{3}}}) \quad (47)$$

Finally, the last constraint is related to the minimum length of stilling basins due to formation of hydraulic jump. This minimum length can be calculated from equation (30) for the last cascade or from equation (37) for preceding cascades.

$$g_5 = l_i \geq l_{i,min} \quad (48)$$

The objective function must reflect the effect of the above-mentioned constraints; therefore, it must be reformulated. Equation (49) is the new objective function in which *Fcost* must be minimized.

[DOI: 10.22068/ijoce.2024.14.3.599]

$$Fcost = f(1 + \varepsilon_1 \sum_{i=1}^{n} \Delta_i)^{\varepsilon_2} \tag{49}$$

where ε_1 and ε_2 are error constants and are considered equal to 1.2 and 1.5, respectively; Δ_i is the normalized value of each constraint, which is not fulfilled; and n is the number of all constraints.

5. DESIGN EXAMPLE

The optimization problem was solved for Tehri dam spillway as an example. Tehri dam is an earth and rock fill dam constructed on the river Bhagirathi, a tributary of the river Ganga in the central Himalayan region of India [32]. The spillway is located on the right abutment of the dam. A single-stage hydraulic jump-type stilling basin as spillway would need a significant riverbed excavation. A chute spillway followed by ski-jump bucket would saturate the side hills and result in sheet landslides. Thus, a series of cascades and stilling basins was adopted as energy dissipation system. Design data and results of VP method are listed in table 1 and 2, respectively [32].

Table 1: Design data for Tehri dam [32]

Design Data	*Value*
Design discharge	11,000 m^3/s
Spillway crest length	95 m
Total height of spillway	218 m
Tailwater depth at design discharge	29.2 m
Cumulative horizontal length in spillway	778 m

Table 2: Results of VP method for three and four cascades [3]

N	P_p (m)	L_p (m)	x_p (m)	Δz_p (m)	L (m)
3	93.55	175.39	58.15	17.8	641.28
4	65.75	156.61	48.06	15.25	788.20

where L is the total length of the structure. The traditional VP method results x_t, L_t, Δz_t and P_t equal to 49.16, 125.04, 2.06 and 68.56 m, respectively [3].

6. RESULTS AND DISCUSSION

The optimization was conducted for two case studies with three and four cascades using a fixed number of iterations (i.e., 50) for each method and was repeated 50 times with different starting random number seeds. The obtained convergence curves for the best and mean solutions for the Tehri dam are shown in Figure 3.

[DOI: 10.22068/ijoce.2024.14.3.599]

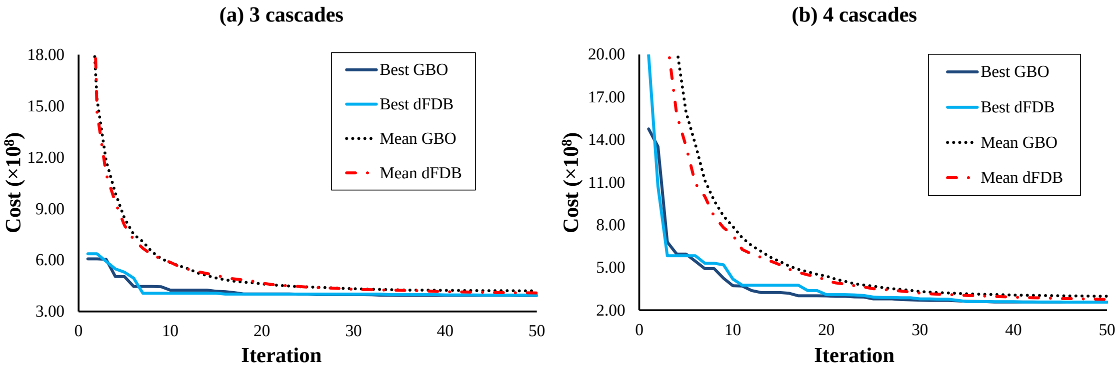


Figure 3. Convergence properties of the GBO and dFDB for (a) three and (b)four cascades obtained from 50 iterations

As shown in Figure 3(a) for the three-cascade structure, both methods converge to the best solution in fewer than 50 iterations. The GBO yields a better best solution, whereas the dFDB method exhibits slightly lower mean values across all iterations. This indicates that the modifications to GBO did not result in significant improvement for this case study. For the four-cascade structure, as shown in Figure 3(b), the convergence behavior is slightly different. Both methods require more iterations to reach the best solution. Although GBO achieves a slightly better best result, dFDB consistently maintains a lower objective function value for the mean of all iterations, demonstrating its robustness.

The comparison of the two methods, focusing on the best, average and the worst costs, is also presented in Tables 3 and 4. In terms of the best cost, the difference between the two methods is negligible. However, in both cases, dFDB shows a better performance in terms of the average and worst costs.

Table 3: Performance comparison for case study 1

Method	*Best cost*	*Average cost*	*Worst cost*
GBO	394,030,283	420,987,605	607,232,548
dFDB	395,460,532	408,674,030	435,678,574

Table 4: Performance comparison for case study 2

Method	*Best cost*	*Average cost*	*Worst cost*
GBO	256,309,109	298,547,179	591,250,501
dFDB	256,769,170	275,767,089	315,780,667

Moreover, Table 5 presents the optimum design obtained from the two methods for both three and four cascades. Both methods significantly improved the objective function, with GBO yielding slightly better results. The objective function improvement for three cascades was 72.5% with GBO and 72.4% with dFDB. For four cascades, the improvement was 71.1% with GBO and 71% with dFDB. These findings suggest that the modification of GBO into dFDB has a minor impact on solving this optimization problem. Additionally, the structure with four cascades is less costly than the one with three cascades.

[DOI: 10.22068/ijoce.2024.14.3.599]

Table 5: Best answer of each method for three and four cadcades

Decision Variable	*N=3*		*N=4*	
	GBO	*dFDB*	*GBO*	*dFDB*
P_1 *(m)*	56.97	56.44	40.00	40.03
P_2 *(m)*	71.84	72.66	42.04	42.02
P_3 *(m)*	86.20	86.17	67.31	67.28
P_4 *(m)*	-	-	80.32	80.44
L_1 *(m)*	295.78	272.11	147.22	172.77
L_2 *(m)*	170.28	191.20	155.65	135.76
L_3 *(m)*	133.10	135.87	159.62	161.30
L_4 *(m)*	-	-	129.97	130.29
Cost	394,030,283	395,460,532	256,309,109	256,769,170

Finally, Table 6 compares the excavation volumes resulting from GBO and dFDB with those obtained from other meta-heuristic methods. Although the exact objective functions were not provided in the referenced articles, the excavation volumes can still be compared. According to Table 6, both GBO and dFDB outperform all other methods in terms of excavation volume, demonstrating that these two methods are highly effective for this engineering problem. For four cascades, this excavation volume is reduced by GBO 58.69%, 21.88%, 14.84% and 13% in comparison with ABC, DP, GA-1 and PSO-1; respectively. This volume is reduced by dFDB 58.61%, 21.74%, 14.68% and 12.85% in comarison with ABC, DP, GA-1 and PSO-1; respectively. The results indicate that both GBO and dFDB deliver superior performance compared to alternative optimization approaches, making them suitable choices for this specific engineering challenge.

Table 6: Excavation volume in unit width resulted from different optimization methods

Method	*N=3*	*N=4*
GBO	**16,947**	**11,091**
dFDB	**16,909**	**11,111**
PSO-1 [2]	-	12,749
GA-1 [2]	-	13,023
DP [1]	21,286	14,198
PSO-3 [21]	27,382	25,492
IPSO [3]	31,574	-
IABC [3]	31,574	-
PSO-2 [4]	31,574	25,187
GA-2 [4]	31,576	27,103
ABC [4]	31,595	26,846
GSA [4]	33,389	27,381
CAA [5]	33,536	20,625
VP [6]	61,946	38,400

[DOI: 10.22068/ijoce.2024.14.3.599]

7. CONCLUSION

In this study, two gradient-based metaheuristic algorithms, Gradient-Based Optimizer (GBO) and dynamic Fitness-Distance Balance (dFDB), were applied to the optimization of cascade stilling basins with the aim of enhancing design efficiency and cost-effectiveness. The results obtained demonstrate promising performance of both algorithms in achieving cost-effective designs. Comparisons were made between the performance of GBO and dFDB across two scenarios, involving structures with three and four cascades.

The convergence curves illustrated consistent convergence to optimal solutions within 50 iterations for both methods. GBO generally yielded slightly better results in terms of objective function values, although dFDB exhibited greater robustness, as indicated by consistently lower objective function values across all iterations. Compared to the traditional VP method, GBO and dFDB showed objective function improvements of 72.5% and 72.4% for three cascades, and 71.1% and 71% for four cascades, respectively. These findings suggest that while both methods are effective, the modification of GBO into dFDB has a minor impact on optimization results.

Additionally, comparison of excavation volumes resulting from GBO and dFDB with those from other optimization methods (e.g., DP, GA, PSO, and ABC) confirmed the superior performance of the utilized metaheuristic approaches. Both GBO and dFDB outperformed alternative methods, demonstrating their effectiveness in achieving cost-efficient designs for cascade stilling basins. In conclusion, this study underscores the efficacy of gradient-based metaheuristic optimization techniques in optimizing the design of cascade stilling basins, offering a robust and efficient means of achieving cost-effective designs while balancing hydraulic performance requirements.

For future research endeavors in this domain, two recommendations are proposed. Firstly, investigating the applicability of other metaheuristic optimization algorithms, such as chaos game optimization [10], colliding bodies optimization [35], and chaotic swarming of particles (CSP) [36] combined with new sampling schemes [37], could provide valuable insights into their effectiveness and comparative performance in optimizing cascade stilling basin designs. Secondly, exploring the integration of machine learning techniques to enhance the predictive capabilities of optimization models and further improve the efficiency and accuracy of design optimization processes represents a promising avenue for future research in this field. By pursuing these avenues, researchers can continue to advance the state-of-the-art in optimizing cascade stilling basin designs, ultimately contributing to the development of more resilient and cost-effective hydraulic infrastructure solutions.

REFERENCES

1. Bakhtyar R, Mousavi SJ, Afshar A. Dynamic Programming Approach to Optimal Design of Cascade Stilling Basins. *J Hydraul Eng*, 2007; **133**: 949–954.
2. Bakhtyar R, Barry DA. Optimization of Cascade Stilling Basins Using GA and PSO Approaches. *JH apc*, 2009; **11**: 119–132.

3. Jazayeri P, Moeini R. Construction Cost Minimisation of the Stepped Spillway Using Improved Particle Swarm Optimisation and Artificial Bee Colony Algorithms. *Water Environ J*, 2020; **34**: 468–480.
4. Jazayeri P, Moeini R. Optimal Design of Cascade Spillway Using Meta-heuristic Algorithms: Comparison of Four Different Algorithms. *Environ Eng Manag J*, 2020; **19**: 687–700.
5. Afshar MH, Daraeikhah M. Cascade Stilling Basin Design Using Continuous Ant Algorithm. *P I Civil Eng - Wat M*, 2008; **161**: 151–159.
6. Chanson H. *Hydraulics of Stepped Chutes and Spillways*, CRC Press, 1st edition, 2002.
7. Kaveh A, Yousefpoor H. *Chaotic Meta-Heuristic Algorithms for Optimal Design of Structures*, Springer Nature, Cham, Switzerland, 2024.
8. Kaveh A, Zaerreza A. *Structural Optimization Using Shuffled Shepherd Meta-Heuristic Algorithm: Extensions and Applications*, Springer Nature, Cham, Switzerland, 2023.
9. Kaveh A, Hamedani, KB. *Advanced Metaheuristic Algorithms and Their Applications in Structural Optimization*, Springer, 1st edition, 2022.
10. Talatahari S, Azizi M. Chaos Game Optimization: A Novel Metaheuristic Algorithm. *Artif Intell Rev*, 2021; **54**: 917–1004.
11. Sheikholeslami R, Kaveh A. A Survey of Chaos Embedded Metaheuristic Algorithms. *Int J Optim Civil Eng*, 2013; **3**: 617–633.
12. Kaveh A, Sheikholeslami R, Talatahari S, Keshvari-Ilkhichi M. Chaotic Swarming of Particles: A New Method for Size Optimization of Truss Structures. *Adv Eng Softw*, 2014; **67**: 136–147.
13 Hashemi Soudmand B, Kaveh A, Sheikholeslami R. Optimal Design of Laminated Composite Structures via Hybrid Charged System Search and Particle Swarm Optimization. *Asian J Civ Eng*, 2013; **14**: 587–604.
14. Kaveh A. Ilchi Ghazaan M. *Meta-Heuristic Algorithms for Optimal Design of Real-Size Structures*, Springer, Cham, Switzerland, 2018.
15. Deb S, Abdelminaam DS, Said M, Houssein EH. Recent Methodology-Based Gradient-Based Optimizer for Economic Load Dispatch Problem. *IEEE Access*, 2021; **9**: 44322–44338.
16. Xiao H, Cheng Y. The Image Segmentation of Osmanthus Fragrans Based on Optimization Algorithms, *4th International Conference on Advances in Computer Technology, Information Science and Communications (CTISC)*, 2022, 1–5.
17. Huang X, Lin Y, Zhang Z, Guo X, Su S. A Gradient-Based Optimization Approach for Task Scheduling Problem in Cloud Computing. *Clust Comput*, 2022; **25**: 3481–3497.
18. Taşci D, Kahraman HT, Kati M, Yilmaz C. *Smart Applications with Advanced Machine Learning and Human-Centred Problem Design,* 2023.
19. Kahraman HT, Bakir H, Duman S, Katı M, ARAS S, Guvenc U. Dynamic FDB Selection Method and Its Application: Modeling and Optimizing of Directional Overcurrent Relays Coordination. *Appl Intell*, 2022; **52**: 4873–4908.
20. Khatsuria RM. *Hydraulics of Spillways and Energy Dissipators*, CRC Press, 2004.
21. Aigner D. Hydraulic Design of Pooled Step Cascades. *21st Century: The new era for hydraulic research and its applications. IAHR Congress, Thema D, Vol. I*, Beijing, China, 2001, pp. 635-640.

22. Thorwarth J. Hydraulisches Verhalten von Treppengerinnen Mit Eingetieften Stufen: Selbstinduzierte Abflussinstationaritäten Und Energiedissipation [Hydraulics of pooled stepped spillways: Self-induced unsteady flow and energy dissipation]. Ph.D. thesis, Univ. of Aachen, Aachen, Germany (in German). 2008.
23. Chinnarasri C, Wongwises S. Flow Patterns and Energy Dissipation over Various Stepped Chutes. *J Irrig Drain Eng*, 2006; **132**: 70–76.
24. Barani GA, Rahnama MB, Bagheri H. Optimization of Stepped Spillway Dimensions and Investigation of Flow Energy Dissipation over a Physical Model. *J Appl Sci*, 2005; **5**: 878–882.
25. Mohammad Rezapour Tabari M, Tavakoli S. Effects of Stepped Spillway Geometry on Flow Pattern and Energy Dissipation. *Arab J Sci Eng*, 2016; **41**: 1215–1224.
26. Frizell K, Renna F, Matos J. Cavitation Potential of Flow on Stepped Spillways. *J Hydraul Eng*, 2013; **139**: 630–636.
27. Roushangar K, Akhgar S, Salmasi F, Shiri J. Modeling Energy Dissipation over Stepped Spillways Using Machine Learning Approaches. *J Hydrol*, 2014; **508**: 254–265.
28. Shahheydari H, Jafari Nodoushan E, Barati R, Azhdary Moghaddam M. Discharge Coefficient and Energy Dissipation over Stepped Spillway under Skimming Flow Regime. *KSCE J Civ Eng*, 2015.
29. Mero S, Mitchell S. Investigation of Energy Dissipation and Flow Regime over Various Forms of Stepped Spillways. *Water Environ J*, 2017; **31**: 127–137.
30. Aal G, Sobeah M, El Deen Y.Helal E, El-Fooly M. Improving Energy Dissipation on Stepped Spillways Using Breakers. *Ain Shams Eng J*, 2017; **9**.
31. Afshoon H, Kamanbedast A, Masjedi A, Heidarnejad M, Bordbar A. Investigation of Energy Loss in Step-Spillway with Obstacles on the Step Using Physical and Flow-3D Model. *Journal of Water and Soil Conservation*, 2019; **26**: 137–156.
32. Vittal N, Porey PD. Design of Cascade Stilling Basins for High Dam Spillways. *J Hydraul Eng*, 1987; **113**: 225–237.
33. Afshar MH, Daraeikhah M, Meraji SH. Application of Particle Swarm Optimization to Optimal Design of Cascade Stilling Basins. *Sci Iran*, 2009; **16**: 50–57.
34. Ahmadianfar I, Bozorg-Haddad O, Chu X. Gradient-Based Optimizer: A New Metaheuristic Optimization Algorithm. *Inf Sci*, 2020; **540**: 131–159.
35. Kaveh A, Mahdavi VR. Colliding Bodies Optimization: A Novel Meta-heuristic Method. *Comput Struct*, 2014; **139**: 18–27.
36. Kaveh A, Sheikholeslami R, Talatahari S, Keshvari-Ilkhichi M. Chaotic Swarming of Particles: A New Method for Size Optimization of Truss Structures. *Adv Eng Softw*, 2014; **67**: 136–47.
37. Sheikholeslami R, Razavi S. Progressive Latin Hypercube Sampling: An Efficient Approach for Robust Sampling-based Analysis of Environmental Models. *Environ Model Softw*, 2017; **93**:109–26.

[DOI: 10.22068/ijoce.2024.14.3.599]