OPTIMAL DESIGN OF MIXED STRUCTURES UNDER COUPLED AND DECOUPLED TIME-HISTORY ANALYSES

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ABSTRACT

This paper deals with the optimum design of the mixed structures that consists of two parts, a lower part made of concrete and an upper part made of steel. Current codes and available commercial software packages do not provide analytical solutions for such structural systems, especially if a decoupled analysis is performed where the lower part is excited by ground motion and its response of total accelerations is used for the upper part. Due to irregular damping ratios, mass and stiffness, dynamic response of each part of a mixed structure differs significantly. The present paper aims at comparing of the optimum design of these structures under the coupled and decoupled models. Toward that goal, the coupled and decoupled time history analyses are performed and the optimum design of the two methods are compared. The results of the two approach show that the cost of the decoupled analysis is higher than the cost of the coupled analysis and the design of the decoupled method may be uneconomical, because the interaction between the two upper and lower parts is neglected.

1. INTRODUCTION

Structures consisting of concrete and steel are introduced as mixed structures, with a lower part called primary structure and an upper part, known as secondary structure. There are inherent differences in the nature of each part since the damping, mass and stiffness of the two parts are different. Therefore dynamic analysis of these structures when it is subjected to stimulation earthquake can be very complicated. In this paper, the substructure and

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superstructure of mixed structures are composed of reinforced concrete and steel respectively.

The seismic design of such structures is not satisfactorily covered by the analysis methods suggested by current design codes. Because the design methods for these structures are iterative and dynamic. The codes recommend only that irregular structures be preferentially designed using dynamic analysis but give no further guidance regarding the expected behavior. Researches have demonstrated that the structures exhibit higher-mode effects and responses that are sensitive to the relative stiffness and mass of the two parts of the structures. Research has observed that higher mode effects are potentially more substantial for irregular structures than regular structures, particularly as the extent of the irregularity increases. Several investigators have proposed methods of modeling and analyzing such irregular structures in the past. All analysis methods are divided into two categories. In the first, introduced as the decoupled method, the structure is divided into two parts, and each part is analyzed separately, but it has no significant accuracy because the interaction of the two parts is neglected. In the second, known as the coupled method, the structure is modeled as a whole, and the interaction of the two parts is considered, but the problem with this method is the irregular damping ratio, mass and stiffness. In both categories of methods, the structure can be analyzed using time history analysis.

Decoupling criteria of secondary systems from their supporting primary systems have been studied by several investigators. Some of the recommended decoupling criteria are presented by Lin and Liu [1]. Hadjian recommended a set of new curves that were developed based on various changes in the frequency of the primary system due to the decoupling of the secondary system [2]. A small variation in frequency, however, cannot assure the response error of the second system within the same tolerance, as indicated directly from numerical simulations on the frequency changes of the primary system and the root-mean-square displacement of the second system in Chen [3]. They are defined by a limited variation of the maximum transfer function of a secondary system that is connected to a single-degree-of-freedom (SDOF) system [3]. Chen and Wu investigated their sufficiency for decoupling when the two-degree-of-freedom (2DOF) primary-secondary system is subjected to a filtered white noise process of the Kanai-Tajimi power spectrum. Verifications for the sufficiency of the new criteria are extended to a secondary system mounted to a multi-degree-of-freedom (MDOF) primary system [4]. Gupta and Tembulkar (1984) extensively studied the change in response to a primary system in addition to the change in frequency due to the decoupling of a secondary system and concluded that criteria related to both of them need to be established. They presented a rational way to extend the frequency-based criteria from the SDOF system into MDOF primary systems but encountered some difficulties in doing so from the response point of view [5]. Spanos et al. presented a dynamic analysis technique that can be used to determine the response of a discrete model of a large linear structural system composed of multiple substructures [6]. To demonstrate the effect of coupling terms on the oscillator response, Adam and Fotiu compared the results from coupled and decoupled analyses of the inelastic primary structure [7]. While the decoupled results are quite accurate at detuned frequencies, there is a substantial overestimation of the peak response at the first tuned frequency and only a slight deviation is noticed at the second tuned frequency. The large differences between coupled and decoupled solutions are effectively demonstrated. Also, they proposed to compute the

response of inelastic mixed systems by decomposition into undamped substructure modes [8]. Papageorgiou and Gantes compared the maximum responses of coupled and decoupled time history analyses and presented in the form of error levels between the two methods [9]. If a coupled method is chosen, the interaction of the two parts is considered, and the method problem is the irregular damping matrix of these structures that are found. The Classical modal analysis does not reach the diagonal matrix and thus complex eigenmodes are required for time history analysis. Lin et al. proposed an alternative inelastic simplification to nonlinear time history analysis. They refer to as uncoupled modal response history analysis [10]. This method is similar to conventional elastic modal analysis but substitution of the traditional SDOF modal system with a new inelastic 2DOF modal system that represents both the stiff-and-strong lower structure and the less stiff, less strong upper structure.

The part of studies for vertical mixed structures concerns the simplified methods for the analysis and design of these structures. The solutions to the problems are divided into two groups. One part of the method is a more approximate and practical, code-specified design [11], while the other part is concerned with simplifying nonlinear analysis rather than immediate application to design [10]. Ugel et al. designed a concrete and steel structure according to Venezuelan seismic codes. They designed all structural elements with the linear analysis but the demands and performance of the elements were calculated with pushover analysis, the calculation of over strength, ductility and displacements with dynamic analysis, and fragility curves with incremental dynamic analysis [12]. Also, Yuan & Xu presented the design of mixed concrete and cold-formed steel. If the lateral stiffness ratio of the lower to upper structures is large, the evaluation of the seismic load is performed by a two-stage lateral force method. They found that the design of the two-stage analysis method may be uneconomical and unsafe [11].

In the past decades, the optimal design of the structures has been investigated that the main goal of the optimization is to use the minimum weight of the materials, optimum size of the large-scale steel structures and minimum cost of the reinforced concrete frames by different metaheuristic algorithms [14-16] for example particle swarm optimization (PSO) [13], enhanced colliding bodies optimization (ECBO) [17], vibrating particles system (VPS) [18], charged system search (CSS) [19-21], plasma generation optimization (PGO) [22], and improved plasma generation optimization (IPGO) [23] but all of them the previous studies are about the optimum design of the steel structures or reinforced concrete (RC) frame structures. Also, the optimum design of the steel-concrete mixed structures is investigated by improved plasma generation optimization (IPGO) [24].

In this paper, the optimum design of the mixed structures is obtained under the coupled and decoupled time history analyses and compare the results of the two methods. After the introduction, the coupled and uncoupled history analysis are includwd in section 2. Section 3 describes the improved metaheuristic and the subsequent section 4 presents the constraints of the steel and concrete frames followed by section 5 that provides the optimization algorithm. Section 6 provides the numerical examples and section 7 concludes the paper.

2. COUPLED AND DECOUPLED TIME-HISTORY ANALYSES

The most rigorous analysis method would be a time history analysis of the complete structure, as shown in Fig. 1(a), using for each part the corresponding damping ratio, mass and stiffness, also referred to as coupled approach. The damping, mass and stiffness matrices are then formed by the union of the two separate matrices of each part [25]. The two damping matrices can be obtained with the Rayleigh method, based on the eigenfrequencies of the complete structure. For design purposes, however, such a procedure must be repeated for several seismic excitations, which is very demanding in terms of computational time and presents significant difficulties in the evaluation of the results. In the exact coupled procedure, the structure is analyzed as a whole with the ground excitation induced at its base which is shown in the following equation:

$$[M]\{\ddot{x}^{coup}\} + [C]\{\dot{x}^{coup}\} + [K]\{x^{coup}\} = -[M]\{r\}\ddot{x}_g \tag{1}$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices of the structure and $\{x^{coup}\}$ is the vector of relative displacements of the DOFs of the structure with respect to its base. Total accelerations at each level are given by the following equation:

$$\{\bar{\ddot{x}}^{coup}\} = \{\ddot{x}^{coup}\} + \{r\}\ddot{x}_g \tag{2}$$

On the other hand, in the decoupled procedure, the primary (p) and secondary (s) subsystems are analyzed separately. Again the ground motion is first induced at the base of the primary subsystem and its response in terms of total accelerations at the mounting level of the secondary subsystem. Then, this response is induced as a new excitation to the secondary structure, the equations of these subsystems are presented in the following:

$$[M_p]\{\ddot{x}_p^{dec}\} + [C_p]\{\dot{x}_p^{dec}\} + [K_p]\{x_p^{dec}\} = -[M_p]\{r\}\ddot{x}_g \tag{3}$$

$$[M_s]\{\ddot{x}_s^{dec}\} + [C_s]\{\dot{x}_s^{dec}\} + [K_s]\{x_s^{dec}\} = -[M_s]\{\bar{\ddot{x}}_p^{dec}\} \tag{4}$$

A different option is to design such structures by a decoupled method that the structure consists of two separate subsystems, shown in Figure 1(b). In this method, the ground motion is induced to the primary subsystem alone, and its response of total acceleration at the support level of the second subsystem is recorded. Then, this response is induced at the base of the secondary structure as a new excitation, and its response, in turn, is obtained. An advantage of that procedure is that the two damping matrices that have to be created now do not have any irregularities. Moreover, it is convenient for the analysis of structures since frequently different teams are responsible for the analysis and design of the concrete and

steel parts of the structure. The disadvantage is that this approach may lead to significant inaccuracies, as in each of the two separate analyses the interaction of the two parts is neglected. The analysis of each part may be either a time history analysis.
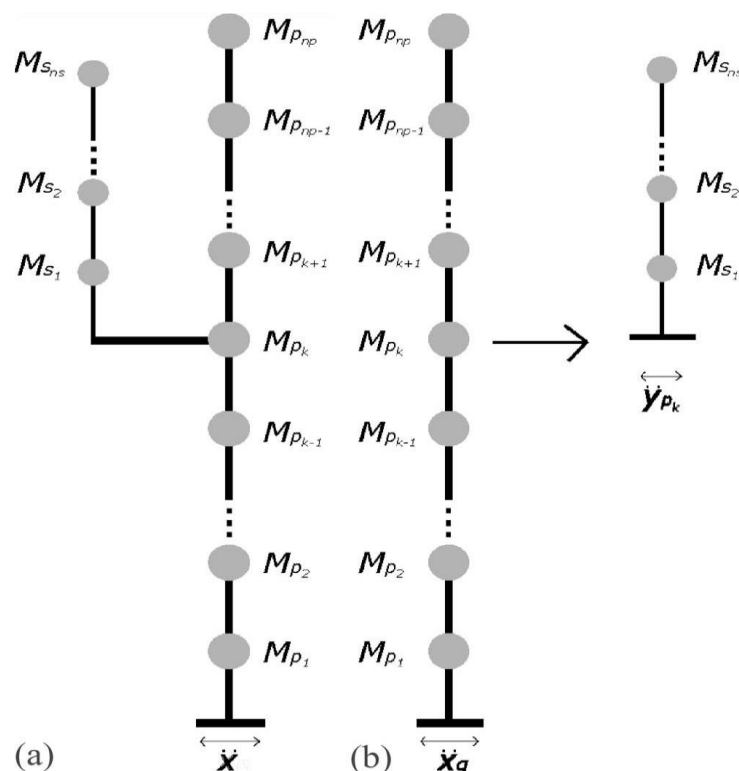
Figure 1. (a) Coupled; (b) decoupled methods [9]

3. IMPROVED METAHEURISTIC ALGORITHMS

The general optimization problem of the structure can be stated as follows:

$$
\begin{aligned}
Find \ \ \{X\} = \{x_1, x_2, \dots, x_{ng}\} \ \ \ x_{i,min} \leq x_i \leq x_{i,max} \\
To \ minimize \ \ \ f(\{X\}) \\
Subject \ to \ \ \ g(\{X\}) \leq 0 \ \ \ \ \ \ \ j = 1, 2, \dots, n
\end{aligned}
\tag{5}
$$

where $\{X\}$ is a vector of design variables; $f(\{X\})$ is the objective function; ng is the number of element groups; $x_{i,min}$ and $x_{i,max}$ are the two vectors of the lower and upper bounds of the design variable x_i, respectively. $g_j(\{X\})$ is the constraints of the design and n is the number of the constraints. In this paper, the objective function is considered the total cost of the mixed structure. It means that the costs of concrete, steel and framework are calculated. Thus, the objective function of the mixed structure can be defined as the following equations [24]:

$$f_{conc} = \sum_{i=1}^{n_{cc}} \{C_c b_i h_i + C_s A_{si} \gamma_s\} + \sum_{i=1}^{n_{cc}} \{2C_f (b_i + h_i)\} L_i \qquad (6)$$

$$f_{steel} = \sum_{i=1}^{n_{cs}} C_s * A_{si} * L_i * \gamma_{si} \qquad (7)$$

$$f_{obj} = f_{conc} + f_{steel} \qquad (8)$$

where f_{obj} is the objective function of the mixed structure (€); f_{conc} is the cost of the concrete elements of the structure; f_{steel} is the cost of the steel elements of the structure; n_{cc} is the number of columns of the concrete elements, respectively; n_{cs} is the number of columns of the steel elements, respectively; C_c, C_f and C_s, are the unit cost of concrete, formwork and steel, respectively; C_t is the unit rate of scaffolding; b, h, L are dimensions of the concrete elements (m); A_{si} is the area of the bars of each section of the concrete elements and the section area of the steel elements (m^2); γ_s is the density of steel as 7849 (kg/m^3). A cross-section database is considered for RC structural elements because the dimensions of the design variables are large, and the computational cost and complexity of the optimization process increase. The IPGO algorithm uses the discrete design variable in the section database to obtain the optimum solution. Also, for steel elements, the 11 discrete design variable is considered discrete design variable that all of them are selected from 267 predetermined W-shaped cross sections. The design variables are defined for calculation of the objective function that includes dimensions of the cross sections, area and number of top and bottom steel bars in the cross-section. The constraints of the concrete structural elements are derived from the ACI 318 building code [26] and the limitation of the steel structural elements are considered according to the AISC-LRFD provisions [27]. For concrete structural elements, the number and diameter of the longitudinal bars varied from four #3 to twenty #11 bars. The database of the cross-sections is sorted in the ascending cost per unit length. For column sections, the bars in the cross-section are symmetrical. The lower bound, upper bound, and increments of dimensions are considered 250, 1200, and 50 mm, respectively [28].

4. THE CONSTRAINTS OF THE STEEL AND CONCRETE FRAMES

Constraints of the concrete elements were obtained from the provisions of the ACI 318 design code [26]. The constraints include the load capacities of the column sections, the limitation of reinforcements in sections, minimum clear spacing between reinforcement bars, and the limitation dimensions of the sections. The constraints of the RC columns are presented in Table 1. In this table f_c' is compressive strength of concrete and f_y is the yield strength of steel; d_b is the diameter of reinforcement bars; M_u and P_u are applied moment and

axial force of columns; and M_n and P_n are nominal flexural and axial strength of columns, respectively. A_g is the total area section; A_s is the area of the longitudinal bars in the section; the depth (h) and width (b) of the column in the top storey (T) should be smaller than the bottom one (B).

Table 1. Constraints of the RC columns

combination of moment and axial force	$l_u = \sqrt{(P_u)^2 + (M_u)^2}$, $l_n = \sqrt{(\emptyset P_n)^2 + (\emptyset M_n)^2}$, $C_1 = (l_u - l_n)/l_n$
minimum longitudinal bars	$C_2 = \dfrac{0.01 \times A_g}{A_s} - 1$
maximum longitudinal bars	$C_3 = \dfrac{A_s}{0.08 \times A_g} - 1$
minimum clear spacing	$s_{min} = \max(1.5d_b, 1.5in), \quad C_4 = \dfrac{s_{min} - s}{s_{min}}$
depth of the column section	$C_5 = \dfrac{b_T}{b_B} - 1$
width of the column section	$C_6 = \dfrac{h_T}{h_B} - 1$

Displacement of the roof and inter-storey displacements and strength constraints of the steel elements are presented according to the LRFD-AISC provisions [27]. The constraints are defined in Table 2.

Table 2. Constraints of the steel columns

displacement of the roof	$\dfrac{\Delta_T}{H} - R \leq 0$
inter-storey displacements	$\dfrac{d_i}{h_i} - R_i \leq 0 \qquad i = 1, 2, \ldots, ns$
strength constraints	$for \ \dfrac{P_u}{2\phi_c P_n} < 0.2 \quad \dfrac{P_u}{2\phi_c P_n} + \left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}} \right) - 1 \leq 0$ $for \ \dfrac{P_u}{2\phi_c P_n} \geq 0.2 \quad \dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}} \right) - 1 \leq 0$

where Δ_T is the lateral displacement of the roof (max); H is the structure height; R is the maximum drift index as 1/300; d_i is the inter-storey drift; h_i is the storey height of the ith storey; ns is the total number of storeys; R_i is the index of inter-storey drift (1/300); P_u is the required strength (tension or compression); P_n is the nominal axial strength (tension or

compression); ϕ_c is the resistance factor ($\phi_c = 0.9$ for tension elements, $\phi_c = 0.85$ for compression elements); M_u is the required flexural strengths; M_n is the nominal flexural strengths; and ϕ_b is the flexural resistance reduction factor ($\phi_b = 0.9$). Due to the good performance of the optimization algorithm, a penalty function $f_{penalty}(\{X\})$ is used to the constraints (g_i) of the optimization problem that is defined in the following equation where m is the number of the constraints and ϑ_i is the penalty parameter corresponding to the ith constraint.

$$f_{penalty}(\{X\}) = W(\{X\}) + \sum_{i=1}^{m} \vartheta_i \times \max(0, g_i)$$

(9)

5. OPTIMIZATION ALGORITHM

Plasma generation optimization (PGO) is a new meta-heuristic algorithm introduced by Kaveh, et al. [22] and its performance of this has been investigated in Ref. [29]. To improve the result of the PGO algorithm, Improved Plasma Generation Optimization (IPGO) is developed to obtain reliable solutions and fast convergence. A comparative study of these algorithms is presented for steel and concrete structures [23]. In the IPGO algorithm, plasma memory (PM) is used to save the best solutions obtained at the previous population in each iteration and their values of the objective function. The electrons of the PM memory are replaced with the worst electrons in the current population. Then, electrons are sorted by their values of the objective function. In the improved version of the PGO algorithm, to determine the step size of each electron, the excitation and de-excitation processes or ionization process should be occurred for each electron and the step size of the electron according to the excitation and de-excitation processes or ionization process is obtained, and the new position of the electrons is calculated, but in the IPGO, $x_{rc,j}$ is considered the best electron in each iteration (x_{best}). Therefore, $\Delta x_{i,j}$ is formulated for the mathematical representation of moving forward to the new position around the best electron.

6. NUMERICAL EXAMPLES

This study designed a five-level moment frame according to ACI and AISC codes. The mixed structure consists of three RC frame storeys and two steel frame storeys. This frame has 20 columns arranged in 10 groups shown in Fig. 2. The model is a pure shear model, all load is resisted in shear only and all storeys are assumed to act as rigid diaphragms. Consequently, vertical displacements and joint rotations are neglected, but this requires a smaller number of DOFs. The mass of the concrete and steel storeys is assumed 200 and 150 tons, respectively, and each storey has one degree of freedom. The height of the storeys and length of the bays is considered 3 (m) and 5 (m). The linear time history analysis is performed to design the elements of the structure. The Imperial Valley earthquake at El Centro station in 1940 is chosen for time history analysis [30]. The detail of the analysis and

a flowchart of the optimization process is provided in Ref. [23] which is about the optimum design of the concrete frame under time history analysis. The damping matrix is calculated by Rayleigh's method and an equivalent damping ratio is calculated based on a semi-empirical error minimization method for mixed structures [31]. Mass and stiffness matrices of the moment frames are presented by Clough and Penzien [32]. The static and dynamic analysis, also optimization processes are programmed in MATLAB [33].
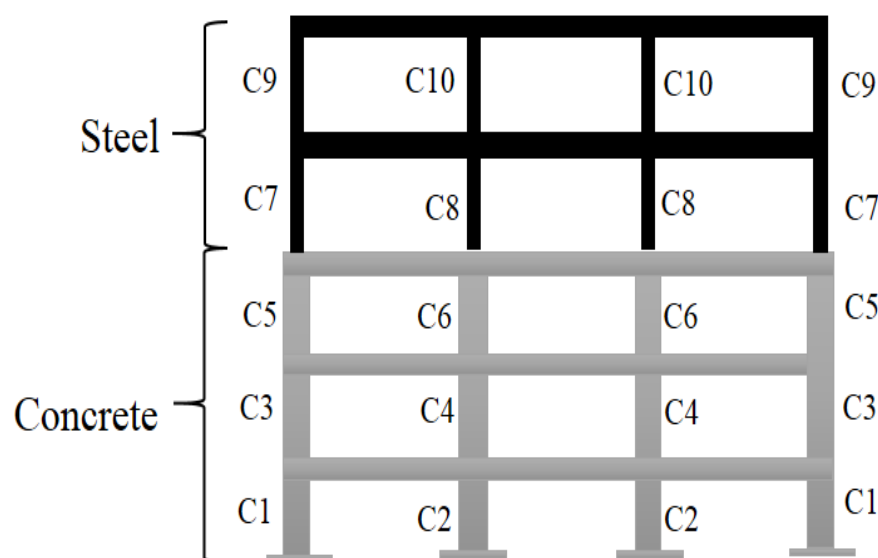

Fig. 2. Steel and concrete structure

For the steel part of the structure, the modulus of elasticity is 200 GPa and the yield stress is 248.2 Mpa. For RC part of the structure, the yield strength of steel (f_y) is 500 MPa; the Compressive strength of concrete (f_c') is 40 MPa; unit weight of steel (γ_s) and concrete (γ_c) are 7849 and 2450, respectively. Limitations and constraints of the steel and RC frames and their sections are said in previous sections and the detail of the costs are presented in Table 3. The population size is selected as 30 and maximum iteration number is 3000 and the parameters of the algorithm include nPM=15, EDR = 0.5, DR = 0.3, and DRS = 0.1. The optimal design of the frame is executed by the IPGO algorithm. To reduce computational effort, the solutions of each iteration are firstly controlled by constraints that do not require structural analysis. Therefore, the optimization procedure found the best solution after a limited number of time history analyses. The optimum design of the IPGO algorithm can be seen in Table 4.

Table 3. The detail of the costs

Costs	Unit	Value
Cost of concrete (C_c)	€/ m³	105.17
Cost of steel (C_s)	€/ton	1300
Cost of formwork for RC frames (C_f)	€/ m²	22.75

Table 4. The optimization result of the mixed structure under the coupled method

Member type	Group	Dimensions		Reinforcements	
		width (mm)	depth (mm)	As top	As bot
Concrete	C1	450	1200	16#8	
	C2	250	250	10#3	
	C3	350	950	20#5	
	C4	250	250	10#3	
	C5	250	950	10#6	
	C6	250	250	10#3	
Steel	C7	W 6×9			
	C8	W 10×68			
	C9	W 6×9			
	C10	W 6×9			
Best cost		4493			

As described in the past, all analysis methods are divided into two groups. In the first, the coupled method, the structure is modeled as a whole, and the interaction of the two parts is considered, but the problem with this method is the irregular damping ratio, mass and stiffness. In the second, the decoupled method, the structure is divided into two parts, and each part is analyzed separately, but it has no significant accuracy because the interaction of the two parts is neglected. The lower substructure is subjected to ground motion, and the absolute acceleration response is applied to the upper substructure, as shown in Fig. 3. The advantage of this method is that it overcomes irregularity in structure analysis. In both categories of methods, the structure is analyzed by time history analysis. The obtained results indicate that the cost of the decoupled method is higher than the cost of the coupled approach and the design of the decoupled method may be uneconomical.
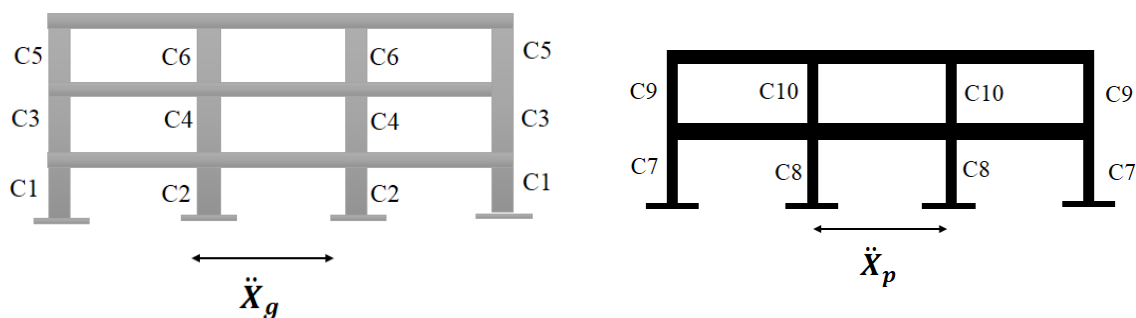

Figure 3. Decoupled model of the mixed structure

Table 5. The optimization result of the mixed structure under the decoupled method

Member type	Group	Dimensions		Reinforcements	
		width (mm)	depth (mm)	As top	As bot
Concrete	C1	400	1050	10#8	
	C2	650	1000	8#10	
	C3	350	750	16#5	
	C4	550	1000	12#8	
	C5	250	700	14#4	
	C6	250	900	8#6	
Steel	C7	W 6×9			
	C8	W 6×9			
	C9	W 6×9			
	C10	W 6×9			
Best cost		5506			

7. CONCLUSIONS

Mixed structures consist of concrete in the lower part and steel in the upper part. There are inherent differences in the nature of each part because the damping properties and material laws of the two parts are different. Thus dynamic analysis of these structures when it is subjected to stimulation earthquake can be very complicated. Current seismic design codes and available commercial software do not have solutions for these structures. Several investigators have proposed methods for the analysis and design. All analysis methods are divided into two categories. In the first, introduced as the decoupled method, the structure is decomposed into two parts, and each part is analyzed separately, but it has no significant accuracy because the interaction of the two parts is neglected. In the second, known as the coupled method, the structure is modeled as a whole, and the interaction of the two parts is considered, but this method is complicated. In this paper, for comparison, the results of the optimum design are presented under the coupled and decoupled analyses. The design constraints of RC and steel frames are imposed according to the standards and limitations of the ACI 318 and LRFD-AISC. The obtained results indicate that the cost of the decoupled method is higher than the cost of the coupled approach, and the design of the decoupled method may be uneconomical. The decoupled method has some disadvantages, since it does not have adequate accuracy and the interaction between the two upper and lower parts is neglected, and the dependence of their modal response is also neglected, which may be significant, especially if the eigenvalues of the two parts are related.

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