



STRUCTURAL DAMAGE QUANTIFICATION USING A CONDENSED FORM OF THE MODAL FLEXIBILITY MATRIX AND CHAOTIC IMPERIALIST COMPETITIVE ALGORITHM

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ABSTRACT

This paper presents an optimization-based model updating approach for structural damage detection and quantification. A new damage-sensitive objective function is proposed using a condensed form of the modal flexibility matrix. The objective function is solved using Chaotic Imperialist Competitive Algorithm (CICA), as an enhanced version of the original Imperialist Competitive Algorithm (ICA), and the optimal solution is reported as the damage detection results. The application of the CICA in vibration-based damage detection and quantification has been successfully investigated in a feasibility study published by the authors of the present paper and herein, its application is generalized for a case in which a complex (but more sensitive) objective function is utilized to formulate the damage detection problem as an inverse model updating problem. The method is validated by studying different damage patterns simulated on three numerical examples of the engineering structures. Comparative studies are carried out to evaluate the accuracy and repeatability of the proposed method in comparison with other vibration-based damage detection methods. The obtained results introduce the proposed damage detection approach as a robust method with high level of accuracy even in the presence of noisy inputs.

Keywords: finite element model; structural damage; condensed modal flexibility; objective function; chaotic imperialist competitive algorithm (CICA).

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1. INTRODUCTION

1.1 Literature review

Although engineering structures are carefully designed to be resistant against different loads, they may experience unexpected natural and/or artificial loads that cause invisible damage in the members or joints which can adversely affect their long-term performance. Obviously, if such damage cases are identified at the initial stages, not only are the unexpected structural collapse prevented, but also the rehabilitation plans can be designed as cost-effective plans. Damages are generally defined as changes in the physical properties of the structures (like mass and stiffness) and finding such changes is the subject of Structural Health Monitoring (SHM) programs, which have been intensively studied during the last few decades [1–3]. In the SHM programs, the structural system is equipped with sensors to record the structural feedback under ambient or synthetically generated excitation. Then, the recorded responses are analyzed to identify damage location and/or severity, which can be used for making decision on the serviceability of the structure [3]. Among different types of the structural feedback that can represent structural main behavior, vibration characteristics (like modal frequencies and shape vectors) are more preferred because they have high level of sensitivity to changes in the physical properties of the structural system [1, 3]. Moreover, they are relatively easy to implement in comparison with static tests [4]. Vibration-based damage detection methods can be classified as index- and model-based methods. In index-based methods, damage can be localized by damage-sensitive indices, like wavelet-based damage indices [5, 6] indices based on the modal frequencies and mode shapes [7] or their derivatives [8]. Moreover, some of such indices are generalized for damage quantification using mathematical and/or statistical methods [9], as well as the pattern recognition techniques by neural networks [10] or machine learning concepts [11, 12] for data training. Although such methods are beneficial in terms of rapid structural health assessment, they face some challenges and difficulties. First, most of the index-based methods needs to have the data collected in at least one degree of freedom (DOF) associated with each node of the finite element model, which means that sensors should be installed based on the element's size. Second, the index-based methods with the quantification capability are mostly applicable to *simple* structures, with special physical characteristics. Third, the pattern recognition methods are based on data training, and this means that the trained system for a specific structure cannot be used for other similar structures with a small modification in the structural properties.

Model-based methods (also known as *model updating methods*) are more convenient in terms of damage localization and quantification, and they can be easily used for a wide range of structures. These methods are based on tuning the physical parameters (as the damage features) of an analytical model to reach the best combination of the variables that can produce the behavior of the monitored system. To perform these methods, different iterative approaches has been developed, which are mostly based on defining the problem as an optimization problem using damage-sensitive objective functions. A wide range of parameters, like free-vibration equilibrium [13], combination of the modal frequencies and mode shape vectors [14–16], static displacement computed by the modal flexibility [17], and model residual force [18] have been used to formulate the damage detection problem as an

inverse problem. Since the problem is appeared as minimization problem with *in-explicit* objective function, to solve it and find global optimal solutions, a robust search algorithm is needed. This issue will be of importance considering the ill-posed nature of the solution domain. To overcome this difficulty, not only a highly sensitive objective function to structural physical properties is required, but also, a robust optimization algorithm should be used to solve the problem. In the last two decades, optimization algorithms have been extensively used in different fields [18–21], and specifically in the field of model updating-based structural damage detection [22–28], which are based on employing random theory-based algorithm to mimic the optimal search technique inspired from a natural phenomenon. Although such algorithms can effectively solve the minimization problem, they might be time-consuming in terms of searching the solution domain for complex problems with many unknown variables. Note that this issue has been addressed in some research works by proposing two-stage method for damage localization and quantification [29, 30]; however, the computational costs as well as the difficulties related to the number of sensors needed for damage localization (discussed former this section, where we mentioned drawbacks of the index methods) make such two-stage methods less-practical for detecting multiple damage in complex structures. Besides, since stages one and two should be checked simultaneously to make decision on the health of a typical element, such methods can be interpreted as *supervised* methods which needs user intervention to judge the health of the elements.

1.2 Scope and novelty

The promising aspects of the optimization-based structural model updating, and parameter estimation were discussed in section 1.1. The present paper proposes a new vibration-based model updating approach to localize and quantify damages in the structural systems in an element-wise scheme. The novelties and main contribution of the present study are itemized as follows:

- (1) Proposing a parameter-sensitive objective function to find the element-wise stiffness: As mentioned in section 1.1, proposing damage-sensitive objective function is of importance in formulating model updating-based damage detection problems. In the present study, we leverage the main concept of model updating approaches to estimate the element-wise stiffness of the structural systems. To satisfy the described criterion, a new objective function is proposed in this paper, which consists of two terms. The first term is based on entry-by-entry inspection of the modal data similarity; however, the second term leverages the geometry-based correlation between vectors to evaluate the match between the behavior of the tested structure and its analytical model.
- (2) Employing fast speed optimization algorithm to solve the problem: In this paper, a chaos theory-based optimization algorithm is used to solve the proposed objective function, named Chaotic Imperialist Competitive Algorithm (CICA) [31]. The CICA is an enhanced version of the original Imperialist Competitive Algorithm (CIA) and compared to ICA, its main advantage is that, it can search much more points of the solution domain comparing with random theory-based optimization algorithms. The CICA has been successfully used for damage detection in a feasibility study that recently conducted by two authors of the present paper [18], and in the present paper it is used to solve a complex (but damage-sensitive) objective function.

The applicability of the proposed method is demonstrated by studying three numerical examples of the engineering structures including two structural frames, representing the

structural systems that are commonly used for the residential and educational serviceability, as well as a 3D frame. The rest of the paper is organized as follows. The proposed algorithm is explained in section 2. Problem formulation and details of the optimization algorithm form this section. Numerical examples and discussion on the obtained results are presented in section 3. Section 4 presents the concluding remarks, which is followed by the list of the references used in this study.

2. THE PROPOSED ALGORITHM

2.1 Problem formulation

Consider a structural system with N elements and N_s DOFs. The free vibration equation of this system is presented as follows:

$$(-(\omega_i^2)\mathbf{M} + \mathbf{K})(\boldsymbol{\varphi}_i) = \mathbf{0} \quad (1)$$

where, \mathbf{M} and \mathbf{K} are the global mass and stiffness matrices, respectively. ω_i and $\boldsymbol{\varphi}_i$ are the i th frequency and mass-normalized mode-shape vector corresponding to the i th mode, respectively. Both the frequencies and mode shape vectors are considered as dynamic properties of a given system which are sensitive to changes in the physical properties [2]. Although such parameters have been separately used for model updating and damage detection, it is more preferred to use a combined version of these parameters to develop a model updating procedure [3, 18]. *Modal flexibility* is a matrix which can represent the modal behavior of a system by combining only the first several modes' data. In this paper, we use flexibility matrix computed by the first several modes to propose an index which is utilized to formulate the objective function. Using the first p modes' data, the modal flexibility matrix, \mathbf{G} , is computed as follows:

$$\mathbf{G}_p = \boldsymbol{\Psi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}^T \quad (2)$$

where, $\boldsymbol{\Psi}$ is the matrix of the first p mode shape vectors and $\boldsymbol{\Omega}$ is a diagonal matrix containing the first p eigenvalues. The objective function is defined in terms of error minimization concept. Although matrices can be fed into the error minimization procedure, they might result in high computational costs because the objective function is evaluated by checking all the members of a matrix in each iteration of the optimization procedure. To tackle this issue without sacrificing the sensitivity of the modal flexibility to structural damage, a condensed version of the flexibility matrix is developed without omitting the entries of the original matrix. In general, the diagonal members of a square matrix can uniquely represent its main characteristics [32]. Defining vector \mathbf{b} as a vector contains the diagonal members of the modal flexibility matrix, a condensed version of the modal flexibility matrix is defined as follows:

$$\mathbf{c} = \mathbf{G}_p \cdot \mathbf{b} \quad (3)$$

Using vector \mathbf{c} , we form the terms needed to formulate the objective function. The main

concept behind the proposed objective function is to minimize the error between the behavior of the tested structure (shown with superscript ‘ t ’) and the analytical model with unknown damage parameters (denoted by superscript ‘ a ’). Note that in this paper the unknown damaged parameters are the element-wise stiffnesses, defined as follows:

$$\mathbf{K}^a(\alpha_1, \alpha_2, \dots, \alpha_N) = \bigcup_{i=1}^N (1 - \alpha_i) \mathbf{k}_i^u, 0 \leq \alpha_i \leq 1.0 \quad (4)$$

in which, \mathbf{k}_i^u and α_i are the stiffness matrix of the i th element in the undamaged state (estimable from as-built maps) and the damage parameter of the i th element, respectively.

Since damage detection problem is inherently an ill-posed problem (with a complex solution domain), the objective function should be formulated in a way that it returns one-to-one relationship between the candidate solutions and the objective function’s value (i.e., cost). To meet this criterion, the objective function is formulated by employing both the entry-by-entry and geometrical search strategies as follows:

$$F(\alpha_1, \alpha_2, \dots, \alpha_N) = \gamma_1 \|\mathbf{c}^t - \mathbf{c}^a(\alpha_1, \alpha_2, \dots, \alpha_N)\|^2 + \gamma_2 |1 - \text{MAC}(\mathbf{c}^t, \mathbf{c}^a(\alpha_1, \alpha_2, \dots, \alpha_N))| \quad (5)$$

in which, $\|\cdot\|$ and $|\cdot|$ denotes the Euclidean length and absolute value of the corresponding arguments, respectively. MAC stands for the modal assurance criterion [33], which is used as a geometrical correlation evaluator and is defined as follows:

$$\text{MAC}(\mathbf{c}^a(\alpha_1, \alpha_2, \dots, \alpha_N), \mathbf{c}^t) = \frac{|(\mathbf{c}^a(\alpha_1, \alpha_2, \dots, \alpha_N))^T \cdot \mathbf{c}^t|^2}{\{(\mathbf{c}^a(\alpha_1, \alpha_2, \dots, \alpha_N))^T \cdot \mathbf{c}^a(\alpha_1, \alpha_2, \dots, \alpha_N)\} \cdot \{(\mathbf{c}^t)^T \cdot \mathbf{c}^t\}} \quad (6)$$

where, a complete correlation between vectors \mathbf{c}^t and \mathbf{c}^a are reported if MAC is 1. Note that in Eq. (5), the first term is aimed at evaluating the entry-wise difference between the analytical and tested models; however, the second term is based on checking the geometrical correlation between \mathbf{c}^t and \mathbf{c}^a . Moreover, γ_1 and γ_2 are the weights for the first and second terms, respectively. Since geometry-based correlation inspection is much more sensitive to update the structural model [22], in this study we consider $\gamma_1=0.25$ and $\gamma_2=0.75$.

2.2 Chaotic imperialist competitive algorithm (CICA)

The original Imperialist Competitive Algorithm (ICA) is a global search optimization technique that is inspired from a socio-political competitive event [34]. This algorithm starts with initial population called *country*, which is defined as:

$$\mathbf{country} = \{y_1 \ y_2 \ \dots \ y_{N_v}\}^T \quad (7)$$

where, y_n is the n -th variable, and N_v denotes the number of variables. The cost of each country, c , is calculated as:

$$c = f(\mathbf{country}) \quad (8)$$

Note that the aim of the optimization algorithm is to find a global extremum of the given

function f . Based on the costs of the countries, they are divided into two parts: Some of the countries with low costs are selected as the *imperialist*, which are the initial candidates for the optimal solution. The remaining countries, however, are considered as *colony* and are divided among the imperialists based on the power of the empires. In the next step, the *assimilation process* begins, in which the imperialists (i.e., the elites of the initial population) attempt to improve their colonies by absorbing new colonies. This process is modeled by moving all the colonies toward the imperialist along different optimization axes. To assure that the process is not followed by a bias caused through a direct line connecting a given country to the imperialists, a random path is induced by a random amount of deviation and is added to the direction of the movement [34]. If during the assimilation process, a colony reaches a position with lower cost than the imperialist, their position will be switched. All the empires try to take the possession of colonies of other empires, and this is can a base for competition, which is mathematically modeled by picking some of the weakest colonies of the weakest empire and making a competition among all empires to possess these colonies. In the next steps, the described process is repeated and the empires with no colony are eliminated in the process. The optimization algorithm is stopped if one empire only is left, or the number of iterations reaches to the defined maximum value. Readers can find more details about the original ICA in [34].

To provide a situation in which many candidate solutions are randomly selected and evaluated in searching highly complex solution domains, Kaveh and Talatahari [35] proposed orthogonal ICA by modifying assimilation process by: (1) using different random values, and (2) considering orthogonal colony-imperialistic contacting line to add some rational deviation in locating final position of the colony in its movements toward the imperialists. Fig. 1 shows assimilation process for the orthogonal ICA in a typical 2D problem. For orthogonal ICA, the assimilation process is defined as below [35]:

$$\mathbf{X}_{new} = \mathbf{X}_{old} + (\beta \times d \times \mathbf{rand} \otimes \mathbf{V}_1) + (Z(-1, +1) \times \tan(\theta) \times d \times \mathbf{V}_2) \quad (9)$$

in which, \mathbf{X}_{old} and \mathbf{X}_{new} are the current and new positions of the colony, respectively, β and d denote the control parameter and the distance between colony and imperialist, respectively. Also, \mathbf{V}_1 is a unit vector, and its start point is the current location of the colony, and its direction is toward the imperialist location, and \mathbf{V}_2 is a unit vector which is perpendicular to \mathbf{V}_1 . Note that \mathbf{rand} is a vector of random numbers and \otimes denotes element-by-element multiplication. By this modification, ICA promoted to an evolutionary optimization approach which can evaluate more points in the solution domain. To provide a situation in which the solution domain is sought much faster, Talatahari *et al.* [31] employed chaos theory and proposed Chaotic Imperialist Competitive Algorithm (CICA) which promoted the speed of orthogonal ICA. Generally, chaos theory-based optimization algorithms can be considered as stochastic search methodologies which differ from the existing evolutionary computation and swarm intelligence methods. Because of the non-repetition of chaos, chaotic optimization algorithms can search the complex solution domains with fast speed comparing with probability-based methods. The assimilation process for CICA is formulated as [31]:

$$\mathbf{X}_{new} = \mathbf{X}_{old} + (\beta \times d \times \mathbf{CM} \otimes \mathbf{V}_1) + (\mathbf{CM} \times \tan(\theta) \times d \times \mathbf{V}_2) \quad (10)$$

in which, **CM** is the chaotic variable (generated based on the chaotic map) which is used instead of random numbers. Readers can find more information about CICA in [18, 31].

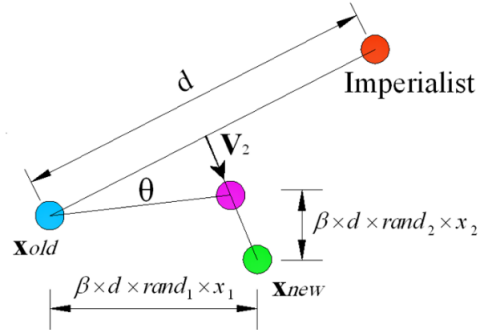


Figure 1. Assimilation process for orthogonal ICA proposed by Kaveh and Talatahari [35]

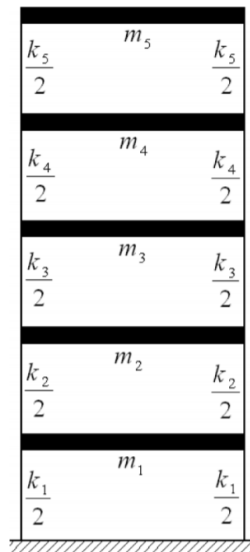


Figure 2. Five-story shear frame

3. NUMERICAL EXAMPLES AND RESULTS

3.1 A Five-story shear building

The first numerical example is devoted to studying a five-story shear building, shown in Fig. 2. The behavior of most of the structures with educational serviceability (like the schools) can be considered as a shear behavior. A 2D shear building structure is a structural system with unidirectional behavior, in which only one DOF is allocated to each floor. Such system is equivalent with spring-mass systems, which are common in developing simplified models of civil and mechanical systems. In the present example, it is assumed that the mass of all

the stories is equal with 2000 kg. Moreover, the stiffness of the stories #1 and #2 are equal to 1800 N/m while the stiffness of the remaining three stories is equal with 1200 N/m. Five damage cases are considered as described in Table 1. In damage case 1, it is assumed that the in-service building has not experienced any damage. In the other cases, however, it is assumed that the stiffness of some of the stories has been changed because of damage. Damage cases 2 and 3 simulates single damage patterns; however, damage patterns 4 and 5 consist of damages in multiple stories. Moreover, damages in different ranges (like 5% damage, representing small damages as well as 30% damage as extreme damage case) have been considered.

In real-world applications, the structural responses of the monitored structure are contaminated with different levels of measurement noises which result in noisy modal data. In this paper, the modal data (i.e., both frequencies and mode shape vectors) associated with the test structure are contaminated with different levels of random noise to simulate a realistic condition. The noisy modal data are produced using equations below [36]:

$$\omega_i^n = \omega_i(1 + \mu r_i) \quad (11a)$$

$$\boldsymbol{\varphi}_i^n = \boldsymbol{\varphi}_i(\mathbf{1} + \mu \mathbf{R}_i) \quad (11b)$$

where, superscript n indicates the noisy parameter, μ is the noise level, and r and \mathbf{R} are randomly generated scalar and vector in interval $[-1,1]$, respectively. Also, i denotes the i th mode. The frequencies are contaminated with 2.5% noise; however, the mode shape vectors are reproduced by adding 5% noises. In real tests, reaching the information of all the modes can be a challenge [21]. Since the model data of the first lower modes can be easily extracted from dynamic tests, in this section the modal data of the first one and three modes are utilized to solve the problem. The selected parameters for CICA are as follows: number of countries = 100; number of imperialists = 10; maximum number of iterations = 1000; $\beta = 2.0$; $\tan(\theta) = 1.0$, and $CM = \text{sinusoidal map: } x_{k+1} = \sin(\pi x_k)$. It should be mentioned that these parameters are selected by trial and error, based on the suggested recommendation in [31] and [18]. Figs. 3 and 4 show the obtained results for the simulated damages patterns when the modal data of the first one and three modes are used for damage detection. As it can be seen, the method is very sensitive to different levels of damage and not only the severe damages, but also minor damages are quantified with high level of accuracy in ideal (i.e., noise free) state. In the noisy state, however, some of the healthy stories are reported as damaged stories, but the damage severity is neglectable. It is worth mentioning that increasing the number of the modes utilized to solve the problem, the noise effects are more evident.

Table 1: Details of the studies cases in the first numerical experiment

Pattern Number	Story Number	Damage (%)
1	---	---
2	1	20
3	3	5
4	1	30
	5	5
5	1	5
	2	10
	5	10

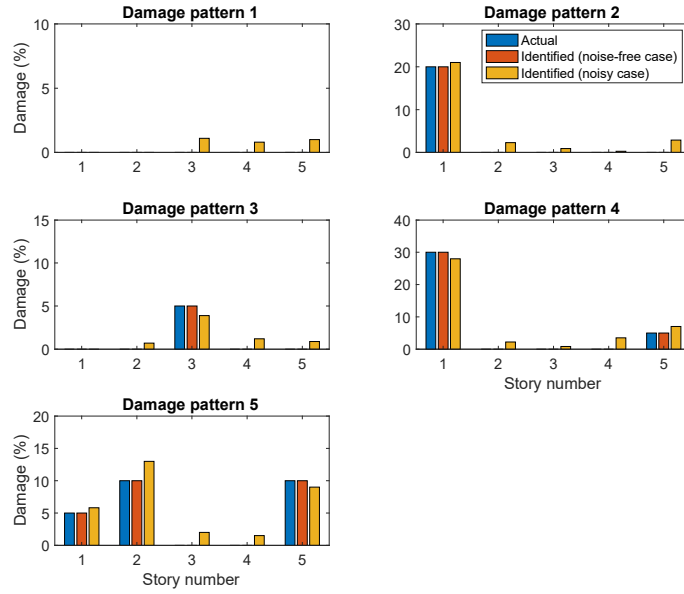


Figure 3. The obtained damage detection results for the five-story shear building structure (the modal data of the first mode is utilized to form the objective function)

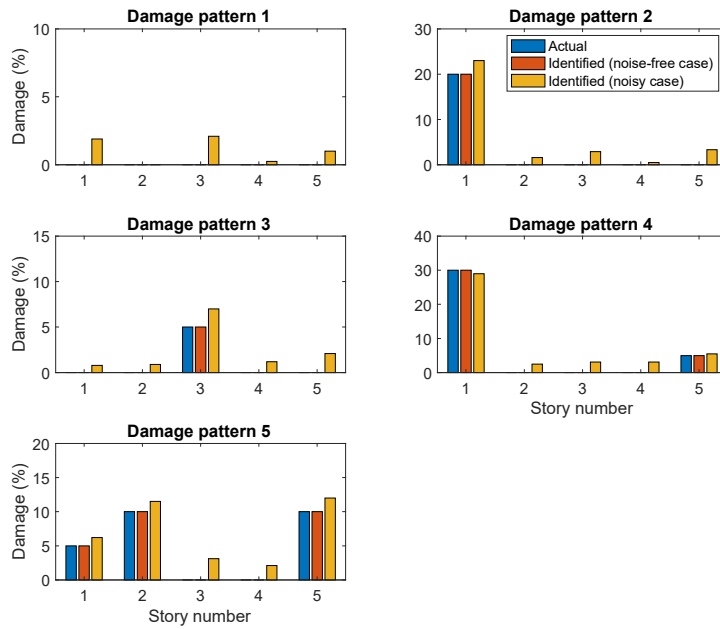


Figure 4. The obtained damage detection results for the five-story shear building structure (the modal data of the first three modes is utilized to form the objective function)

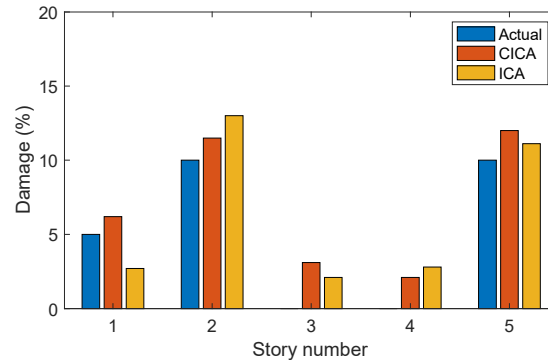


Figure 5. Comparative study to evaluate the robustness of CICA and ICA in solving the proposed objective function (damage pattern 5, $p = 3$).

In the following, the robustness of the CICA is compared with the original ICA for the case #5, which contains multiple damage cases. The number of the countries, imperialists, and maximum iterations are equal to 100, 10 and 1000, respectively. Fig. 5 shows the obtained results for a state in which the first 3 modes' data are used. Based on this figure, CICA is able to identify all stiffness reduction factors. However, ICA identify the small damages with some errors. The reason is the complexity of the solution domain which has caused many different local extremums around the location of the global extremum. Since CICA seeks more points of the domain in comparison with the original ICA, it can perform better in converging to the global optimum point.

3.2 2D steel frame

The second example is a 2D three-bay by five-story steel frame studied in [22] (Fig. 6). This structure can represent the typical residential buildings in the urban areas. The Young's elasticity and mass density for all elements are $E = 200$ GPa and $\rho = 7850$ kg/m³, respectively. The cross-sections of the columns and beams are W250 × 80 and S200 × 34, respectively. Each free node of the system has three DOFs and the system contains 35 members (see Fig. 6). Table 2 describes the details of the damage cases studied in this example. In case 1, the stiffness of only one element has been changed. In case 2, however, three members with different levels of damages are considered. Similar to the previous example, not only the ideal case, but also two noisy cases are studied:

- Noise I: 2.5% noise in frequencies and 5% noise in the mode shapes;
- Noise II: 3% noise in frequencies and 6% noise in the mode shapes.

The selected parameters for CICA are same as the selected parameters in section 3.1 except the number of the countries, the number of imperialists, and the maximum number of iterations which are equal with 500, 50, and 2000, respectively.

Figs. 7 and 8 show the obtained damage detection results for the cases that $p = 5$ and $p = 8$, respectively. Based on the results, the proposed method can identify the structural damages with high level of accuracy even though noisy inputs are used to solve the problem.

In general, by increasing the number of the modes utilized to solve the problem, the noise effects are intensified because the number of the noisy data fed into the method increases.

The damage detection results (which in this paper are basically the estimated stiffness of the monitored structures) can be used for different purposes. The static resilience indicator is one of the simplest but practical concepts can be useful in terms of developing rehabilitation plans. For this purpose, the deflections of the structure are numerically computed under a unit force for both the pristine structure and the updated model using the modal stiffness matrix. The feasibility of this idea is studied in this section using the obtained results for damage case 2. For this purpose, the structural system was simulated in ABAQUS [37] (using B21 elements and mesh size of 2 cm) and a set of concentrated static forces were applied to the joints (see Fig. 9). Fig. 10 shows the plot of the static deflections. Comparing the deflection values reveal that although in the static displacements of the damaged structure is greater than those of the healthy structure, all the drifts are small enough and the structure still is in the safe zone. Therefore, a very easy and fast evaluation of the resilience of the monitored structure is carried out to develop initial status report of the monitored structure. Regardless this, it should be noted that the damage has inversely affected dynamic behavior of the building and a rehabilitation plan is required to prevent progressive damage in the case of earthquake occurrence.

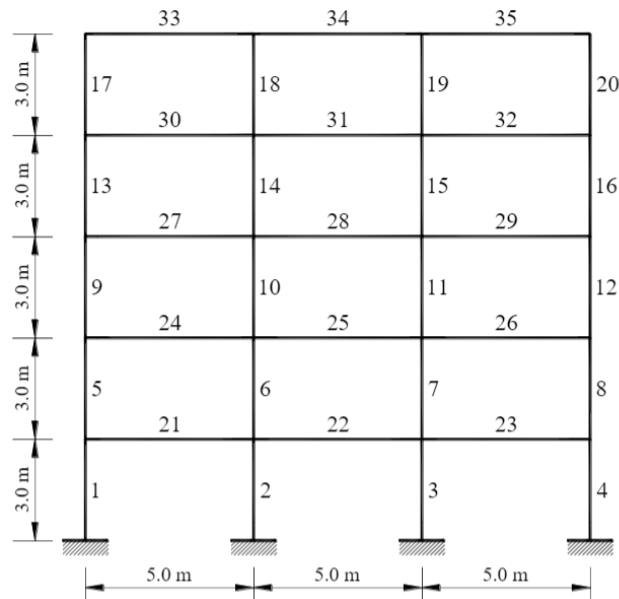


Figure 6. Finite element model of the 2D steel frame (Zare Hosseinzadeh *et al.* [22])

Table 2: Details of the studies cases in the second numerical experiment

Pattern Number	Story Number	Damage (%)
1	22	25
	5	10
2	11	10
	34	20

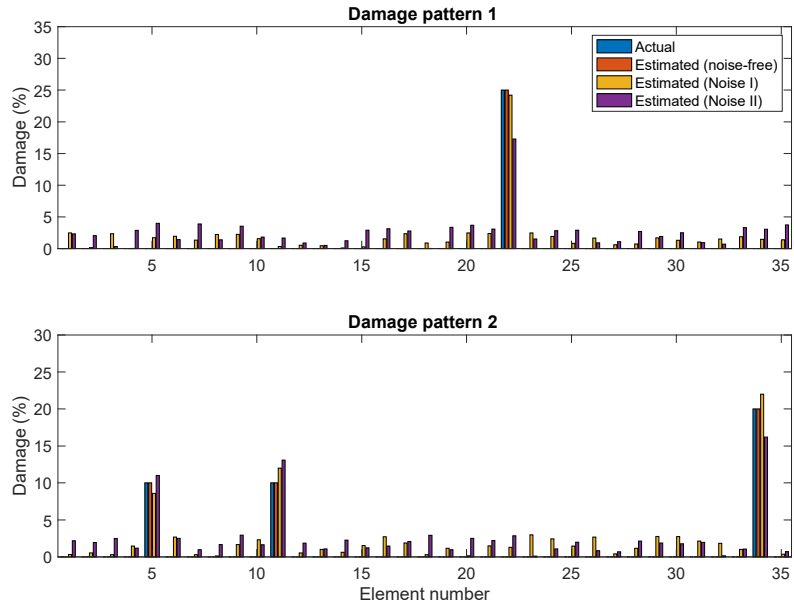


Figure 7. The obtained damage detection results for the 2D steel frame ($p = 5$)

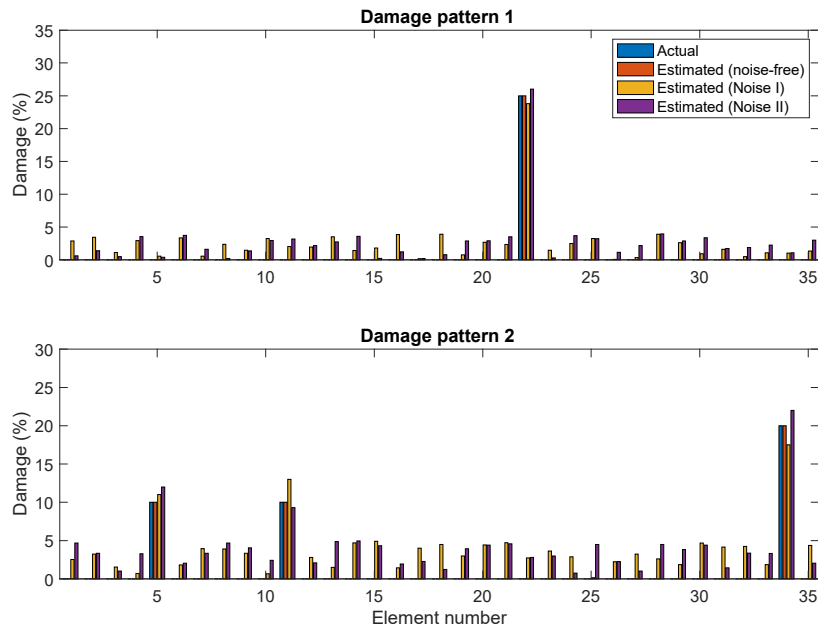


Figure 8. The obtained damage detection results for the 2D steel frame ($p = 8$).

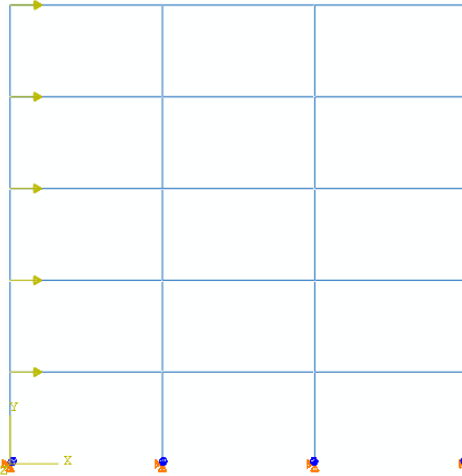


Figure 9. The modeled structure in ABAQUS

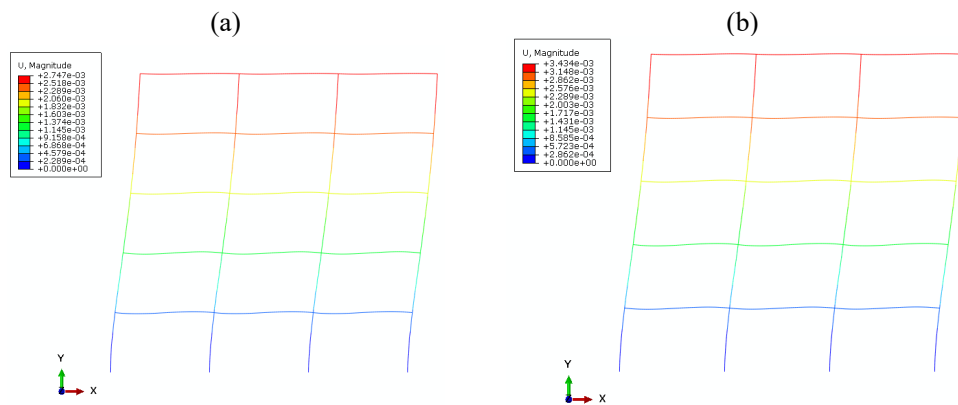


Figure 10. The static deflections of the 2D steel frame: (a) undamaged structure and (b) damaged structure (based on damage pattern 2).

3.3 ASCE benchmark structure

In this section, the proposed method is validated by studying the first phase of the benchmark problem provided by the International Association for Structural Control (IASC)-American Society of Civil Engineers (ASCE) Task Group on Structural Health Monitoring. The benchmark structure is a four-story steel frame, two-bay by two-bay and quarter-scale model structure, constructed in the Earthquake Research Laboratory at the University of British Columbia (see Fig. 11). This building can represent the model of the common buildings in the urban areas with either educational or residential serviceability. Material properties and geometrical details of this structure can be found in Johnson *et al.* [38]. A finite element model of this structure with 12 DOFs (as a three-dimensional shear building structure with three DOFs at each floor) is employed to form the structural model. Damage is introduced by cutting one-third of the area of one brace at the first story [37]. The optimization parameters are similar to what we selected in section 3.2. Here not only the

noise-free state, but also two noisy states are considered, which are defined as follows:

- Noise I: 3% noise in the frequencies and 5% noise in the mode shapes,
- Noise II: 2% noise in the frequencies and 8% noise in the mode shapes.

Moreover, the objective function is formed using the first two and four modes' data (i.e., $p = 2$ and 4). The obtained results are summarized in Table 3. The general trend of the obtained results can confirm the robust performance of the proposed method in structural model updating. Note that measurement noises can adversely affect the accuracy of the obtained results; however, there is no false alarm that can lead to incorrect results, and this proves the ability of the presented method in dealing with ill-posed problems. The other point is that by increasing the number of the utilized modes the accuracy of the results in noise-free state slightly increases; however, in the noisy states, the errors go up.

To evaluate the static resilience of this building, a resilience index, ϑ , is defined as below:

$$\vartheta_j = \frac{\delta_j^d}{\delta_j^u} \quad (14)$$

where, δ_j denotes the static deflection of the damaged (shown by d) and healthy (shown by u) structure for the j th DOF. The maximum resilience index that we computed in this study was 0.84 (for Noise B), which means that the damaged structure has good static resilience, and the structure is stable even though it has experienced some kind of damage. Note that although this index cannot quantify the resilience, it can provide the engineers to design suitable rehabilitation plans ahead of time.

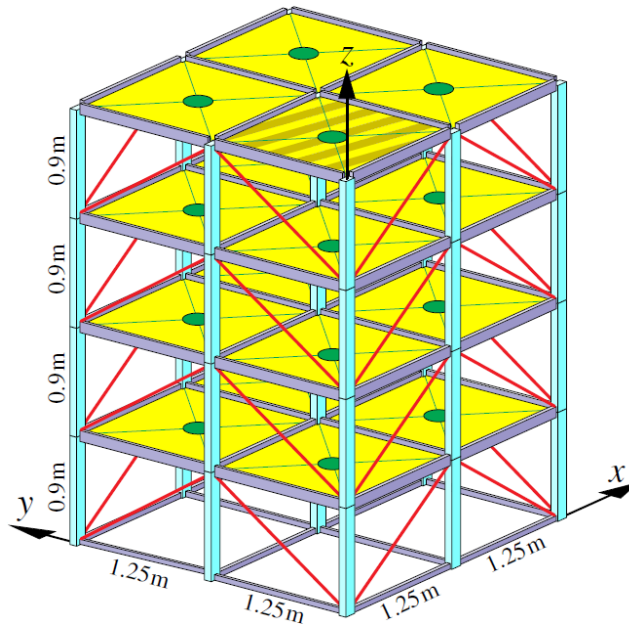


Figure 11. Geometry of the benchmark structure constructed in the Earthquake Research Laboratory at the University of British Columbia

Table 3: Obtained results (%) for the benchmark problem

Story	DOF	Actual	Without noise		Noise A		Noise B	
			$p = 2$	$p = 4$	$p = 2$	$p = 4$	$p = 2$	$p = 4$
1	x	0	0.000	0.001	0.023	0.174	0.911	0.101
2	x	0	0.000	0.000	0.121	0.612	0.762	0.333
3	x	0	0.000	0.000	0.315	0.124	0.111	0.334
4	x	0	0.000	0.000	0.031	0.741	1.235	0.741
1	y	5.92	5.921	5.920	5.672	6.105	8.140	7.141
2	y	0	0.000	0.000	0.303	0.666	0.325	0.505
3	y	0	0.000	0.000	0.021	0.961	0.762	0.971
4	y	0	0.000	0.001	0.514	0.471	0.333	0.981
1	θ_z	2.71	2.709	2.711	2.666	2.101	2.511	3.201
2	θ_z	0	0.000	0.000	0.211	0.711	0.981	1.001
3	θ_z	0	0.000	0.000	0.201	0.321	0.782	1.008
4	θ_z	0	0.000	0.000	0.401	0.914	1.741	1.753

4. CONCLUSION

A model-based approach for damage detection and quantification in the building structures was proposed. The method leveraged the promising aspects of the model updating approaches to define the problem as an optimization problem. A new objective function was proposed using entry- and geometry-wise inspection of a modal flexibility-based vector. The CICA was employed to solve the problem and determine the global extremum as the optimal point. The CICA is an enhanced version of the original ICA which uses chaos theory to search the solution domain of the highly ill-posed problems with high speed and high accuracy. The estimated stiffnesses then can be used to evaluate the static resilience of the monitored structure. For this purpose, the static deflections of a baseline model should be extracted under a known virtual force. Note that the mentioned resilience criterion is based on the physical properties of the tested structure. To find more realistic criterion, a risk-based resilience factor should be developed and added to the mentioned factor, which is the subject of the ongoing research by the authors of the present paper. The proposed method was evaluated by studying three examples of the typical building structures with residential and education serviceability in the urban areas. The results revealed high accuracy of the proposed approach and introduced it as a robust method for structural damage detection and quantification.

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