Optimizing Setup Time Reduction Rate in an Integrated JIT Lot-Splitting Model by Using PSO and GS Algorithms for Single and Multiple Delivery Policies

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ABSTRACT

This article develops an integrated JIT lot-splitting model for a single supplier and a single buyer for only one product. The relationship between optimal lot size and setup time reduction is an important subject in such problems. In this model we analyze the effect of setup time reduction in the integrated lot splitting strategy. Two cases, Single Delivery (SD) case, and Multiple Delivery (MD) case are investigated before and after setup time reduction. The Gradient Search (GS) and Particle Swarm Optimization (PSO) are used in proposed model to determine the optimal order quantity (Q*), optimal rate of setup reduction (R*), and the optimal number of deliveries (N*) -just for multiple deliveries case. These optimum values are calculated by minimizing the total cost for both buyer and supplier. Finally numerical example and sensitivity analysis are provided to compare the aggregate total cost for two cases and effectiveness of the considered algorithms. The results show that which policy for lot-sizing is leading to lower total cost. Results show that the aggregate total cost in Single delivery policy is obtained 1.3% lower when we used the optimized setup time reduction rate.

1. Introduction

Joint economic lot sizing: (JELS), the problem of determining production and procurement quantities is one that has to face when the supplier and the buyer has agreed to cooperate in a production system network. Goyal [1] has considered an integrated inventory model for a single product and perhaps it is the first contribution in this field.

A seminal work in the area of "integrated inventory models" is that of Banerjee [2] who proposed the concept of a joint economic lot size. Also the studies of Goyal [3] are related to this concept. He has expanded this model, where a buyer's order quantity is delivered by the supplier in equal several shipments, as well as Kim and Ha [4].

Earlier works focused on the potential saving for both parties (the vendor and the buyer) simultaneously. A comprehensive literature review of this work is

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presented in Goyal & Gupta [5], Abad [6], Parlar & Wang [7], Aderohunmu & others [8], Lu [9], Goyal [10], Hill [11], Viswanathan [12], Bylka [13], and Goyal & Nebebe [14].

Since Goyal [1] introduced the integrated inventory model between a supplier and a buyer, many researchers have developed this concept for various cases, such as Banerjee [2], Goyal [3] and Hill [15].

The first study on setup reduction is due to Porteus [16] with an economic order quantity (EOQ) model. Spence & Porteus [17] found the optimal rate of setup reduction in a multi-product EOQ model and Kim and others [4] the economic manufacturing quantity (EMQ) model, respectively.

Recently, there has been an investigation an EOQ model which considers setup cost reduction in the variable lead time environment.

Eyler et al. [18] extended the previous research in two areas. First, the EOQ model with setup cost reduction in the variable lead time environment. Second, investigation to a more realistic situation where there is only a finite number of opportunities for setup cost reduction investment.

Denizel et al. [19] developed a dynamic lot-sizing model M where the values of the setup costs can be reduced by various amounts depending on the level of funds R committed to this reduction. Yang and Deane researched on dependence of setup time reduction and competitive advantage in a closed manufacturing cell [20].

Particle swarm optimization (PSO): is a population-based swarm intelligence algorithm. It was first introduced by Kennedy and Eberhart [21] as a simulation of the social behavior of social organisms, such as bird flocking and fish schooling. PSO uses the physical movement of the individuals (particles) in the swarm and has a flexible and well-balanced mechanism to enhance and adapt to global and local exploration in continuous space, while some work has been done recently in discrete domains. Recent complete surveys for PSO can be found in [22, 23, 24].

Several successful applications of PSO to unclear problems reported in [25-28] motivated us to use PSO in this work.

In [25-28], the advantages of PSO are demonstrated over other well-established PBM (Population Based Metaheuristic). PSO has been applied successfully to scheduling problems such as job shop scheduling, [29, 30], flow shop scheduling [31,32], assembly scheduling [33], and resource-constraint project scheduling [34].

The wide use of PSO mainly during the last few years is due to the number of advantages of this method compared with other optimization methods. Some of the key advantages are as follows.

This optimization method does not require the calculation of derivatives. The knowledge of good solutions is retained by all particles and the particles in the swarm share information among themselves. Furthermore PSO is less sensitive to the nature of the objective function, which can be used for stochastic objective functions and also can easily escape from local minima.

The rest of the paper is organized as follows: Section 2 addresses the notations and assumptions of the proposed model. The description of the setup time reduction formulation is given in section 3. Section 4 describes the PSO algorithm.

The joint economic lot-sizing model, setup time reduction, and Gradient search algorithm are described for a single supplier and a single buyer with single delivery and multiple deliveries in sections 5 and 6, respectively. In section 7, numerical examples and sensitivity analysis are presented. Conclusions are summarized in section 8.

2. Notations and Assumptions

Joint economic lot sizing model allows the supplier and the buyer to reduce their total costs. At the other hand, small lot sizing is a way to implementing successful JIT leading to minimum supply chain costs. In this study we extend Kim & Ha's model [4] by considering setup time reduction as a decision variable in a joint economic lot-sizing (JELS) model with both Single delivery and several deliveries.

2.1. Notations:
Following notations are considered in this paper:
D: buyer's demand rate per unit time, deterministic
P: supplier's production rate per unit time, (P>D)
A: buyer's ordering cost per order
S: supplier's setup time
C: unit cost for supplier's setup time
Q: buyer's order quantity (production lot size)
H_B: buyer's holding cost per unit time
H_S: supplier's holding cost per unit time
F: fixed transportation cost per trip
V: unit variable cost for order handling and receiving
N: number of deliveries per batch cycle (integer number)
q: delivery size per trip, $q = Q/N$

2.2. Assumptions:
1) We consider single supplier and single buyer for only one product.
2) All necessary information of the buyer and supplier are given to both sides.
3) Backorders and shortages are not allowed.
4) The buyer pays transportation and order handling cost to facilitate frequent deliveries.
5) Product is manufactured with a finite production rate P and P>D. (if P<D, we cannot satisfy buyer's demand and the problem would be infeasible.)
6) All cost parameters are known and constant.

7) No quantity discount is allowed and unit price is fixed. (demand rate and production rate are known, constant and deterministic.

8) $H_B > H_S$, therefore it is not optimal to send any shipment when the buyer has some inventory.

9) The number and size of transportation vehicles has no constraints.

10) The transportation and receiving cost of each shipment is a linear function of the shipped quantities at a fixed cost.

11) There is no lead time.

In the Single Delivery case:

12) Every time the buyer requests an order, the supplier make the production set up on a lot for lot basis.

In the Multiple Deliveries:

13) When the buyer places an order, the supplier splits the order quantity into small lot sizes and send them in equal shipments.

3. Setup Time Reduction

At first Porteus [16] introduces the relationship between optimal lot size and setup time reduction. Following Porteus, to reduce the setup time the cost equation have been modified by Kreng and Wu [35] as follows:

$$C_S(t'_S) = x - y \ln(t'_S) \quad \text{for} \quad 0 \le t'_S \le t_S \qquad (1)$$

The rate of setup time reduction is calculated by Kreng and Wu [35]:

$$R = 1 - \frac{t'_S}{t_S} \qquad \text{for} \quad 0 \le R \le 1 \qquad (2)$$

x, y and t_S are positive constants. t_S and t'_S are the original setup time before reduction the setup time and after reduction, respectively.

A fixed cost is needed to reduce setup time by a fixed percentage. This fixed percentage is θ, and the cost to reduce the increment of fixed percentage setup reduction is M, which has constant value. Therefore, the Eq.(1), C_S, redefined as:

$$C_S(t'_S) = \frac{M}{\ln(1-\theta)} \ln(1-R) \qquad (3)$$

where R is considered as the decision variable.

4. PSO Algorithm

In the implementation of the PSO, the population is referred to as a swarm and each individual as a particle. It is initialized with a random particles group:
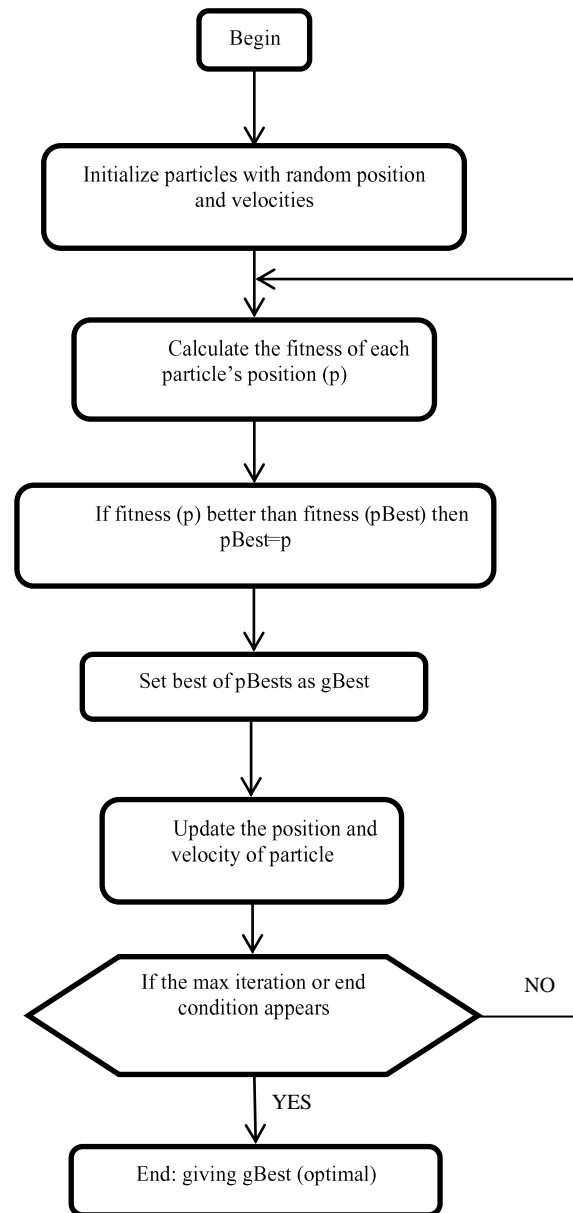

Fig.1. the flowchart of general PSO Algorithm

and then searches the solution space for optimal value by updating generations. The general PSO algorithm is represented in Figures 1 and 2.

In PSO, each particle in a social structure keeps in mind its best position and uses this as a factor for affecting its speed.

A particle gains speed toward its individual best position considering how far away from that point. It also shows the same behavior for the global best position. In other words, while it is scanning the surface, it is affected by the global best position and adjusts its own speed. If the particle is far from the global best position, there will be a higher chance in its speed and direction. Individuals (particles) of a swarm show inclination to change their movements by using the information below.

- Position of the *i*th particle in *k*th iteration is x_i^k (*k*=0,… *iter*$_{max}$ and *i*=1,…,N).
- Speed of the particle *i* in iteration *k* is V_i^k .
- Best position of the particle *i* (local best) is *pBest$_i$*.
- Best position of the particle group (global best) is *gBest*.

Each individual's speed changes according to the formula in Eq.(4);

$$V_i^{k+1} = (w * V_i^k + C_i * (Rnd) * (Pbest_i - x_i^k)$$
$$+ C_2 * (Rnd) * (Gbest_i - x_i^k)) \qquad (4)$$

Where w and C_i are inertia function and *i*th inertia factor, respectively. Rnd is a random number between 0 and 1.

Inertia value of the equation changes in each iteration. This changing is based on the logic of decreasing from the value determined to minimum value according to inertia function.

The objective is to converge the created speed by diminishing on the further iterations; hence more similar results can be obtained [33].

Inertia function is obtained as follows:

$$w = w_{max} - (\frac{w_{max} - w_{min}}{iter_{max}}) * k \qquad (5)$$

w_{max} first inertia force
w_{min} minimum inertia force
$iter_{max}$ maximum iteration number

The values of C_i inertia factor and w_{max} and w_{min} inertia forces are investigated by Shi and Eberhart [37,38]. It is found that these values should not be changed from a problem to another. They fixed the values of these parameters as; C_i=2, w_{max}=0.9 and w_{min}=0.4. Therefore, we use these values in our study.

Positions of the particles change by speeds as shown in Eq.(6)

$$x_i^{k+1} = x_i^k + v_i^{k+1} \qquad (6)$$

Same procedure is reiterated for each dimension.
As it can be seen above, the advantages of the PSO are easiness to implement and having few parameters to adjust. However, there are some difficulties related with applying PSO on constricted models. But the PSO has been successfully applied in many areas, such as function optimization, artificial neural network training, fuzzy system control, and other areas [39].
in our model, we also use this algorithm for optimizing obtained functions.

Initialization (for k=0)

   For i=1 to N

     Assign particles randomly in solution space (x_i^k)

     Generate initial solutions S(x_i^k)

     Assign pBest$_i$ = initial solutions S(x_i^k)

     Assign gBest$_i$ = the obtained best solution among all particles

     Generate initial velocities randomly (V_i^k)

     Add velocities to the corresponding particles (x_i^{k+1})

Initialization (for k=0)

   Determine the inertia weight (w_k)

   For i=1 to N

     Update velocities (V_i^k)

     Modify the current positions (x_i^{k+1})

   Update the pBest$_i$

   Update the gBest$_i$

Finalize the algorithm (k=iter$_{max}$)

   Assign the gBest$_i$=uBest and stop

Fig.2. Algorithmic schema for general PSO

5. Single Delivery (SD) case
5.1. Joint Economic Lot Sizing (JELS) Model:

In this section we first present a lot for lot inventory policy. In lot for lot model, the supplier produces optimal lot size at one setup and delivers it to the buyer at one shipment. The buyer's total cost is composed of ordering cost, holding cost, transportation cost and order receiving cost:

$$TC(Q)_{Buyer} = \frac{D}{Q}A + \frac{Q}{2}H_B + \frac{D}{Q}(F + VQ) \qquad (7)$$

The supplier's total cost consists of setup cost and holding cost:

$$TC(Q)_{Supplier} = \frac{D}{Q}CS + \frac{Q}{2}H_S(\frac{D}{P}) \qquad (8)$$

The total cost function for a joint economic lot sizing model consists of all costs of both buyer and supplier. Hence, by adding Eq.(7) and Eq.(8), aggregate total cost function will be found.

$$TC(Q)_{aggregate} = \frac{D}{Q}(A + CS) + \frac{Q}{2}(H_B + H_S(\frac{D}{P}))$$

$$+ \frac{D}{Q}F + DV \qquad (9)$$

By taking the first derivative of Eq.(9), with respect to Q and set it equal to zero, optimal order quantity Q* will be obtained.

$$Q* = \sqrt{\frac{2D(A + CS + F)}{H_B + H_S(D/P)}} \qquad (10)$$

5.2. Setup Time Reduction for SD Policy

We know S is the supplier's setup time and C is the unit cost for setup time. By considering s as the setup cost per unit time and t_S as the setup time per production run before reduction, following equations are yield: CS=$s\,t_S'$ and $t_S' = t_S(1-R)$

Then, the aggregate total cost for single delivery policy with considering the setup time reduction can be redefined as follows:

$$TC(Q,R)_{aggregate} = \frac{D}{Q}(st_s(1-R) + A) + \qquad (11)$$

$$\frac{Q}{2}(H_B + H_S(\frac{D}{P})) + \frac{D}{Q}F + DV + K\frac{M}{\ln(1-\theta)}\ln(1-R)$$

K is the amortization of the setup reduction capital and a fixed reduction percentage, θ, can be achieved whenever the unit incremental cost of M is made.

5.3. Gradient Search for SD Policy

In order to obtain optimal Q* and R* in single delivery case after setup time reduction, we should take the partial derivatives of Eq.(11) with respect to Q and R, By setting the derivatives equal to zero the optimum values for Q and R are obtained as follows:

$$Q* = \sqrt{\frac{2D(A + st_S(1-R) + F)}{H_B + H_S(\frac{D}{P})}} \qquad (12)$$

$$R* = 1 - \frac{QKM}{Dst_S.\ln(1-\theta)} \qquad (13)$$

6. Multiple Delivery (MD) Case
6.1. Joint Economic Lot Sizing (JELS) Model:

In multiple deliveries case, the order which is produced by quantity of Q, is delivered to buyer over N times, in small quantities q. So we have: $Q = Nq$.

Small lot sizing is a way to implementing successful JIT. The buyer's total cost is:

$$TC(Q,N)_{Buyer} = \frac{D}{Q}A + \frac{Q}{2N}H_B + \qquad (14)$$

$$\frac{DN}{Q}(F + V\frac{Q}{N})$$

The supplier's total cost consists of setup cost and holding cost:

$$TC(Q,N)_{Supplier} = \frac{D}{Q}CS +$$

$$\frac{QH_S}{2N}\left\{(2-N)\frac{D}{P} + N - 1\right\} \qquad (15)$$

Adding Eq.(14) and Eq.(15) yields the total cost for the supplier and the buyer as follows:

$$TC(Q,N)_{aggregate} = \frac{D}{Q}(A + CS) +$$

$$\frac{Q}{2N}\left\{H_B + H_S(\frac{(2-N)D}{P} + N - 1)\right\} \qquad (16)$$

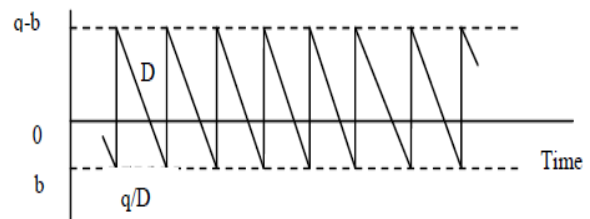$$+ \frac{DN}{Q}F + DV$$

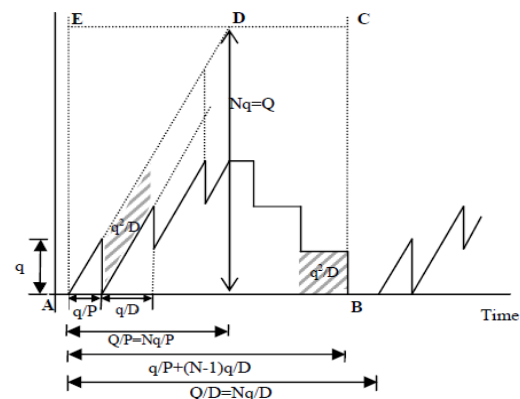**Fig 3. Inventory-Time plot of buyer for MD case**

Fig 4. Inventory-Time plot of supplier for MD case

Note that if the number of deliveries, N, in Eq.(16) is equal to one, the MD case becomes identical to Eq.(9) for SD policy. Hence, in this case, we assume that $N \geq 2$.

According to calculations of Kim & Ha [4], by taking the first derivatives of Eq.(16) with respect to Q and N, we obtain following optimum values for N and Q:

$$N^* = \sqrt{\frac{(A+CS)\left[P(H_B - H_S) + 2DH_S\right]}{F(P-D)H_S}} \qquad (17)$$

$$Q^* = \sqrt{\frac{2ND(A+NF+CS)}{H_S + H_B((2-N)\frac{D}{P} + N - 1)}} \qquad (18)$$

N* denote the optimum integer value of N and Q* is the optimum value of Q. If N* in the Eq.(17) is not an integer number, we should choose N, which yields $\min\{TC(N^+), TC(N^-)\}$ in Eq.(16), where N$^+$ and N$^-$ represent the nearest integers larger and smaller than the N* respectively. The minimum aggregate total cost is obtained by substituting N* and Q* into Eq.(16).
The optimal delivery size q*, which remains the same over multiple deliveries policy, is obtained by dividing Q* by N* from Eq.(17) and Eq.(18).

6.2. Setup Time Reduction for MD Policy
Integrating Eq.(2) and Eq.(3) to Eq.(16), the aggregate total cost for multiple deliveries with considering setup time reduction is obtained as follows:

$$TC(Q,N,R)_{aggregate} = \frac{D}{Q}(st_s(1-R) + A) +$$

$$\frac{Q}{2N}\left\{H_B + H_S(\frac{(2-N)D}{P} + N - 1)\right\} + \frac{DN}{Q}F + \qquad (19)$$

$$DV + K\frac{M}{\ln(1-\theta)}\ln(1-R)$$

The above equation consisted of buyer's ordering cost, buyer's holding cost, transportation cost, order receiving cost, supplier's setup cost after setup time reduction, supplier's holding cost, and total setup reduction capital. The objective is to minimize the sum of these costs.

6.3. Gradient Search Algorithm for MD Case
We can determine the optimal order quantity, Q*, optimal rate of setup time reduction, R*, and optimal number of deliveries from the aggregate total cost after setup time reduction from Eq.(19) with regarding that TC(Q,N,R)$_{aggregate}$ is a convex function. The optimum values are found by taking partial derivatives of Eq.(19) with respect to R, N, and Q, and setting the derivatives equal to zero.

$$N^* = \sqrt{\frac{(A+st_s(1-R))\left[P(H_B - H_S) + 2DH_S\right]}{F(P-D)H_S}} \qquad (20)$$

$$Q^* = \sqrt{\frac{2ND(st_s(1-R) + A + NF)}{H_B + H_S((2-N)\frac{D}{P} + N - 1)}} \qquad (21)$$

$$R^* = 1 - \frac{QKM}{Dst_s\ln(1-\theta)} \qquad (22)$$

7. Numerical Examples
In this section, we use an example which originally comes from Banerjee [2]. It was modified by Kim & Ha [4] and we gathered additional values from example of Kreng & Wu [35] for analyzing our model.
We consider a buyer, a supplier and a single product. Buyer's annual demand is 4800 units and the order cost for each order is 25$. Fixed transportation cost which buyer pays for each trip is 50$ and the unit variable cost for order handling and receiving is 1.00$/unit. Annual production capacity of supplier is 19220 units. The cost of supplier's setup time is 400$ per unit. We assume that H$_B$ and H$_S$ are 7$ and 8$ per unit per year. The setup time before reduction policy is 4 and the setup cost per unit time is 100$, where the amortization of the setup reduction capital is 0.35. It is assumed that a rate of 30% of fixed reduction can be achieved whenever the unit incremental cost of 2000$ is made. In summary:

A=	20	C×S=	400
D=	4800	s=	100
t_S=	4	F=	60
M=	2000	V=	1
P=	19200	K=	0.35
H$_S$=	6	H$_B$=	7
θ =	0.3		

7.1. Example for SD Policy Before and After Setup Time Reduction
Table 1 presents the effect of the rate of setup time reduction on Q* and aggregate total cost. For single delivery policy, by using PSO algorithm and also Gradient Search algorithm (Eq.(11) and Eq.(19))
From this table, we can interfere that if the setup time is reduced from t_S=4 to 0 and all other parameters
remain unchanged, then by using GS algorithm, the optimal solution will be R*= 0.4, $t'_S = 2.4$, Q*= 596.46, and TC*(Q,R)=10872.44; and by using PSO algorithm, the optimal solution will be R*= 0.4, $t'_S = 2.4$, Q*= 592.89, & TC*(Q,R)=10872.53.

7.2. Example for MD Policy Before and After Setup Time Reduction
Using the given parameters in PSO and GS algorithms, we calculate N* and Q*. Then by decreasing t_S from 4

to 0, we find the optimal value of R. table 2 presents this results.

From the results shown in table 2, we can interfere that by using the GS algorithm, optimal value of R is 0.1. In other word, if the setup time is reduced from $t_S=4$ to 0 and all other parameters remain unchanged, the optimal solution will be R*= 0.1, N*=2, t'_S = 3.6, Q*= 859.34, and TC*(Q,R,N)= 10592.47.

From the results of Particle Swarm Optimization Algorithm presented in table 2, we can interfere that by using this algorithm, optimal value of R is 0.1. In other word, if the setup time is reduced from $t_S=4$ to 0 and all other parameters remain unchanged, the optimal solution will be R*= 0.1, N*=2, t'_S = 3.6, Q*= 852.23, and TC*(Q,R,N)= 10592.67.

Tab. 1. The effects of the rate of setup time reduction on Q* and aggregate total cost in single delivery

	R=	t_s	t_s'	t_s'/t_s	Gradient Search				PSO			
	1-t'/t				Q*	ΔQ	TC(Q)	ΔTC	Q*	ΔQ	TC(Q)	ΔTC
before	0	4	4	1	732.44	-	11025.75	-	736.18	-	11025.83	-
After setup time reduction	0.1	4	3.6	0.9	700.92	31.52	10964.63	61.12	703.37	32.81	10964.67	61.17
	0.2	4	3.2	0.8	667.92	64.52	10915.26	110.49	661.04	75.14	10915.56	110.27
	0.3	4	2.8	0.7	633.20	99.24	10882.19	143.56	634.54	101.64	10882.20	143.63
	0.4	4	2.4	0.6	**596.46**	135.98	**10872.44**	153.31	**592.89**	143.29	**10872.53**	153.30
	0.5	4	2.0	0.5	557.30	175.14	10897.44	128.31	559.17	177.01	10897.46	128.37
	0.6	4	1.6	0.4	515.18	217.26	10977.33	48.43	515.76	220.42	10977.33	48.50
	0.7	4	1.2	0.3	469.29	263.15	11151.87	-126.11	468.11	268.07	11151.88	-126.05
	0.8	4	0.8	0.2	418.40	314.04	11515.04	-489.29	416.93	319.25	11515.06	-489.23
	0.9	4	0.4	0.1	360.39	372.05	12382.32	-1356.6	358.47	377.71	12382.36	-1356.5
	1	4	0	0.0	291.04	441.40	-	-	292.69	443.49	-	-

Tab. 2. The effects of R & Q on TC in multiple delivery policy using GS and PSO algorithm

R=1-t'/t	t_s	t_s'	t_s'/t_s	N*	N	Gradient Search						PSO					
						Q*	ΔQ	TC(Q,N)	min TC(Q,N)	opt(N)	ΔTC	Q*	ΔQ	TC(Q,N)	min TC(Q,N)	opt(N)	ΔTC
before					2	893.05	-	10604.83				891.65	-	10604.83			
	0	4	4	1	2.49				10597.83	3	0.0				10599.408	3	-
					3	973.06	-	10597.83				962.98	-	10599.41			
After setup time reduction					2	**859.34**	113.72	10592.47				**852.23**	39.42	10592.67			
	0.1	4	3.6	0.9	2.37				**10592.47**	**2**	5.4				**10592.666**	**2**	6.7
					3	940.07	32.99	10608.01				926.98	-35.33	10610.21			
					2	824.25	148.81	10595.55				818.06	73.59	10595.70			
	0.2	4	3.2	0.8	2.24				10595.55	2	2.3				10595.6987	2	3.7
					3	905.87	67.19	10635.42				898.65	-7.00	10636.50			
					2	787.60	185.47	10619.37				779.18	112.47	10619.67			
	0.3	4	2.8	0.7	2.11				10619.37	2	-21.5				10619.6704	2	-20.3
					3	870.33	102.73	10685.73				862.29	29.36	10686.97			
					2	749.15	223.91	10672.03				732.76	158.89	11012.95			
	0.4	4	2.4	0.6	1.96				10672.03	2	-74.2				10691.9811	2	-92.6
					3	833.28	139.78	10767.49				820.11	71.54	10769.78			
					1	588.93	304.12	10945.41				601.63	290.02	10951.21			
	0.5	4	2	0.5	1.81				10766.43	2	-161.6				10766.8611	2	-167.5
					2	708.63	264.43	10766.43				699.04	192.61	10766.86			
					1	545.24	427.82	11028.39				539.53	352.12	11026.48			
	0.6	4	1.6	0.4	1.63				10924.95	2	-327.1				10925.1792	2	-325.8
					2	665.64	307.42	10924.95				658.79	232.86	10925.18			
					1	497.74	475.33	11206.99				491.48	400.17	11204.96			
	0.7	4	1.2	0.3	1.44				11190.79	2	-593.0				11191.1373	2	-591.7
					2	619.68	353.38	11190.79				611.54	280.11	11191.14			
					1	445.19	527.87	11575.80				438.18	453.47	11573.61			
	0.8	4	0.8	0.2	1.22				11575.80	1	-978.0				11573.6055	1	-974.2
					2	570.02	403.04	11663.77				561.24	330.41	11664.21			
	0.9	4	0.4	0.1	0.94	1	385.55	587.52	12451.54	12451.54	1	-1853.7	377.43	514.22	12449.18	12449.1752	1	-1850
	1	4	0	0	0.54	1	314.80	658.27	-	-	-	-	308.58	583.07	-	-	-	-

7.3. Sensitivity Analysis

A sensitivity analysis is performed to study the effects of changes in the parameters system on the optimal order quantity, rate of setup time reduction, and number of deliveries. This analysis is performed by increasing or decreasing the parameters by 10%, 20%, and 30% taking one at a time, keeping the remaining parameters at their original values. The effects of changes in parameters on SD case and MD case are investigated. Following ratios are being calculated for different quantity of these parameters:

$$r_1 = \frac{TC^*_{GradientSearch} - TC^*_{PSO}}{TC^*_{PSO}} \times 100 \qquad (23)$$

$$r_2 = \frac{TC^*_{SD(PSO)} - TC^*_{MD(PSO)}}{TC^*_{MD(PSO)}} \times 100 \qquad (24)$$

$$r_3 = \frac{TC^*_{SD(GS)} - TC^*_{MD(GS)}}{TC^*_{MD(GS)}} \times 100 \qquad (25)$$

which r_1 shows the deviation of PSO solution from Gradient Search solution. r_2 and r_3 determine the difference of optimal cost function in MD and SD case, by using PSO and GS algorithms, respectively.

Tab. 3. Sensitivity analysis over parameter D

Change (%)	D	SD GS Q*	R*	TC*(Q)	SD PSO Q*	R*	TC*(Q)	r_1 (*10³)	MD GS Q*	R*	TC*(Q,N)	MD PSO Q*	R*	TC*(Q,N)	r_1 (*10³)	r_2	r_3
-30%	3360	606.1	0.1	8445.5	601.2	0.1	8445.7	-0.02	747.2	0	8216.67	761.46	0	8217.54	-0.11	0.03	0.03
-20%	3840	612.1	0.2	9296.9	618.3	0.2	9297.2	-0.03	798.8	0	9031.99	801.29	0	9032.02	0.00	0.03	0.03
-10%	4320	610.3	0.3	10116.3	614.6	0.3	10116.4	-0.01	815.2	0.1	9825.83	809.73	0.1	9825.96	-0.01	0.03	0.03
0%	4800	601.2	0.4	10912.5	592.9	0.4	10913.0	-0.05	859.3	0.1	10592.5	852.23	0.1	10592.7	-0.02	0.03	0.03
10%	5280	625.0	0.4	11689.0	632.4	0.4	11689.4	-0.03	1025.5	0	11334.6	1017.6	0.1	11335	-0.03	0.03	0.03
20%	5760	605.4	0.5	12448.1	591.6	0.5	12449.6	-0.11	1039.8	0.1	12049.7	1035.2	0.1	12050.7	-0.09	0.03	0.03
30%	6240	624.8	0.5	13192.7	629.4	0.5	13192.9	-0.01	1087.6	0.1	12750.3	1096.2	0.1	12749.6	0.06	0.03	0.03

Tab. 4. Sensitivity analysis over parameter P

Change (%)	P	SD GS Q*	R*	TC*(Q)	SD PSO Q*	R*	TC*(Q)	r_1 (*10³)	MD GS Q*	R*	TC*(Q,N)	MD PSO Q*	R*	TC*(Q,N)	r_1 (*10³)	r_2	r_3
-30%	13440	579.7	0.4	11102.2	595.4	0.4	11104.1	-0.170	993.7	0	10490.1	976.03	0	10492.9	-0.27	0.06	0.06
-20%	15360	588.3	0.4	11024.0	601.3	0.4	11025.3	-0.112	984.9	0	10535.2	966.42	0	10538.4	-0.30	0.05	0.05
-10%	17280	595.4	0.4	10962.4	614.7	0.4	10965.0	-0.241	978.3	0	10570.1	953.04	0	10575.1	-0.48	0.04	0.04
0%	19200	601.2	0.4	10912.5	592.9	0.4	10913.0	-0.045	859.3	0.1	10592.5	852.23	0.1	10592.7	-0.02	0.03	0.03
10%	21120	606.1	0.4	10871.4	624.6	0.4	10873.7	-0.212	859.3	0.1	10592.5	876.19	0.1	10593.5	-0.10	0.03	0.03
20%	23040	610.2	0.4	10836.8	608.4	0.4	10836.8	-0.003	859.3	0.1	10592.5	871.69	0.1	10593	-0.05	0.02	0.02
30%	24960	613.8	0.4	10807.4	631.7	0.4	10809.5	-0.191	859.3	0.1	10592.5	865.96	0.1	10593.5	0.00	0.02	0.02

The following inferences can be made from the results of table 1 and 2. and sensitivity analysis based on table 2 and 3.

- By using PSO algorithm, which is a meta-heuristic algorithm and gives an approximate solution, and Gradient Search algorithm, which gives an exact solution, and comparing the results of these methods by considering r_1, we can interfere that the values computed for the aggregate total cost are approximately similar.

- To compare effectiveness of different delivery policies, SD and MD, we should compare joint total costs of SD and MD case in tables 1 and 2. Since TC*$_{MD}$<TC*$_{SD}$, consequently, the policy of frequent shipment results in less total cost than the single shipment policy. TC*$_{MD}$=10592.5 by N=2, while TC*$_{SD}$=10872.5.

- From table 4 we can interfere that by increasing the value of D, while other parameters remain unchanged, the optimal joint total cost of MD policy increases, while the optimal joint total cost of SD policy decreases.

- Increasing the parameter P results to more joint total cost for both single delivery and multiple delivery policies.

8. Conclusions

The effects of setup time reduction in the integrated lot splitting strategy have been analyzed in this study. The proposed model determines optimal order quantity, optimal rate of setup reduction, and optimal number of deliveries on the integrated total relevant cost for single delivery and multiple deliveries policies, the results were inferred by comparing the optimal values of these two policies which obtained by using PSO and GS algorithms.

The results show that which policy for lot-sizing is leading to lower total cost. Results show that the aggregate total cost in Single delivery policy is obtained 1.3% lower when we used the optimized setup time reduction rate.

The proposed model can be extended in future studies by considering multiple products, multiple buyers and suppliers, or probabilistic parameters.

Appendix:

The expression of holding cost is derived by Joglekar (1988). From Fig.3 the holding cost of supplier is derived as follows:

$$H_S \frac{D}{Nq}\left\{ S_{ABCE} - S_{ADE} - \left(\frac{q^2}{D} + 2\frac{q^2}{D} + ... + (N-1)\frac{q^2}{D}\right)\right\} =$$

$$H_S \frac{D}{Nq}\left\{ Nq\left(\frac{q}{P} + (N-1)\frac{q}{D}\right) - \frac{N^2 q^2}{2P} - \left(\frac{q^2}{D}(1+2+...+(N-1))\right)\right\} =$$

$$H_S \frac{D}{Nq}\left\{ \frac{Nq^2}{2D}\left((2-N)\frac{D}{P} + N - 1\right)\right\} =$$

$$H_S \frac{Q}{2N}\left\{(2-N)\frac{D}{P} + N - 1\right\}$$

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