



Optimal Capacities in Discrete Facility Location Design Problem

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ABSTRACT

Network location models comprise one of the main categories of location models. These models have various applications in regional and urban planning as well as in transportation, distribution, and energy management. In a network location problem, nodes represent demand points and candidate locations to locate the facilities. If the links network is unchangeably determined, the problem will be an FLP (Facility Location Problem). However, if links can be added to the network at a reasonable cost, the problem will then be a combination of facility location and NDP (Network Design Problem). In previous studies, capacity of facilities was considered to be a constraint while capacity of links was not considered at all. The proposed MIP model considers capacity of facilities and links as decision variables. This approach increases the utilization of facilities and links, and prevents the construction of links and facility locations with low utilization. Furthermore, facility location cost (link construction cost) in the proposed model is supposed to be a function of the associated facility (link) capacity. Computational experiments as well as sensitivity analyses performed indicate the efficiency of the model.

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1. Introduction

Network location models form a major category of location models. In the network location problem, nodes represent demand points and candidate locations to locate facilities. The objective of the problem is to determine the optimum number and locations of the required facilities. Facility location problem (FLP) and capacitated facility location problem (CFLP) are two basic and generic problems that can be formulated as network location problems. In FLP, the capacity of a facility is considered to be unlimited while in CFLP, facilities have a limited capacity.

Most studies of FLP and CFLP have focused on the development of efficient solution algorithms [1, 2, 3, and 4]. Approximation algorithms based on greedy heuristics were the first to be proposed for facility location problems by Hochbaum [5].

A modified greedy algorithm for uncapacitated facility location problem was analyzed by Jain et al. [6] using factor revealing LP which exploits the special properties of the heuristic and also the structure of the problem.

Algorithms based on rounding the fractional optimal solution to the LP relaxation of the original integer programs were proposed by Shmoys et al. [7]. They used the filtering idea proposed by Lin and Vitter [8] to round the fractional solution to the LP and obtain constant factor approximations for many facility location problems. This idea was also combined with randomization by Chudak and Shmoys [9]. Solution

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algorithms based on primal-dual techniques were proposed by Jain and Vazirani [10]. They solved the uncapacitated facility location problem using a two-phase primal-dual scheme. Their technique's novelty was in relaxing the primal conditions while satisfying all the complimentary slackness conditions. Approximation algorithms based on local search are perhaps the most versatile.

Local search heuristics have been used for many years by practitioners and one such heuristic was proposed by Kuehn and Hamburger [11]. However, Korupolu et al. [12] showed for the first time that a worst case analysis of the local minima computed by these heuristics was possible and they showed constant factor approximations to many facility location problems which were comparable to those obtained by other techniques. A hybrid algorithm which combines Lagrangian heuristic and Ant colony System (ACS) to solve the single source capacitated facility location problem was proposed and tested by Chen and Ting [13].

A model that integrates the tasks of facility location and network design has been presented by Berman et al. [14], Campbell [15], Melkote and Daskin [16], [17] and Berger et al. [18]. In this kind of problems, a set of nodes is given that represents the demand nodes, as well as candidate facility locations, and a set of uncapacitated links.

Each link has a fixed construction cost as well as a per unit transportation cost, and each node is associated with a fixed charge for building an uncapacitated facility at that node. The objective is to find the network design and the set of facility locations that minimize the total system cost (fixed + operational). This model is reported to be used in the design of pipeline distribution systems, inter-modal transportation systems, power transmission networks and all the hub location problems.

Uncapacitated facility location/network design problem (UFLNDP) was motivated by the simple observation that it may be more economical in a network to add some links instead of locating new facilities to improve service levels. In other words, the assumption of unchangeable links network in the location problem was relaxed. Generally speaking, UFLNDP is applicable to and useful in modeling a number of situations in which tradeoffs must be made between facility location costs, network design costs, and operating costs. Instances of this combined model have been solved with up to 40 nodes and 160 candidate links in polynomial time [16].

In UFLNDP, it is assumed that facilities may serve an infinite amount of demand. This is valid and logical when it is known in advance that the facilities will operate significantly below their capacity. However, such an assumption in many situations e.g. in power transmission and telecommunication networks, is not valid and an upper limit should be considered on demands that a facility can handle.

Melkote and Daskin [16] propose a capacitated facility location/network design problem (CFLNDP). This problem is derived from the classical capacitated facility location problem (CFLP) that has been discussed by many researchers. They give a mixed integer programming formulation of CFLNDP and solve it using branch-and-bound method. In the CFLNDP model, the capacity of a facility is known and taken as the input parameter while that of links is assumed to be unlimited.

Drezner et al. [19] introduced new network design problems where the objective is to minimize the total construction and transportation costs. Links could be either constructed, at a given cost, or not constructed. Each link could be designed either as a one-way link or a two-way link. Four basic problems of the model were solved heuristically by applying a descent algorithm, simulated annealing, tabu search, and a genetic algorithm.

In the network model proposed in this paper, the capacities of links and facilities are considered not as constraints but as decision variables. Here, the value of capacities are also not continuous and depend on the special applications, which can be optimally selected among several candidate discrete values. So the problem becomes more flexible and facility and link utilization can be improved. The objective function is to minimize the sum of facility location costs, link construction costs, and operating costs (transportation costs). Link construction and facility location costs are considered as functions of the associated link or facility capacity in order for them to conform to real conditions.

The rest of this paper is organized as follows. In section 2, we present a mixed integer programming formulation for the problem at hand. In section 3, numerical examples are investigated and the behavior of the model will be explored via sensitivity analysis. Finally in Section 4, we draw conclusions and suggested areas for future study.

2. Model Formulation

Prior to formulating the problem, the following assumptions regarding the underlying network must be made. The network includes N nodes each of which represents a demand point. Facilities may be located only on the nodes of the network. Only one facility can be located per node. The network is a customer-to-server system, in which demands themselves travel to facilities to be served. The demand is only for a single service or commodity. The capacity of links and facilities are considered as decision variables. Some links which are candidates to be constructed connect nodes of the network to each other. For each node, the demand and travel cost per unit flow on the link (i, j) are known. Links can be constructed in different "ranks". The simplest interpretation of the "rank" is its width such that if increased, the link capacity will

increase correspondingly. For example, a four-lane road can be considered as a link with rank 4. We assume the construction cost of a unit length link with rank 1 to be known and the same for all links. Construction cost of each link is a linear function of its length and rank. The link rank is an integer variable with an upper bound. When link rank increases by one unit, the link capacity will be increased by a specific amount. Facility location cost contains two components: (1) the initial cost for locating a basic facility (a facility with minimum capacity), and (2) a variable cost that is related to the capacity of that facility. As the capacity of the facility increases (or its rank increases), the variable facility location cost also increases as a linear function of facility rank. A facility rank is also an integer variable with an upper bound. When the rank of a facility is increased by one unit, its capacity also increases by a specified quantity. To formulate the problem we define the following notations:

To formulate the problem we define the following notations:

N	Set of nodes in the network
L	Set of candidate links in the network
D_i	Demand at node i
$M = \sum_{i \in N} D_i$	Total network demand
W_i	Total demand served by a facility at node i
d_{ij}	Distance between nodes i and j
e_{ij}^k	Travel cost per unit of demand of node k from node i to j
p^k	Travel cost per unit of demand of node k in unit of length ($e_{ij}^k = p^k d_{ij}$)
u	Construction cost of link per unit of length with rank 1
v	Cost of increasing capacity of a facility up to n (increasing rank of facility by one unit)
S	Link capacity with rank 1
m	Initial facility capacity (capacity of basic facility)
n	The amount of facility capacity increase when its rank is increased by one unit
IC_i	Location cost with capacity m (basic facility)
b_1	Upper bound for $Capl_{ij}$
b_2	Upper bound for $Capf_i$
E_i	Equals unity if a facility with basic capacity is located at node i , otherwise it is 0

The decisions variables are:

Y_{ij}^k	Fraction of demand of node k that flows on link (i, j)
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W_i^k	Fraction of demand of node k served by a facility at node i
$Capl_{ij}$	Rank for link (i, j)
$Capf_i$	Rank of facility that is located at node i

According to the above assumptions and notations, the problem can be formulated as follows:

$$\min \sum_{(i,j) \in L} \sum_{k \in N} e_{ij}^k D_k Y_{ij}^k + \sum_{(i,j) \in L} [(u d_{ij}) Capl_{ij}] + \sum_{i \in N} (IC_i E_i + v Capf_i) \quad (1)$$

$$\sum_{j \in N} Y_{ji}^k = \sum_{j \in N} Y_{ij}^k + W_i^k, \quad i, k \in N, i \neq k \quad (2)$$

$$W_i^i + \sum_{j \in N} Y_{ij}^i = 1, \quad i \in N, D_i > 0 \quad (3)$$

$$\sum_{i \in N} W_i^k = 1, \quad k \in N, D_k > 0 \quad (4)$$

$$Capf_i \leq b_2 E_i, \quad i \in N \quad (5)$$

$$\sum_{k \in N} D_k W_i^k \leq (m E_i + n Capf_i), \quad i \in N \quad (6)$$

$$\sum_{k \in N} Y_{ij}^k D_k \leq S \cdot Capl_{ij}, \quad (i, j) \in L \quad (7)$$

$$Y_{ij}^k \leq capl_{ij}, \quad (i, j) \in L, k \in N \quad (8)$$

$$W_i^k \leq E_i, \quad i \in N, k \in N \quad (9)$$

The objective function is the sum of transportation, link construction, and facility location costs. Eq. (2) is a conservation of flow Equation, stating that input flow to a node must be equal to the output flow from that node. Eq. (3) guarantees that each demand entering node i is either shipped out or served at i . And finally, (4) states that all demands on each node must be served completely. Eq. (5) states that the rank of the facility at node i can be positive, only if an initial facility is located at this node. It also takes into account the upper bound of the facility rank. Eqs. (6) and (7), respectively, imply facility and link capacity constraints. Eq. (8) states that commodities will be moved from node i to node j only if the link (i, j) is constructed at least with rank 1. Eq. (9) states that demand on node k can be served at node i only if a facility is located at this node. Eqs. (10) and (11) are standard nonnegativity and integrality constraints:

$$capl_i \in \{1, 2, \dots, b_1\}, Capf_i \in \{1, 2, \dots, b_2\}, \forall i \in N \quad (10)$$

$$\begin{aligned} E_i &\in \{0, 1\}, W_i \geq 0, \forall i \in N \\ Y_{ij}^k &\geq 0, \forall (i, j) \in L \end{aligned} \quad (11)$$

If it is desirable to control the number of facilities (p), the following constraint can be applied:

$$\sum_{i=1}^N E_i \leq p \quad (12)$$

If the budget for locating facilities and constructing links is limited (B), the following constraint will be used:

$$\sum_{(i,j) \in L} [(u d_{ij}) Capl_{ij}] + \sum_{i \in N} (IC_i E_i + v Capf_i) \leq B \quad (13)$$

The above formulation is regarded to be 'strong' in the sense that constraints including big M are avoided. The presence of constraints including big M in the formulation would require longer CPU time [20].

3. Numerical Results

Numerical results are organized in three separate sections. In section one, the structure of the solutions are investigated and compared to the solutions of CFLNDP using a benchmark problem. In the next section, sensitivity analyses will be conducted with respect to link construction and facility location cost. Finally, in the third section, the efficiency of the model

with respect to CPU time is investigated. The effect of the parameter u on CPU time will also be investigated in the same section. A CPLEX solver in GAMS software on a Pentium 4 computer with a 3.4 GHz processor and 512 MB of memory is used to solve numerical examples.

3.1. Solution Configuration and Comparison of Results

To examine the structure of the solutions and to make comparisons and sensitivity analyses, a 21-node network has been used. This test problem is a well-known one commonly employed in several papers ([21]; [22]; [23]).

In the present paper, a number of changes have been made in the problem due to differences in assumptions. Locations of nodes and their distances as well as demands on each node are the same as in the original problem. Initial facility location costs are generated randomly from a Uniform [2000, 6000] distribution and are normalized so that their average is 4000 (according to Balakrishnan *et al.* [24]). In Figure 1, quantity of demands, initial facilities location cost, and distances between nodes have been shown. Table 1 shows the values for other parameters used in the model.

Tab. 1. Input Parameters

$p^k, \forall k \in N$	u	v	S	n	m	b_1	b_2
1	30	200	20	20	100	4	5

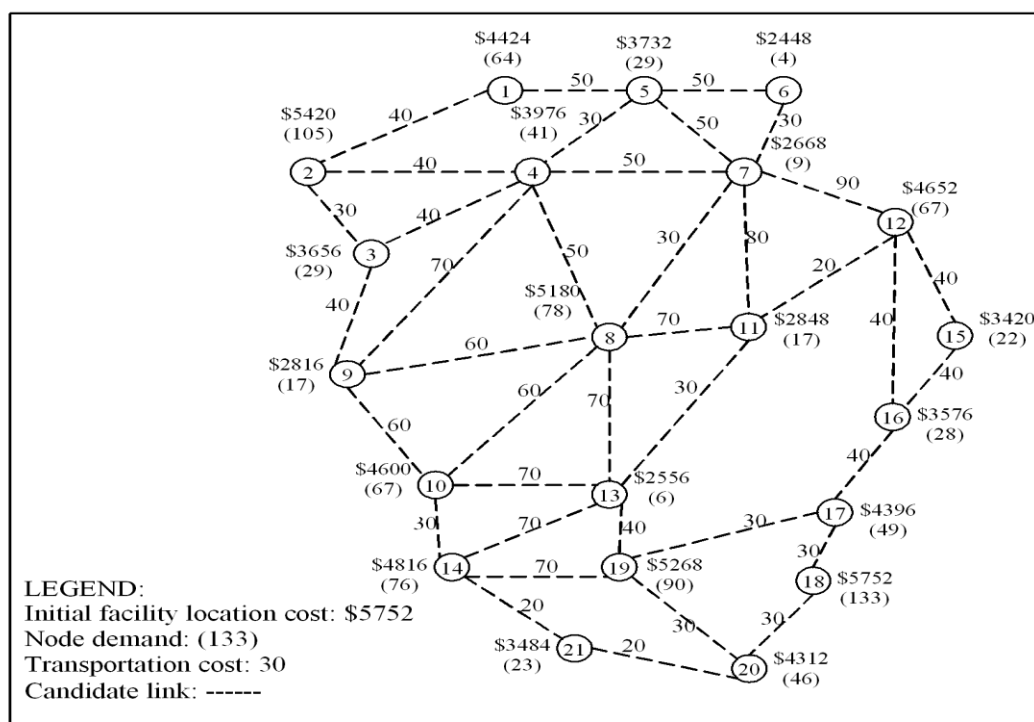


Fig. 1. Network with 21 nodes

Figure 2 shows the optimal solution to the problem. The located facilities have been shown with squares. The facilities located at nodes 2, 12, 18, and 21 have gained ranks of 2, 2, 5, and 3, respectively. However, the other facilities have no increase in their capacity. This Figure also shows constructed links. Line thicknesses imply link ranks. For example, the rank of links (7, 8), (12, 16) and (14, 21) are 1, 2, and 4, respectively. The optimum total cost obtained for this problem was \$70736. In Figure 2, costs are shown separately.

To investigate the effect of considering facilities and links capacities as decision variables, the results are compared with those of CFLNDP. To do such comparisons, some customizations are needed. In CFLNDP, link capacities are supposed to be infinite but links are capacitated in our model. Therefore, Eq. (14) is added to CFLNDP. In this equation, X_{ij} is equal to 1 if the link (i, j) is constructed; otherwise, it equals 0.

$$\sum_{k \in N} Y_{ij}^k D_k \leq S.Capl'_{ij} . X_{ij} \quad , (i, j) \in L \quad (14)$$

Based on this observation in the example, b_1 is equal to 4, a number from the set $\{1, 2, 3, 4\}$ is randomly selected for $Capl'_{ij}$. For example, if 2 is selected for the

link (i, j) , this link will not be constructed, or in case of construction, its capacity will be $2 * 20 = 40$. This is while in the proposed model; each link can select its capacity from the set $\{20, 40, 60, 80\}$. To determine the capacity of facilities for each node (given that $b_2 = 5, n = 20, m = 100$), a number from the set $\{100, 120, 140, 160, 180, 200\}$ is randomly selected. These values are proportional to the values of variables E_i and $Capf_i$ in our proposed model.

In CFLNDP, for example, if 140 is selected for node 5, no facility or a facility with capacity 140 will be located in this node, while in our model the capacity of each facility in each node can be each of the above values. Using this method, 20 random problems will be generated whose solutions obtained from both CFLNDP and the proposed model can be compared as shown in Table 2.

The results show that our model simultaneously decreases the total cost and clearly increases utilization of facilities and links. In fact, by optimum use of facilities and links capacity, the link construction and facility location costs are decreased. Although this observation can be expected due to the flexibility of the solution space in our proposed model compared to that in CLENDP, this example reveals that the improved values (average values of 63% and 27%, respectively, for increases in facility and link utilizations and 18% for total cost reduction) are quite considerable.

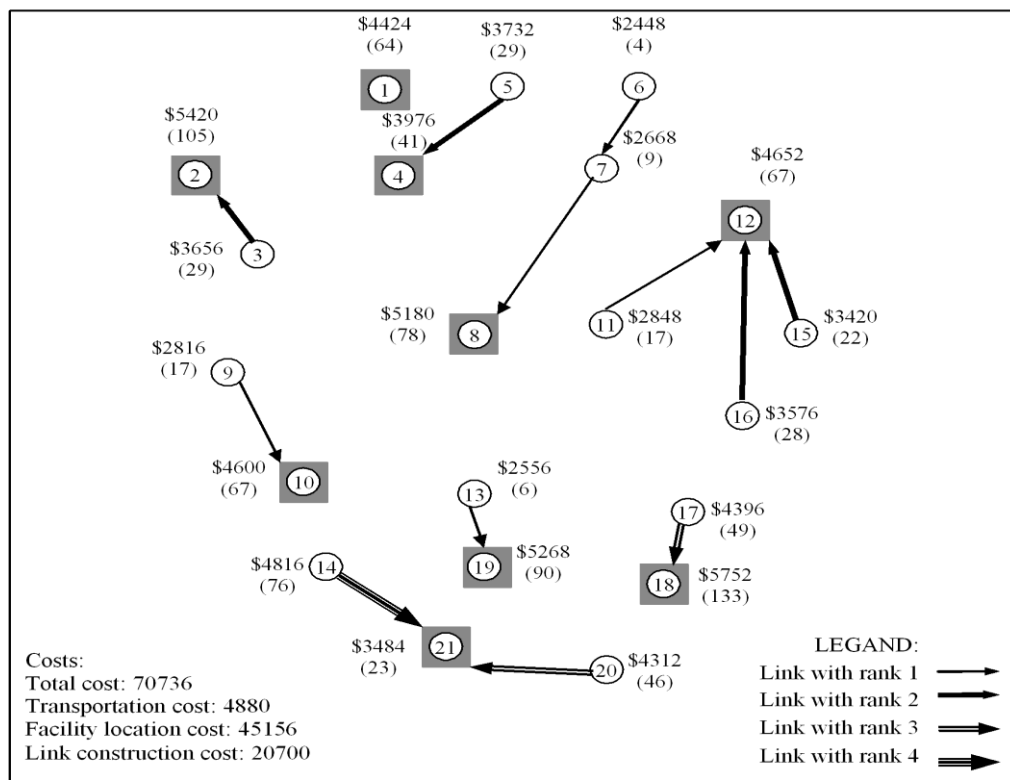


Fig. 2. Optimal solution

Tab. 2. Comparison of results obtained from proposed model and CFLNDP

CFLNDP	Prob.	Number of candidate						Number of located						Avg. facilities	Number of candidate				Number of constructe d				Avg. links	Total
	Num.	facilities with capacity:						facilities with capacity:						utilization (%)	links with capacity:				links with capacity:				utilization (%)	cost
		100	120	140	160	180	200	100	120	140	160	180	200		20	40	60	80	20	40	60	80		
	1	0	4	5	4	6	2	0	2	5	1	4	0	55.15	8	12	13	5	4	5	1	0	57.42	84246
	2	1	4	8	1	6	1	1	3	5	1	2	0	61.46	5	11	14	8	2	5	3	0	51.42	86914
	3	2	2	3	5	6	3	1	2	3	2	5	1	50.27	5	13	16	4	1	5	3	0	39.17	92678
	4	5	2	6	3	3	2	3	2	5	1	1	1	56.89	12	10	12	4	3	4	3	0	51.50	86258
	5	2	5	4	6	1	3	2	2	3	5	1	1	48.48	4	13	13	8	0	6	1	0	50.48	85174
	6	0	5	1	5	6	4	0	2	0	2	6	3	44.68	7	10	15	6	4	3	2	1	54.96	92266
	7	0	6	6	2	3	4	0	1	4	2	1	3	56.12	8	13	11	6	4	4	3	0	58.79	85110
	8	3	6	2	0	7	3	2	5	2	0	6	2	42.02	6	17	10	5	2	3	0	0	58.00	88568
	9	5	1	2	4	6	3	4	1	1	2	3	2	53.42	8	11	15	4	2	4	2	0	57.81	83564
	10	3	1	2	5	8	2	2	1	2	0	6	2	49.58	9	10	16	3	5	4	1	0	66.00	89920
	11	0	3	4	6	5	3	0	3	4	2	3	1	52.64	6	12	13	7	4	5	1	0	46.83	89064
	12	2	4	4	4	7	0	2	4	3	1	3	0	57.22	6	12	14	6	4	4	0	1	58.33	84890
	13	1	5	3	7	3	2	0	4	1	6	1	0	57.85	8	11	14	5	4	4	2	0	56.92	83566
	14	3	1	7	5	4	1	2	1	4	3	3	0	53.42	5	14	13	6	4	4	1	0	56.76	82418
	15	5	3	8	3	2	0	1	2	6	2	2	0	56.92	6	14	13	5	1	6	2	0	43.06	86558
	16	1	5	7	4	3	1	0	4	4	4	1	0	55.30	3	12	15	8	0	5	3	0	41.35	86732
	17	1	4	2	5	7	2	0	2	2	4	3	1	52.42	3	13	15	7	1	4	3	1	52.04	86344
	18	3	6	3	4	3	2	3	4	3	1	2	2	50.51	5	19	9	5	1	6	0	0	55.71	87444
	19	0	5	6	6	2	2	0	4	3	4	1	1	52.50	9	11	13	5	4	3	3	0	54.08	86924
	20	2	4	5	6	2	2	2	3	3	4	1	0	55.72	9	11	14	4	2	5	2	0	46.94	84998
	avg.	2	3.8	4.4	4.3	4.5	2.1	1.3	2.6	3.2	2.4	2.8	1	53.13	6.6	12	13	5.6	2.6	4.5	1.8	0.2	52.88	86681.8
Proposed Model	-	-	-	-	-	-		5	0	2	1	0	1	86.45	-	-	-	-	5	4	2	1	67.36	70736

3.2.2. Sensitivity Analysis with Respect to Facilities Location Cost (IC, ν)

To study the effect of increasing facilities location cost, (IC, ν) in the above example is multiplied by various coefficients (horizontal axes in the charts). By increasing (IC, ν), more links are constructed and demands are shipped out to other nodes. So, link construction and transportation costs are increased (Figures 4.d & 4.b). By increasing (IC, ν), the total facility location costs will increase with relevant fluctuations. When (IC, ν) is increased, the model initially suggests previous facilities or their equivalents to be located. If this trend continues, locating some facilities may no longer be economical (Figure 4.f).

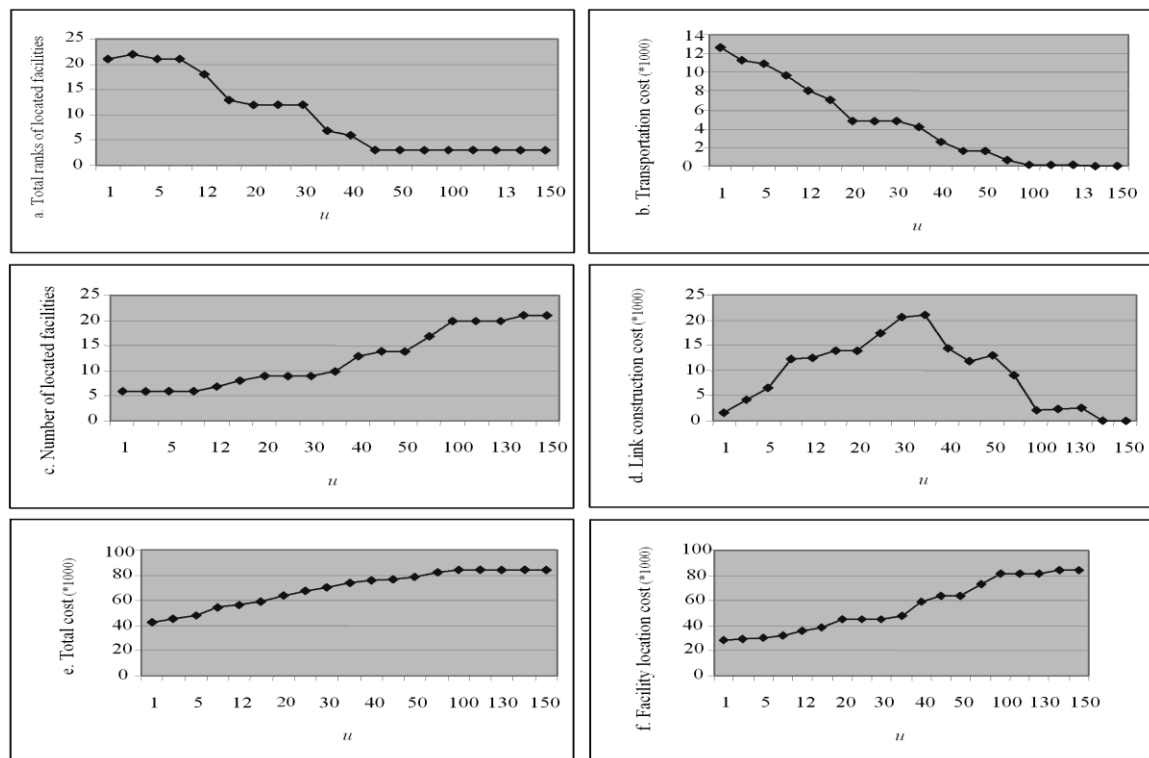
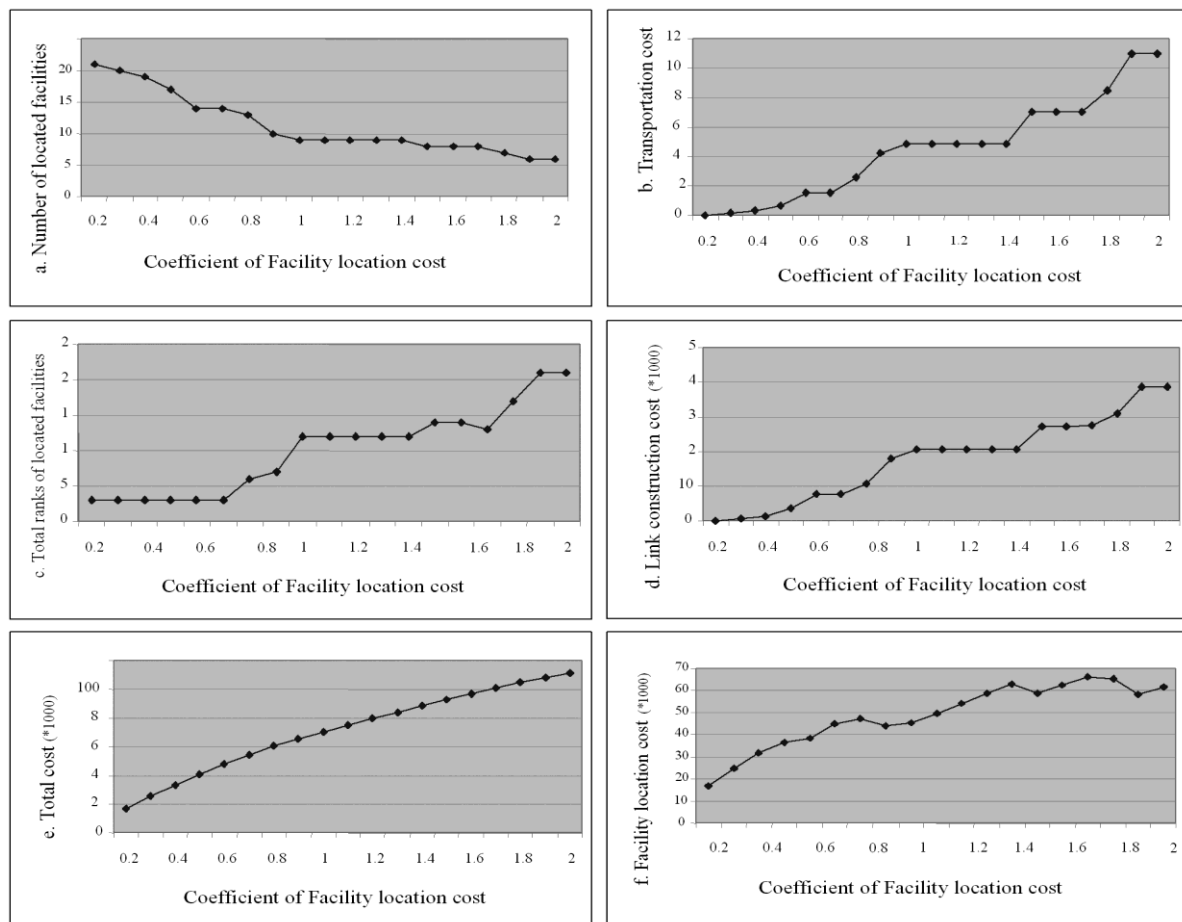
Figures 4.a and 4.c show that increasing (IC, ν) decreases the number of located facilities and the sum of facilities rank enhancements increases. Figure 4.e shows that by increasing (IC, ν), the total cost is increased.

3.3. Investigation of Model Efficiency with Respect to Computational Solution Time

In this part, we will employ almost the same approach to test problems as used by Balakrishnan et al. [24] for the UFLNDP problem. This approach can be explained as follows. At first, the location of each node is randomly determined in a network of the size 500*500 (For each node, x and y are randomly generated in the range [1,500]).

² Average facilities utilization in CFLNDP model is obtained from $\sum_i \sum_k [D_k W_i^k / K_i] / \sum_i Z_i$ (where K_i is capacity of facility that is located at node i , Z_i is equal to 1 if a facility is located at node i and otherwise equal to 0) and in proposed model from $\sum_{i \in \{N: E_i \neq 0\}} \sum_k [D_k W_i^k / (m E_i + n \text{Cap} f_i)] / \sum_i E_i$.

³ Average links utilization in CFLNDP model is calculated from $\sum_{(i,j) \in L} \sum_{k \in N} [Y_{ij}^k D_k / S \cdot \text{Cap} l'_{ij}] / \sum_{(i,j) \in L} X_{ij}$ (where X_{ij} is equal to 1 if link (i, j) is constructed and $i < j$, otherwise equal to 0) and in proposed model from $\sum_{(i,j) \in \{L: \text{Cap} l_j \neq 0\}} \sum_{k \in N} [Y_{ij}^k D_k / S \cdot \text{Cap} l_{ij}] / \sum_{(i,j) \in L} \min\{1, \text{Cap} l_{ij}\}$.

Fig. 3. Sensitivity analysis with respect to u Fig. 4. Sensitivity analysis with respect to (IC, v)

Then, node distances are calculated as *Euclidean* distances and the number of candidate links that are connectable to other nodes is generated from a Uniform [2,5] distribution. In other words, for each node an integer random number is generated which represents the number of candidate links connectable from that node to others in the network. Also, the demand on each node is generated randomly from a Uniform [50, 200] distribution. Basic facilities location cost is generated and normalized using a Uniform [2000, 6000] distribution so that their average is 4000. Other parameters are used as described in Table 1. Figure 5 shows the CPU time versus the number of nodes and the value of u . For node numbers less than 40, the CPU time is less than 1 second and for number of nodes less or equal to 200, the computation time is less than 1000 seconds. For a fixed value of u , as we can see, CPU time is almost exponentially increased with increasing number of nodes. Sensitivity analysis of the CPU time with respect to the value of u as an important parameter was also performed since the ratio of link construction cost to facility location cost can be controlled by changing the value of u . Numerical results show that CPU time generally decreases when the value of u increases.

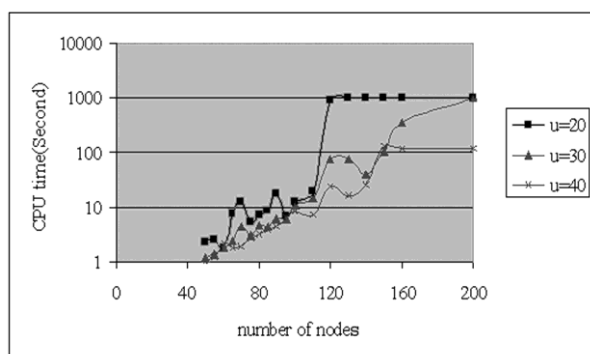


Fig. 5. CPU time versus number of nodes and values of u

4. Conclusion

In this paper, we proposed a MIP model to determine discrete optimal facilities and links capacities in FLNDP. The major difference between the present model and previous studies of FLNDP is that facilities and links capacity are here considered as decision variables, not as constraints. This approach increases the utilization of facilities and links, and prevents construction of links and facility locations of low utilization. Also to conform to real conditions, link construction (facility location) cost was considered as function of that link (facility) capacity. The proposed model was exhaustively investigated via numerical examples. Compared to CFLNDP, computational results showed that the proposed model would locate facilities and construct links of higher utilization, so that the total cost would decrease. The model is well sensitive to changes in input parameter

values. The numerical results also show that the CPU time varies almost exponentially with the number of nodes and, further, that problems up to 200 nodes are solvable in less than 17 minutes. With regard to the kind and nature of this model and its applications, it can be said that a problem with 200 nodes is a rather large one and the CPU time used is reasonable. Several extensions of the model are possible to enhance its applicability to a variety of real life transportation and distribution network planning scenarios. Some suggestions are: (1) the consideration of multi-commodity networks, and (2) the consideration of two directional links.

References

- [1] Wu, L.Y., Zhang, X., Zhang, J., *Capacitated Facility Location Problem with General Setup Cost*, Computers & Operations Research 2006; 33: pp. 1226-1241.
- [2] Sun, M., *Solving the Uncapacitated Facility Location Problem using Tabu Search*, Computers and Operations Research 2006; 33(9): pp. 2563 - 2589.
- [3] Mauricio, G.C., Resende, Renato F., Werneck, *A Hybrid Multistart Heuristic for the Uncapacitated Facility Location Problem* European Journal of Operational Research 2006; 174(1):54-68.
- [4] Klose, A., Görtz, S., *A Branch-and-Price Algorithm for the Capacitated Facility Location Problem*, Eur J Oper Res, 179, 2007, pp. 1109-1125.
- [5] Hochbaum, D.S. *Heuristics for the Fixed Cost Median Problem*. Mathematica Programming 1982; 22(1): pp. 148-162.
- [6] Jain, K., Mahdian, M., Saberi, A., *A New Greedy Approach for Facility Location Problems*. In Proceedings of the 34th Annual ACM Symposium on Theory of Computing, 2002.
- [7] Shmoys, D., Tardos, E., Aardal, K., *Approximation Algorithms for Facility Location Problems*. In Proceedings of the 29th Annual ACM Symposium on Theory of Computing, 1997, pp. 265-274.
- [8] Lin, J., Vitter, J., *ϵ Approximation with Minimum Packing Constraint Violation*. In Proceedings of 24th Annual ACM Symposium on Theory of Computing 1992; pp. 771-782.
- [9] Chudak, F., Shmoys, D., *Improved Approximation Algorithms for Capacitated Facility Location Problem*. In Proceedings of the 10th Annual ACM-SIAM Symposium on Discrete Algorithms, pages 1999; pp. 875-876.
- [10] Jain, K., Vazirani, V., *Approximation Algorithms for Metric Facility Location and k Median Problems using the Primal-Dual Schema and Lagrangian Relaxation*. Journal of ACM, 48(2): 2001, pp. 274-296.
- [11] Kuehn, A., Hamuburger, M.J., *A Heuristic Program for Locating Warehouses*. Management Sciences, 1963; 9: pp. 643-666.

- [12] Korupolu, M., Plaxton, C., Rajaraman, R., *Analysis of a Local Search Heuristic for Facility Location Problems*. Technical Report 1998; 98-30, DIMACS. Rutgers University, Piscataway, NJ.
- [13] Chen, C., Ting, C., *Combining Lagrangian Heuristic and Ant Colony System to Solve the Single Source Capacitated Facility Location Problem*, *Transportation Research Part E: Logistics and Transportation Review* 44, 2008, pp. 1099-1122.
- [14] Berman, O., Ingco, DI., Odoni, AR., *Improving the Location of Minisum Facilities Through Network Modification*. *The Annals of Operations Research* 1992; 40(1-4):pp. 1-16.
- [15] Campbell, JF., *A survey of Network Hub Location*. *Studies in Locational Analysis*, 6:31-49, 1994.
- [16] Melkote, S., Daskin, MS., *Capacitated Facility Location/Network Design Problems*, *European Journal of Operations Research* 2001; 129(3): pp. 481-495.
- [17] Melkote, S., Daskin, MS., *An Integrated Model of Facility Location and Transportation Network Design*, *Transportation Research Part A* 2001; 35(6): pp. 515-538.
- [18] Berger, RT., Coullard, CR., Daskin, MS., *Location-Routing Problems with Distance Constraints*. *Transportation Science* 2007; 41(1): pp. 29-43.
- [19] Drezner, Z., Wesolowsky, GO. *Network Design: Selection and Design of Links and Facility Location*, *Transportation Research Part A* 2003; 37(3): pp. 241-256.
- [20] Nemhauser, GL., Wolsey, LA., *Integer and Combinational Optimization*, Wiley, New York, 1988.
- [21] Hodgson, MJ., Rosing, KE., *A Network Location-Allocation Model Trading off Flow Capturing and p-Median Objectives*, *Ann of Oper Res* 1992; 40(1): pp. 247-260.
- [22] Simchi-Levi, D., Berman, O., *A heuristic Algorithm for the Traveling Salesman Location Problem on Networks*, *Oper Res* 1988; 36(3): pp. 478-484.
- [23] Daskin, M.S., Hurter, A.P., Van Buer, M.G. *Toward an Integrated Model of Facility Location and Transportation Network Design*, *Working Paper*, The Transportation Center, Northwestern University, Evanston, IL. 1993.
- [24] Balakrishnan, A., Magnanti, T.L., Wong, R.T., *A Dual Ascent Procedure for Large-Scale Incapacitated Network Design*, *Oper Res* 1989; 37(5): pp. 716-740.

