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Application of Decision on Beliefs for Fault Detection in uni-variate Statistical Process Control

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KEYWORDS

Statistical Quality Control,
Bayesian Rule,
Decision on Belief.

ABSTRACT

In this research, the decision on belief (DOB) approach was employed for fault detection in uni-variate process control. The concept of DOB and its application in decision making problems were introduced, and then methodology of modeling fault detection in statistical process control by DOB approach was discussed. In this iterative approach, the belief of being a fault in the process was updated by taking new observations on a quality characteristic using Bayesian rule and prior beliefs. If the beliefs are more than a specific threshold, then the system will be classified as an out-of-control condition. Finally, a numerical example and simulation study were provided for elaborating the application of the proposed methodology and evaluating the performance of the proposed method.

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1. Introduction

The value of Bayes' theorem, as a basis for statistical inference, has swung between acceptance and rejection since its publication in 1763, so that Bayes' mode of reasoning, which was finally buried on so many occasions, has recently risen again with astonishing vigor [1]. This research focuses on fault detection using decision on beliefs technique in statistical process control. Traditional SPC methods provide a group of statistical tests for a general hypothesis, in which the mean value for the quality characteristic of a process, or a process mean, for short, is consistent with its target level. A variety of graphical tools have been developed for monitoring a process mean by Shewhart charts [2], CUSUM charts [3], and EWMA charts [4]. The process mean is desired to be kept at its target level; however, random process errors can shift the process mean to an unknown level. A control chart is needed to detect this fault as soon as possible. At the same time, it should not signal too many false alarms when the process mean is on the target. These criteria are usually defined in terms of the Average Run Length of the control chart for the in-control operation and out-of-control operation of the process, i.e., in-control ARL and out-of-control ARL, respectively [5].

For the quality control of a manufacturing process, one essential task is to detect any possible abnormal change in the process mean and remove it. Consider a manufacturing process, in which detecting off-target state is very important and a control charting method is not sufficient. We present a dynamic programming approach for theses types of processes. This approach, named Decision on Belief (DOB), was firstly presented by Eshragh and Modarres [6]. Decision on Belief is a new optimization tool for decision making problems that is based on the Bayesian Inference. They applied this concept in distribution fitting problem, also Eshragh and Niaki applied DOB concept in Response Surface methodology [7 & 8]. Fallahnezhad and Niaki [9] applied this concept in the problem of determining the best binomial distribution. Also they applied this approach in acceptance sampling plans and production systems [10, 11].

Some researchers have applied Bayesian analysis in Quality control [12, 13]. Chun and Rinks [14] assumed that the proportion defective is a random variable that follows a Beta distribution and regarding Bayesian inference, they derived Bayes producer's and

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consumer's risks. Fallahnezhad and Niaki [14] proposed a new monitoring design for uni-variate statistical quality control charts based on an updating method. Marcellus [15] proposed a Bayesian statistical process control and compared it with CUSUM charts.

In this research, a control threshold policy along with dynamic programming approach and Bayesian inference is applied to detect the faults in uni-variate Statistical Process Control. Fallahnezhad and Nasab [16] applied control threshold policy in sampling plans. Naeini et al [17] applied Bayesian inference along with control threshold concept in control charts. The proposed method is based on the Normality of Data. Noorossana et al. [18] Proposed monitoring methods for Non-Normal data**.** The rest of the paper is organized as follows: DOB modeling for SPC is presented in Section 2. Section 3 provides the belief formulation and its updating method. A Decision on Beliefs Approach is discussed in section 4. The numerical demonstration on the proposed methodology is provided in section 5. a simulation experiment comes in Section 6. we concluded the paper in Section 7.

2. DOB Modeling for SPC Problems

In a uni-variate quality control environment, if we limit ourselves to apply a control charting method, most of the information obtained from data behavior will be ignored. The main aim of a control charting method is to detect quickly undesired faults in the process. However, we may calculate the belief for the process being out-of-control applying Bayesian rule at any iteration in which some observations on the quality characteristic are gathered. Regarding these beliefs and a stopping rule, we may find and specify a control threshold for these beliefs and when the updated belief in any iteration is more than this threshold, an out-of-control signal is observed. In Decision on Beliefs, first, all probable solution spaces will be divided into several candidates (the solution is one of the candidates), then a belief will be assigned to each candidate considering our experiences and finally, the beliefs are updated and the optimal decision is selected based on the current situation. In a SPC problem, a similar decision-making process exits. First, the decision space can be divided into two candidates; an in-control or out-of-control production process. Second, the problem solution is one of the candidates (in-control or out-of-control process).

Finally, a belief is assigned to each candidate so that the belief shows the probability of being a fault in the process. Based upon the updated belief, we may decide about states of the process (in-control or out-of-control process).

3. Learning: the Beliefs and Approach for Its Improvement

For simplicity, individual observation on the quality characteristic of interest in any iteration of data gathering process was gathered. At iteration k of data gathering process, $O_k = (x_1, x_2, ..., x_k)$ was defined as the observation vector where $x_1, x_2, ..., x_k$ resemble observations for previous iterations 1, 2, …, *k*. After taking a new observation, x_k, the belief of being in an out-of-control state is defined as $B(x_k, O_{k-1}) = \Pr\{Out-of-control | x_k, O_{k-1}\}$. At this iteration, we want to update the belief of being in out-of-control state based on observation vector O_{k-1} and new observation x_k. If we define $B(O_{k-1}) = B(x_{k-1}, O_{k-2})$ as the prior belief of an out-of-control state, in order to update the posterior belief $B(x_k, O_{k-1})$, since we may assume that the observations are taken independently in any iteration, then we will have:

$$\Pr\{x_k | Out-of-control, O_{k-1}\} = \Pr\{x_k | Out-of-control\}$$

With this feature, the updated belief is obtained using Bayesian rule:

$$B(x_k, O_{k-1}) = \Pr\{Out-of-control | x_k, O_{k-1}\} = \frac{\Pr\{Out-of-control, x_k | O_{k-1}\}}{\Pr\{x_k | O_{k-1}\}} = \frac{\Pr\{Out-of-control | O_{k-1}\}\Pr\{x_k | Out-of-control, O_{k-1}\}}{\Pr\{x_k | O_{k-1}\}} \quad (1)$$

Since in-control or out-of-control state partition the decsin space, we can write equation (1) as:

$$B(x_k, O_{k-1}) = \frac{\Pr\{Out-of-control | O_{k-1}\}\Pr\{x_k | Out-of-control\}}{\Pr\{Out-of-control | O_{k-1}\}\Pr\{x_k | Out-of-control\} + \Pr\{In-control | O_{k-1}\}\Pr\{x_k | In-control\}} = \frac{B(O_{k-1})\Pr\{x_k | Out-of-control\}}{B(O_{k-1})\Pr\{x_k | Out-of-control\} + (1-B(O_{k-1}))\Pr\{x_k | In-control\}} \quad (2)$$

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Assuming the quality characteristic of interest follows a normal distribution with mean μ and variance σ^2, we use equation (2) to calculate both beliefs for occurring positive or negative shifts in the process mean μ.

- **Positive shifts in the process mean**

The values of $B^+(O_k)$, showing the probability of occurring a positive shift in the process mean, will be calculated applying equation (2) recursively. $\Pr\{x_k | In-control\}$ is defined by the following equation,

$$\Pr\{x_k | In-control\} = 0.5$$

For positive shift, the probability of being a positive shift in the process at iteration *k*, $\Pr\{x_k | Out-of-control\}$, is calculated using equation (3).

$$\Pr\{x_k | Out-of-control\} = \varphi(x_k) \tag{3}$$

where $\varphi(.)$ is the cumulative probability distribution function for the normal distribution with mean μ and variance σ^2. Above probabilities are not exact probabilities and they are a kind of belief function to ascertain good properties for $B^+(O_k)$.

Therefore $B^+(O_k)$ is determined by the following equation,

$$B^+(O_k) = \frac{B^+(O_{k-1})\varphi(x_k)}{B^+(O_{k-1})\varphi(x_k) + 0.5(1-B^+(O_{k-1}))} \tag{4}$$

- **Negative shifts in the process mean**

The values of $B^-(O_k)$ denotes the probability of being a negative shift in the process mean that is calculated using equation (2) recursively. In this case, $\Pr\{x_k | In-control\}$ is defined by the following equation,

$$\Pr\{x_k | In-control\} = 0.5$$

Also $\Pr\{x_k | Out-of-control\}$ is calculated using equation (5).

$$\Pr\{x_k | Out-of-control\} = 1-\varphi(x_k) \tag{5}$$

Thus $B^-(O_k)$ is determined by the following equation,

$$B^-(O_k) = \frac{B^-(O_{k-1})(1-\varphi(x_k))}{B^-(O_{k-1})(1-\varphi(x_k)) + 0.5(1-B^-(O_{k-1}))} \tag{6}$$

4. A Decision on Beliefs Approach

We present a decision making approach in terms of Stochastic Dynamic Programming approach. Presented approach is like an optimal stopping problem.

Suppose *n* stages for decision making is remained and two decisions are available.

1. A positive shift is occurred in the process mean
2. No positive shift is occurred in the process mean

Decision making framework is as follows:

1. Gather a new observation.
2. Calculate the posterior Beliefs in terms of prior Beliefs.
3. Order the current Beliefs as an ascending form and choose the maximum.
4. Determine the value of the least acceptable belief ($d^+(n)$ is the least acceptable belief for detecting the positive shift and $d^-(n)$ is the least acceptable belief for detecting the negative shift)
5. If the maximum Belief in step 3 was more than the least acceptable belief, $d^+(n)$, select the belief candidate with maximum value as a solution else go to step 1.

In terms of above algorithm, the belief with maximum value is chosen and if this belief was more than a control threshold like $d^+(n)$, the candidate of that Belief will be selected as optimal candidate else the sampling process is continued. The objective of this model is to determine the optimal values of $d^+(n)$. The result of this process is the optimal strategy with *n* decision making stages that maximize the probability of correct selection.

Suppose new observation x_k is gathered. (*k* is the number of gathered observations so far). $V(n, d^+(n))$ is defined as the probability of correct selection when *n* decision making stages are remained and we follow $d^+(n)$ strategy explained above also $V(n)$ denotes the maximum value of $V(n, d^+(n))$ thus,

$$V(n) = \underset{d^+(n)}{Max} \{V(n, d^+(n))\}$$

CS is defined as the event of correct selection. S_1 is defined as selecting the out-of-control condition (positive shift) as an optimal solution and S_2 is defined as selecting the in-control condition as an optimal decision and *NS* is defined as not selecting any candidate in this stage.

Hence, using the total probability law, it is concluded that:

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$$V\left(n,d^{+}(n)\right)=Max\{\Pr\{CS\}\}=\Pr\{CS|S_1\}\Pr\{S_1\}+\Pr\{CS|S_2\}\Pr\{S_2\}+\Pr\{CS|NS\}\Pr\{NS\} \quad (7)$$

$\Pr\{CS|S_1\}$ denotes the probability of correct selection when candidate S_1 is selected as the optimal candidate and this probability equals to its belief, $B^{+}(O_k)$, and with the same discussion, it is concluded that $\Pr\{CS|S_2\}=1-B^{+}(O_k)$

1. $\Pr\{S_1\}$ is the probability of selecting out of control candidate (positive shift) as the solution thus following the decision making strategy, we should have $B^{+}(O_k)=\max(B^{+}(O_k),1-B^{+}(O_k))$ and $B^{+}(O_k)>d^{+}(n)$ that is equivalent to following,
$\Pr\{S_1\}=\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}$, $d^{+}(n)\in[0.5,1]$
With the same reasoning, it is concluded that,
$\Pr\{S_2\}=\Pr\left\{1-B^{+}(O_k)>d^{+}(n)\right\}$, $d^{+}(n)\in[0.5,1]$
2. $\Pr\{CS|NS\}$ denotes the probability of correct selection when none of candidates has been selected and it means that the maximum value of the beliefs is less than $d^{+}(n)$ and the process of decision making continues to latter stage. As a result, in terms of Dynamic Programming Approach, the probability of this event equals to maximum of probability of correct selection in latter stage *(n-1)*, $V(n-1)$, but since taking observations has cost, then the value of this probability in current time is less than its actual value and by using the discounting factor α, it equals $\alpha V(n-1)$
3. Since the entire solution space is partitioned, it is concluded that $\Pr\{NS\}=1-(\Pr\{S_1\}+\Pr\{S_2\})$

By the above preliminaries, the function $V(n)$ is determined as follows:

$$V(n)=\max_{0.5<d^{+}(n)<1}\left[\begin{array}{l}B^{+}(O_k)\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}+\\(1-B^{+}(O_k))\Pr\left\{1-B^{+}(O_k)>d^{+}(n)\right\}\\+\Pr\{CS|NS\}\\\left(\begin{array}{l}1-\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}-\\\Pr\left\{1-B^{+}(O_k)>d^{+}(n)\right\}\end{array}\right)\end{array}\right]$$

$$=\max_{0.5<d^{+}(n)<1}\left[\begin{array}{l}B^{+}(O_k)\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}+\\\left(1-B^{+}(O_k)\right)\Pr\left\{1-B^{+}(O_k)>d(n)\right\}\\+\alpha V(n-1)\left(\begin{array}{l}1-\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}\\-\Pr\left\{1-B^{+}(O_k)>d^{+}(n)\right\}\end{array}\right)\end{array}\right] \quad (8)$$

In terms of above equation, $V\left(n,d^{+}(n)\right)$ is obtained as follows:

$$V\left(n,d^{+}(n)\right)=\left[\begin{array}{l}B^{+}(O_k)\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}+\\\left(1-B^{+}(O_k)\right)\Pr\left\{(1-B^{+}(O_k))>d^{+}(n)\right\}\\+\alpha V(n-1)\left(\begin{array}{l}1-\Pr\left\{B^{+}(O_k)>d^{+}(n)\right\}-\\\Pr\left\{1-B^{+}(O_k)>d^{+}(n)\right\}\end{array}\right)\end{array}\right] \quad (9)$$

Calculation method for $V\left(n,d^{+}(n)\right)$:

$B^{+}(gr,O_k)$ and $B^{+}(sm,O_k)$ are defined as follows:

$B^{+}(gr,O_k)=\max\left\{B^{+}(O_k),1-B^{+}(O_k)\right\}$

$B^{-}(sm,O_k)=\min\left\{B^{+}(O_k),1-B^{+}(O_k)\right\}$

Now equation (9) is rewritten as follows:

$$V\left(n,d^{+}(n)\right)=\left(B^{+}(gr,O_k)-\alpha V(n-1)\right)\Pr\left\{B^{+}(gr,O_k)>d^{+}(n)\right\}+\left(B^{+}(sm,O_k)-\alpha V(n-1)\right)\Pr\left\{B^{+}(sm,O_k)>d^{+}(n)\right\}+\alpha V(n-1) \quad (10)$$

There are three conditions:

1. $B^{+}(gr,O_k)<\alpha V(n-1)$:

In this condition, both $B^{+}(gr,O_k)-\alpha V(n-1)$ and $B^{+}(sm,O_k)-\alpha V(n-1)$ are negative, thus we should have $d^{+}(n)=1$ in order to maximize $V\left(n,d^{+}(n)\right)$. since $B^{+}(gr,O_k)<d^{+}(n)=1$, we don't select any candidate in this condition and sampling process continues.

2. $B^{+}(sm,O_k)>\alpha V(n-1)$:

In this condition, both $B^{+}(gr,O_k)-\alpha V(n-1)$ and $B^{+}(sm,O_k)-\alpha V(n-1)$ are positive, thus we should have $d^{+}(n)=0.5$ in order to maximize $V\left(n,d^{+}(n)\right)$.

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Since $B^{+}(gr,O_k) > d^{+}(n) = 0.5$, we select the candidate of belief $B^{+}(gr,O_k)$ as the solution.

3. $B^{+}(sm,O_k) < \alpha V(n-1) < B^{+}(gr,O_k)$:

In this condition, one of the probabilities in equation (10) has positive coefficient and one has negative coefficient, to maximize $V\left(n,d^{+}(n)\right)$, optimality methods should be applied.

Definition: $h\left(d^{+}(n)\right)$ is defined as follows:

$$h\left(d^{+}(n)\right) = \frac{d^{+}(n)\left(1-B^{+}(O_{k-1})\right)}{\left(1-d^{+}(n)\right)B^{+}(O_{k-1})} \tag{11}$$

First the value of $\Pr\left\{B^{+}(O_k) > d^{+}(n)\right\}$ is determined as follows:

$$\begin{aligned}&\Pr\left\{B^{+}(O_k) > d^{+}(n)\right\} = \\ &\Pr\left\{\begin{matrix}\dfrac{\varphi(x_k)B^{+}(O_{k-1})}{\varphi(x_k)B^{+}(O_{k-1}) + \left(1-B^{+}(O_{k-1})\right)0.5} \\ > d^{+}(n)\end{matrix}\right\} \\ &= \Pr\left\{\varphi(x_k) > h\left(d^{+}(n)\right)0.5\right\} = \\ &1-0.5h\left(d^{+}(n)\right)\end{aligned} \tag{12}$$

Since $\varphi(x_k)$ is a cumulative distribution function thus it follows a uniform Distribution function in interval [0, 1], thus the above equality is concluded. With the same reasoning, it is concluded that:

$$\begin{aligned}&\Pr\left\{1-B^{+}(O_k) \ge d^{+}(n)\right\} = \\ &\Pr\left\{1-d^{+}(n) \ge B^{+}(O_k)\right\} \\ &= 0.5h\left(1-d^{+}(n)\right)\end{aligned} \tag{13}$$

Now equation (8) can be written as follows:

$$V(n) = \max_{0.5<d^{+}(n)<1}\left[\begin{matrix}B^{+}(O_k)\left(1-0.5h\left(d^{+}(n)\right)\right) + \\ \left(1-B^{+}(O_k)\right)0.5h\left(1-d^{+}(n)\right) \\ +\alpha V(n-1) \\ \left(\begin{matrix}1-0.5\left(1-h\left(d^{+}(n)\right)\right) - \\ 0.5h\left(1-d^{+}(n)\right)\end{matrix}\right)\end{matrix}\right] \tag{14}$$

And equation (10) can be written as follows:

$$\begin{aligned}&V\left(n,d^{+}(n)\right) = \\ &\left(B^{+}(O_k) - \alpha V(n-1)\right)\left(1-h\left(d^{+}(n)\right)0.5\right) + \\ &\left(1-B^{+}(O_k) - \alpha V(n-1)\right)0.5h\left(1-d^{+}(n)\right) \\ &+\alpha V(n-1)\end{aligned} \tag{15}$$

Since $V^{*}(n) = \underset{0.5<d^{+}(n)<1}{Max}\left[V\left(n,d^{+}(n)\right)\right]$ thus it is sufficient to maximize the real value function $V\left(n,d^{+}(n)\right)$, therefore; we should find the function value in points where The first derivative is equated to zero as follows,

$$\begin{aligned}&\frac{\partial V\left(n,d^{+}(n)\right)}{\partial d^{+}(n)} = 0 \Rightarrow \\ &-\frac{\left(B^{+}(O_k) - \alpha V(n-1)\right)}{\left(1-B^{+}(O_k) - \alpha V(n-1)\right)} = \frac{\left(1-d^{+}(n)\right)^2}{d^{+^2}(n)} \\ &\Rightarrow d^{+}(n) = \frac{1}{\sqrt{-\dfrac{\left(B^{+}(O_k) - \alpha V(n-1)\right)}{\left(1-B^{+}(O_k) - \alpha V(n-1)\right)}} + 1}\end{aligned} \tag{16}$$

The optimal threshold $d^{+}(n)$ is determined by the above equation. Since the optimal value of $d^{+}(n)$ should be in the interval [0.5, 1] thus it is concluded that the optimal value of $d^{+}(n)$ will be determined as follows:

$$d^{+}(n) = Max\left[\frac{1}{\sqrt{-\dfrac{\left(B^{+}(O_k) - \alpha V(n-1)\right)}{\left(1-B^{+}(O_k) - \alpha V(n-1)\right)}} + 1}, 0.5\right] \tag{17}$$

The above method is presented for detecting the positive shifts for the process mean and can be adapted for detecting the negative shifts with the same reasoning.

The general decision making algorithm is summarized as follows:

1. Set k=0 and the initial beliefs $B^{+}(O_0) = 0.5, B^{-}(O_0) = 0.5$.
2. Gather an observation and set $k = k+1, n = n-1$.
3. If $n < 0$, then no shift is occurred in the process mean and decision making stops.
4. Update the values for the beliefs $B^{-}(O_k), B^{+}(O_k)$ by equation (2).

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5. If $Min\left(B^{+}(O_k),1-B^{+}(O_k)\right)>\alpha V\ (n-1)$, then if $Max\left(B^{+}(O_k),1-B^{+}(O_k)\right)=B^{+}(O_k)$, it is concluded that a positive shift is occurred in the process mean and decision making stops, also if $Max\left(B^{+}(O_k),1-B^{+}(O_k)\right)=1-B^{+}(O_k)$, then no positive shift is occurred in the process mean and decision making stops.
6. If $Max\left(B^{+}(O_k),1-B^{+}(O_k)\right)<\alpha V\ (n-1)$, then data is not sufficient for detecting the positive shift and go to

- stage 2.

7. If $Min\left(B^{-}(O_k),1-B^{-}(O_k)\right)>\alpha V\ (n-1)$ then if $Max\left(B^{-}(O_k),1-B^{-}(O_k)\right)=B^{-}(O_k)$ it is concluded that a negative shift is occurred the process mean and decision making stops and if $Max\left(B^{-}(O_k),1-B^{-}(O_k)\right)=1-B^{-}(O_k)$, then no negative shift is occurred in the process mean and decision making stops.
8. If $Max\left(B^{-}(O_k),1-B^{-}(O_k)\right)<\alpha V\ (n-1)$, then data is not sufficient for detecting the negative shift and go to

- stage 2.

9. If $Max\left(B^{+}(O_k),1-B^{+}(O_k)\right)> \alpha V\ (n-1)>Min\left(B^{+}(O_k),1-B^{+}(O_k)\right)$, then determine the value for $d^{+}(n)$ (least acceptable belief for detecting the positive shift) by the following equation:

$$d^{+}(n)= Max\left(\frac{1}{\sqrt{-\frac{\left(B^{+}(O_k)-\alpha V\ (n-1)\right)}{\left(1-B^{+}(O_k)-\alpha V\ (n-1)\right)}+1}},0.5\right) \quad (18)$$

10. If $Max\left(B^{-}(O_k),1-B^{-}(O_k)\right)>\alpha V\ (n-1)> Min\left(B^{-}(O_k),1-B^{-}(O_k)\right)$,

then determine the value for $d^{-}(n)$ (least acceptable belief for detecting the negative shift) by the following equation:

$$d^{-}(n)= Max\left(\frac{1}{\sqrt{-\frac{\left(B^{-}(O_k)-\alpha V\ (n-1)\right)}{\left(1-B^{-}(O_k)-\alpha V\ (n-1)\right)}+1}},0.5\right) \quad (19)$$

11. If $B^{+}(O_k)>d^{+}(n)$, then a positive shift is occurred and decision making stops, and if $\left(1-B^{+}(O_k)\right)>d^{+}(n)$, then no positive shift is occurred and decision making stops, else go to stage 2.
12. If $B^{-}(O_k)>d^{-}(n)$, then a negative shift is occurred and decision making stops, and If $\left(1-B^{-}(O_k)\right)>d^{-}(n)$, then no negative shift is occurred and decision making stops, else go to stage 2.

The approximate value of $\alpha V\ (n-1)$ based on the discount factor α in the stochastic dynamic programming approach is $\alpha^{n}V\ (0)$.

5. Numerical Example

A numerical example is provided to detect the positive and negative shift of mean for the process. We assume that a quality characteristic in a process follows the standard normal distribution and a shift $\delta=1$ is occurred in the process mean. It was assumed that sampling has a cost thus 0nly 15 observations could be sampled (n=15), also the values of $\alpha=0.97$ and $V\ (0)=0.95$ were selected as the parameter values.
The initial value for the out-of-control or in-control beliefs is equal to 0.5.

$$B^{+}(O_0)=0.5, B^{-}(O_0)=0.5$$

To show the process of decision making on Beliefs, random numbers of normal distribution with parameters $\mu=0,\delta=1,\sigma=1$ were generated.

5.1. First Observation:

Step 1: Observation $x_1=0.25$ is gathered.

Step 2: Posterior belief are calculated.

$$B^{+}(O_1)=0.59$$
$$B^{-}(O_1)=0.41$$

Step 3: Order the belief.

$$B^{+}(gr,O_1)=B^{+}(O_1)>1-B^{+}(O_1)=B^{+}(sm,O_1)$$
$$B^{-}(gr,O_1)=1-B^{-}(O_1)>B^{-}(O_1)=B^{-}(sm,O_1)$$

Step 4: Since $B^{+}(gr,O_1),B^{-}(gr,O_1)$ are less than $\alpha^{15}V\ (0)=0.6$, thus we are in the condition 1 of decision making method. Hence, $d^{+}(15)=1$.

Step 5: Since $B^{+}(gr,O_1),B^{-}(gr,O_1)$ are less than $d^{+}(15)=1$, then there is not any selection. Therefore, another observation will be selected.

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5.2. Second Observation:

Step 1: Observation $x_2 = 0.24$ is gathered

Step 2: Posterior beliefs are calculated.

$$B^+(O_2) = 0.67$$
$$B^-(O_2) = 0.33$$

Step 3: Order the beliefs.

$$B^+(gr,O_2) = B^+(O_2) > 1 - B^+(O_2) = B^+(sm,O_2)$$
$$B^-(gr,O_2) = 1 - B^-(O_2) > B^-(O_2) = B^-(sm,O_2)$$

Step 4:

Since $B^+(gr,O_2) > \alpha^{14}V(0) = 0.620 > B^+(sm,O_2)$ and $B^-(gr,O_2) > \alpha^{14}V(0) = 0.620 > B^-(sm,O_2)$, thus we are in the condition 3 of decision making process, hence first, the values of $d^+(14)$ and $d^-(14)$ are determined by equation (17):

$$d^+(14) = \max\left(\frac{1}{\sqrt{-\frac{B^+(O_2) - \alpha^{14}V(0)}{1 - B^+(O_2) - \alpha^{14}V(0)} + 1}} = \frac{1}{\sqrt{-\frac{0.67 - 0.62}{0.33 - 0.62} + 1}}, 0.5 \right) = 0.7$$

$$d^-(14) = \max\left(\frac{1}{\sqrt{-\frac{B^-(O_2) - \alpha^{14}V(0)}{1 - B^-(O_2) - \alpha^{14}V(0)} + 1}} = 0.29, 0.5 \right) = 0.5$$

Step 5:

Since $B^+(gr,O_2) = B^+(O_2) = 0.67 < d^+(14) = 0.7$ and $B^-(gr,O_2) > d^-(14) = 0.5$, then it is concluded that no negative shift is occurred in the process mean but additional observations are needed for decision making about the positive shifts**.**

5.3. Take the Third Observation:

Step 1: Observation $x_3 = 0.26$ is gathered.

Step 2: Posterior beliefs are calculated.

$$B^+(O_3) = 0.75$$

Step 3: the beliefs are ordered.

$$B^+(gr,O_3) = B^+(O_3) > 1 - B^+(O_3) = B^+(sm,O_3)$$

Step 4:

Since $B^+(gr,O_3) > \alpha^{13}V(0) = 0.639 > B^+(sm,O_3)$, thus we are in the condition 3 of decision making process, hence first, the value of $d^+(13)$ is determined by equation (17):

$$d^+(13) = Max\left[\frac{1}{\sqrt{-\frac{B^+(O_3) - \alpha^{13}V(0)}{1 - B^+(O_3) - \alpha^{13}V(0)} + 1}} = \frac{1}{\sqrt{-\frac{0.75 - 0.639}{0.25 - 0.639} + 1}}, 0.5 \right] = 0.64$$

Step 5:

Since $B^+(gr,O_3) = B^+(O_3) = 0.75 > d^+(13) = 0.64$, thus it is concluded that a positive shift is occurred in the process mean.

This numerical example is solved by using simulated data but the decision making steps are the same when data from real case studies are available.

6. Simulation Experiment

For the simulation methodology, the standard normal observations were generated. Then, the proposed procedure was implemented for the simulated data in different iterations.

Table 1 shows the estimated probabilities of detecting a positive shift in the proposed methodology. Five different parameter sets with 10,000 independent replications of the process were selected.

Table (1) shows that the overall values of probabilities are favorably large.

Further, as the magnitudes of the mean shifts increase, the probabilities become larger. Also, when the value of α increases, the probability of first type error, denoting the probability of detecting a positive shift while no shift is occurred in the process mean, decreases and the probability of second type error decreases in most of the cases .

Also as the number of decision making stage (n) decreases, the probability of first type error decreases but the probability of second type error increases simultaneously therefore the optimal value of n should be selected based on the tradeoff between second type error and first type error. Also as $V(0)$ decreases, the probability of first type error increases but the probability of second type error decreases in small value of shifts and increases in large values of shifts therefore the optimal value of $V(0)$ should be selected based on out of control value of mean that should be detected.

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Tab. 1. The Probabilities of detecting a positive shift in simulation experiments

Mean Shifts	$\alpha=0.95$, $V(0)=0.99$, $n=10$	$\alpha=0.99$, $V(0)=0.99$, $n=10$	$\alpha=0.9999$, $V(0)=0.99$, $n=10$	$\alpha=0.9999$, $V(0)=0.99$, $n=7$	$\alpha=0.99$, $V(0)=0.99$, $n=20$	$\alpha=1$, $V(0)=0.999$, $n=10$	$\alpha=1$, $V(0)=0.999$, $n=20$	$\alpha=1$, $V(0)=0.99$, $n=20$
0	0.465853	0.429657	0.29557	0.20298	0.486551	0.15088	0.307369	0.436856
0.25	0.60144	0.649935	0.555644	0.39816	0.679532	0.35556	0.687731	0.767023
0.5	0.725527	0.832617	0.79952	0.624738	0.816418	0.63674	0.927207	0.944806
0.75	0.818818	0.934007	0.938206	0.820218	0.905709	0.86181	0.993401	0.991701
1	0.890411	0.978402	0.989001	0.933607	0.956804	0.96480	0.9997	0.998
1.25	0.934307	0.991801	0.997	0.983002	0.978402	0.99440	0.9997	0.9997
1.5	0.960104	0.9962	0.9995	0.9956	0.990001	0.99930	0.9999	0.9997
1.75	0.976602	0.998	0.9999	0.9996	0.9955	0.99970	0.9999	0.9999
2	0.989501	0.9994	0.9999	0.9999	0.9974	0.99990	0.9999	0.9999
2.25	0.994201	0.9997	0.9999	0.9998	0.9991	0.99990	0.9999	0.9999
2.5	0.9973	0.9997	0.9999	0.9999	0.9995	0.99990	0.9999	0.9999
2.75	0.9987	0.9999	0.9999	0.9999	0.9999	0.99990	0.9999	0.9999
3	0.9995	0.9999	0.9999	0.9999	0.9999	0.99990	0.9999	0.9999

Tab. 2. The ARL values in simulation experiments

Mean Shifts	$\alpha=0.95$, $V(0)=0.99$, $n=10$	$\alpha=0.99$, $V(0)=0.99$, $n=10$	$\alpha=0.9999$, $V(0)=0.99$, $n=10$	$\alpha=0.9999$, $V(0)=0.99$, $n=7$	$\alpha=0.99$, $V(0)=0.99$, $n=20$	$\alpha=1$, $V(0)=0.999$, $n=10$	$\alpha=1$, $V(0)=0.999$, $n=20$	$\alpha=1$, $V(0)=0.99$, $n=20$
0	1.54	3.29	5.05	3.90	2.77	6.13	10.36	7.90
0.25	1.54	3.30	4.96	3.84	2.72	6.01	9.64	7.41
0.5	1.45	3.04	4.60	3.72	2.45	5.68	8.01	5.90
0.75	1.38	2.62	4.01	3.45	2.04	5.11	6.03	4.39
1	1.29	2.19	3.24	3.09	1.79	4.40	4.67	3.43
1.25	1.21	1.85	2.74	2.68	1.54	3.71	3.77	2.76
1.5	1.15	1.60	2.33	2.32	1.37	3.11	3.11	2.35
1.75	1.09	1.43	2.03	2.02	1.26	2.70	2.69	2.04
2	1.06	1.29	1.79	1.81	1.17	2.36	2.36	1.80
2.25	1.04	1.20	1.62	1.61	1.11	2.10	2.11	1.63
2.5	1.02	1.13	1.47	1.47	1.07	1.89	1.89	1.47
2.75	1.01	1.08	1.34	1.35	1.04	1.72	1.72	1.36
3	1.01	1.05	1.24	1.25	1.02	1.57	1.56	1.25

Table (2) shows that the overall values of average run length (ARL) are favorably small. Further, as the magnitudes of the mean shifts increase, the ARL values become smaller. Also, when the value of α increases, both values of ARL_0 and ARL_1 increases. Also as the number of decision making stage (n) decreases, both values of ARL_0 and ARL_1 decreases.

Also as $V(0)$ decreases, both values of ARL_0 and ARL_1 decreases**.** Therefore it is concluded that optimal parameter should be selected based on the required application and performance.

7. Conclusions

In this paper, the concept of decision on belief (DOB) approach is employed to analyze and classify the states of uni-variate quality control systems. In this method, we tried to update the beliefs of being a fault in system by taking new observations on the given quality characteristic. Decision making is based on comparing the values of beliefs with a control threshold. The value of control threshold is determined by solving an optimization problem. The optimal values of control thresholds are determined in order to maximize the belief of selecting correct decision. A numerical example is presented to illustrate how the proposed procedure can be applied

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to design a control method. Also, in simulation experiment, the performance of proposed method for detecting the positive shifts of the process mean is evaluated and it is denoted that the performance of proposed method is satisfactory in detecting the shifts of the process mean. As shown in simulation study, the probabilities of first and second type error and ARL values for out-of-control state are favorably small. Since these performance criteria are the most important characteristics of any control chart therefore we can justify the applicability and usefulness of proposed approach.

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