**Cellular manufacturing systems considering the cooperation of workers applying a bi-objective programming approach**

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**Abstract**

Generally, human resources play an important role in manufacturing systems as they can affect the work environment. One of the most important factors affecting the human resources is being an interactional interest among the workers in the workshops. If the workers in a manufacturing cell have the highest surface of the interactional interest level, it causes a significant raise in coordination and cooperation indicators and in long time periods. In this paper, a new concept of being an interactional interest between workers in a manufacturing cell besides the ability to work with its machines is proposed and a bi-objective mathematical model to carry out this new point of view in cellular manufacturing systems is presented. Applying the ε-constraint method as an optimization tool for multi-objective mathematical programming, a comprehensive numerical example is solved to exhibit the capability of the presented model.

**Keywords:** Cellular manufacturing; Worker; Bi-objective programming approach; ε-constraint method

**1. Introduction**

Group technology (GT) is a manufacturing method that has attracted a lot of attention because of its positive effects in batch-type production. GT, as a philosophy that exploits similarities/ dissimilarities in product design and product processes, was first introduced by Mitrofanov (1966). Cellular manufacturing is one of the applications of the concept of GT in manufacturing systems. In the design of a cellular manufacturing system, similar parts are grouped into part families and dissimilar machines into manufacturing cells so that one or more part families can be processed within a single machine group. The major advantages of cellular manufacturing reported in the literature contain reduction in setup time, reduction in throughput time, reduction in work-in-process inventories, reduction in material handling costs, better quality and production control, increment in flexibility, etc. (Heragu 1994 and Wemmerlov and Hyer 1989). The cell formation problem (CFP) is one of the major issues in the designing of cellular manufacturing systems.

CFP in a given 0–1 part-machine incidence matrix includes a rearrangement of rows and columns of the matrix to make part families and machine cells. The main objective of cell formation is to create machine cells and part families, and then dispatch part families to machine cells. In the last three decades of research, many solution methods have mainly used zero–one machine component incidence matrix as the input data for the CFP such as hierarchical methods, non-hierarchical methods, production flow analysis, genetic algorithms, simulated annealing, neural networks, mathematical model, meta-heuristics algorithms and etc. which comprehensive summaries and taxonomies were shown in Mansouri et al. (2000), Yin and Yasuda 2006, Ghosh et al. (2010 a,b) and Papaioannou and Wilson (2010).

Mahdavi et al. (2009) developed a mathematical model for the cell formation problem based on cell utilization concept in the cell formation problem. An efficient algorithm based on genetic algorithm was designed to solve the mathematical model. Wu et al. (2009) proposed a hybrid heuristic algorithm employing both the Boltzmann function from simulated annealing and the mutation operator from the genetic algorithm to explore the unvisited solution region and expedite the solution searching process for the cell formation problem, so that grouping efficacy is maximized. Noktehdan et al. (2010) proposed a grouping version of differential evolution algorithm and its hybridized version with a local search algorithm to solve benchmarked instances of cell formation problem posing as a grouping problem. Pailla et al. (2010) presented two approaches to solve the manufacturing cell formation problem. Firstly, an evolutionary algorithm was introduced that improves the efficiency of the standard genetic algorithm by considering cooperation with a local search around some of the solutions it visits. Secondly, an approach based on simulated annealing was implemented that uses the same representation scheme of a feasible solution. Díaz et al. (2010) proposed a greedy randomized adaptive search procedure (GRASP) heuristic to obtain lower bounds for the optimal solution of the cell formation problem. Their method consists of two phases. In the first phase an initial partition of machines into machine-cells or parts into part families is obtained, while in the second phase the assignment of parts to machine cell or machines to part-families is considered. Paydar et al. (2010) formulated the cell formation problem as a single depot multiple travelling salesman problem (SDmTSP). Arkat et al. (2011) presented a multi-objective programming model with the aim of minimizing the number of exceptional elements and the number of voids, simultaneously. They have also developed a bi-objective genetic algorithm for large-scale problems. Feng and Pheng (2011) proposed an exact schema theorem that is able to predict the expected number of copies of schemas in the next GA generation which applied for machine cell formation.

Despite a large number of published researches on CFP, all are limited to a two-dimensional representation of CFP and consider machines and parts. In a real factory situation, not only grouping of machines and parts may lead to a better productivity, assignment of workers also has a major role in the utilization of manufacturing resources.

Min and Shin (1993) created a prototype of three-dimensional GT. Their method was to insert the third factor, operator, into the sorted incidence matrix of parts and machines. Parkin and Li (1997) proposed an algorithm for N-dimensional GT. Their algorithm focused on each incidence matrix, sorting each separately. Li (2003) showed a method of solving multi-dimensional GT problem. This method can consider all incidence matrices at the same time without considering each incidence matrix separately. Mahdavi et al. (2012) addressed a mathematical model for cell formation problem based on a three-dimensional machine-part-worker incidence matrix which minimizes total number of exceptional elements and voids in a cellular manufacturing system.

Generally, human resources play an important role in manufacturing systems as they can affect the work environment and vice versa they can be influenced by that. One of the most important factors affecting the human resources is being an interactional interest among the workers in the workshops. If the workers in a manufacturing cell have the highest surface of the interactional interest level, it cause a significant raise in coordination and cooperation indicators and in long time periods we can see also interactional training aspects. We represent the interactional interest through a binary square matrix called *interest matrix* having up to the number of workers for rows and columns. The interest matrix shows which pair workers have interactional interest and which pair’s don’t. Moreover, the relationship between the workers and the machines is represented by a binary matrix called *task matrix* having up to the number of workers for rows and the number of machines for columns. The task matrix shows each worker can work with which machines. With respect to these preambles in a flexible cellular manufacturing system, we seek to find an optimal assignment of the workers and machines to the cells so as to both workers could work with their entire cell machines and have the highest level of interactional interest simultaneously. So, the objective function consists of two parts, the first part contains the number of voids corresponding to the worker-machine incidence matrix or the same task matrix. A void in the worker-machine incidence matrix represents a gap from ideal conditions where the cell reliability is in a high level. If all workers could work with entire their cell machines, in absence worker situation, other workers can support the idle machine, hence, the cell reliability increases and jobs with high priority can be continued on that machine likewise. The Second part contains the number of voids corresponding to the worker-worker incidence matrix or the same interest matrix. Ideally we seek to conditions workers have the highest surface of interactional interest in their own cells. This cause increase in cooperation and coordination measurements and finally cell synergy rises. So the first objective function leads to increase in the cell reliability measure and the second function causes the raise in the cell synergy measure, thus it is obvious that these two objectives are nonhomogeneous. This issue illustrates the necessity of simultaneously minimizing the number of voids corresponding to task and interest matrixes; hence in this paper a bi-objective optimization approach has been investigated to reach the optimal solutions.

The reminder of the paper is organized as follows. In section 2, the mathematical notations are used throughout the paper are presented. In sections 3, a bi-objective mathematical model for minimizing the number of voids in both worker-machine and worker-worker incidence matrixes simultaneously is proposed. In section 4, exact ε-constraint method for bi-objective combinatorial problems has been described as solution method. In section 5 an example is presented in medium scale to show the performance of the proposed bi-objective mathematical model. Finally we conclude this paper in section 6.

**2. Mathematical notation**

The notations, parameters and decision variables applied throughout in this paper are presented below.

**2.1. Indices**

: Number of workers

 : Number of machines

 : Number of cells

 : Index of workers

 : Index of machines

 : Index of cells

**2.2. Parameters**

: 1 if worker can work on machine; 0 otherwise

: 1 if worker and  have interactional interest; 0 otherwise

: The lower bound for workers to be assigned to each cell

: The lower bound for machines to be assigned to each cell

**2.3. Decision variables**

: 1 if worker  is assigned to cell; 0 otherwise

: 1 if machine is assigned to cell; 0 otherwise

**3. Mathematical model**

In this section, a bi-objective mathematical model that the first objective function computes the total number of voids corresponding to the task matrix and the second objective function computes the total number of voids corresponding to the interest matrix is developed. The nonlinear programming model is proposed below.

|  |  |
| --- | --- |
| *Min* | (1-1) |
| *Min* | (1-2) |

*Subject to*

(2)

(3)

(4)

(5)

(6)

(7)

The first objective function corresponds to the total number of cases that a worker cannot work with a machine and the second objective function refers to the total number of cases that a pair of workers has no interactional interest. Constraints (2) ensure that each worker is assigned to only one cell. Constraints (3) guarantee that each worker is assigned to only one cell. Constraints (4) enforce the lower bound on the number of workers to be assigned to each cell. Constraints (5) enforce the lower bound on the number of machines to be assigned to each cell. Constraints (6) ensure that each worker be able to work with at least one machine on his/ her cell. Constraints (7) specify that decision variables are binary.

**3.1. Linearization of the proposed model**

Here, we linearize the objective functions and constraint (6) of the mathematical model proposed in section 3.The nonlinear terms are multiplication of binary variables which can be linearized using the auxiliary binary variables and. The validity of each linearization is established by lemmas.

**Lemma1.** First nonlinear part of the objective function and constraint (6) can be linearized with, under the following sets of constraints:

(8)

(9)

**Proof1.** Consider the following two cases:

Case1., 

Such a situation arises when. So, constraint (8) implies, ensuring that.

Case2.,

Such a situation arises under one of the following three subcases:

1.  and
2. and 
3. and 

In all of these subcases, we have, because constraint (9) implies, to ensure that.

Since does not have a strictly positive cost coefficient, the minimizing objective function does not ensure that. Thus, constraint (9) should be added to the mathematical model.

**Lemma2.** Second nonlinear part of the objective function can be linearized with, under the following sets of constraints:

(10)

(11)

**Proof2.** The proof is similar to proof1.

**3.2. The linearized model**

We now present the linear mathematical model as follows:

Min  (1-1)

Min (1-2)

Subject to

(2) – (5), (8)-(11) and

(12)

(13)

**4. Solution method**

Bi-objective combinatorial optimization (BOCO) problems are a special case of the multi-objective combinatorial optimization (MOCO) problems. General BOCO problems are formulated as

Min  such that,

where  is the set of feasible solutions, or the solution space. The reader is referred to Ehrgott and Gandibleux (2002) for a review of the literature on multi-objective combinatorial optimization (MOCO) problems. Among the methods to find the Pareto front of MOCO problems, weighted sum scalarization is the most popular according to Ehrgott and Gandibleux (2002). This method solves different single objective sub problems generated by a linear scalarization of the objectives. By varying the weights of this linear function, all supported non-dominated points can be found. Besides weighting sum algorithms, the ε-constraint method is the best known approach for solving MOCO problems, according to Ehrgott and Gandibleux (2002). This method generates single objective sub problems, called ε-constraint problems, by transforming all but one objectives into constraints. The upper bounds of these constraints are given by the ε-vector and, by varying it; the exact Pareto front can theoretically be generated. In practice, because of the high number of sub problems and the difficulty to establish an efficient variation scheme for the ε-vector, this approach has mostly been integrated within heuristic and interactive schemes. It can however generate the exact Pareto front in particular situations.

**4.1. Exact ε-constraint method for BOCO problems**

The ε-constraint is probably the best known technique to solve multi-objective discrete optimization problems. It guarantees the exact set of the efﬁcient solutions. It solves ε-constraint problemsobtained by transforming one of the objectives into a constraint. This method was introduced by Haimes et al. (1971), and an extensive discussion can be found in Chankong and Haimes (1983).For the bi-objective case, the problemsand are:

Min 

Such that



Min 

Such that

.

**Theorem3.**is an efficient solution of a BOCO problem if and only ifsuch that **** solves or  such that ****solves.

**Throrem4.** If **** solves or and if the solution is unique, then **** is an efficient solution of a BOCO problem.

Theorems 1 and 2 have been proved for general multi-objective problems (see Chankong and Haimes1983 and Miettinen1999) and are therefore valid for the BOCO problems. These theorems mean that efficient solutions can always be found by solving ε-constraint problems, as long asis such that is feasible or  is such that  is feasible. Let the objective space be defined by and being the ideal points and being the nadir points defining the lower and upper bounds on the value of efficient solutions, respectively. Algorithm 1 below finds the Pareto front of BOCO problems with integer objective values through a sequence of ε-constraint problems. In the algorithm, is decreased by a constant value (here set to 1).As explained later, may sometimes be larger to strengthen the ε-constraint.

**Algorithm1.**Exact Pareto front of BOCO problems with integer objective values.

1. Set  or.
2. Compute the ideal and nadir points.
3. Set and.
4. Whiledo
   1. Solveby branch-and-cut and add the optimal solution to.
   2. Set.
5. Remove dominated points from if required.

**5. Computational results**

In this section an example is presented to illustrate the proposed bi-objective model using branch-and-cut method by the Lingo 9 software package on an Intel® Core (TM) i52.4 GHz Personal Computer with 4 GB RAM.

Consider three cells, nine workers and nine machines. The data set related to the task matrix and interest matrix is shown in Tables 1 and 2, respectively. Table 1 indicates capabilities of workers in working on different machines. For example, worker 9 is able to work on machine types 1– 4 and 7. Table 2 indicate sexist of interactional interest between workers. For example, worker type 4 has interest to cooperate with workers 1, 2 and 6.Moreover, the minimum size of each cell in terms of the number of workers and machines is assumed to be 1.

**Table1. Worker-Machine incidence matrix**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Machine | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Worker | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 5 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 8 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 9 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |

**Table2. Worker-Worker interest matrix**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Worker | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Worker | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 4 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 6 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 7 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 8 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 9 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |

After performing Algorithm 1, the Pareto optimal solutions are found to be:

1. The first Pareto optimal point isandwith the solution shown in table3.
2. The second Pareto optimal point is and  with the solution shown in table4.
3. The third Pareto optimal point is  and  with the solution shown in table5.
4. The forth Pareto optimal point is  and  with the solution shown in table6.
5. The final Pareto optimal point is  and  with the solution shown in table7.

Figure 1 shows the primal Pareto front solution generated form steps (1) – (4) of Algorithm 1 and in Figure 2 dominated points according to step (5) are removed.

For this small size example the optimal Pareto solutions set are reached. Every point in this set can be applied by the decision maker as an optimal strategy and there is no priority to selection.

As discussed in section 4, ε-constraint method is an exact approach to solve the bi-objective problems and find the efficient Pareto solutions set. In this method  plays an important role in convergence of the obtained Pareto solutions set to the optimal Pareto set. Whatever  decreases, more convergence to the optimal Pareto set. Two objective functions in this paper have a discrete solution space and integer values because of their no fractional variables coefficient (all variables coefficient are one), so we can confident setting  leads to the optimal Pareto solutions set, because of the upper bound on second function decreases one by one and thus all the solution space points can be reached. Instead setting  causes more iteration for ε-constraint algorithm and on the other hand most of iterations lead a dominated point. So for large scale examples a balance between Pareto front with high quality and low iterations must be established.

To illustrate setting  decreases the quality of Pareto solutions set, we execute the ε-constraint method with  for the example mentioned before again. After removing dominated points, the Pareto solutions set are obtained with points set. These points are shown in figure 3. As we can see, Just points set  are common in two Pareto solution sets. Point (0, 11) is dominated by point (0, 10) and also the Pareto archive in figure 3 is sparse against Pareto front in figure2 because of absence of point (3, 3). Thus we conclude the ε-constraint with lower value in  leads to more efficient Pareto solution sets and for the problem discussed in this paper,  cause the optimal Pareto front because of its discrete solution space and no fractional variables coefficients in two objective functions.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Worker | | | | | | | | |
| 1 | 2 | 5 | 6 | 8 | 9 | 4 | 3 | 7 |
| Worker | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 5 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 8 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 4 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 7 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

**Table3. The first Pareto optimal solution**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Machine | | | | | | | | |
| 3 | 1 | 2 | 4 | 8 | 5 | 6 | 7 | 9 |
| Worker | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 2 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 6 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 9 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 4 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 3 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

**Table4. The second Pareto optimal solution**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Worker | | | | | | | | |
| 2 | 4 | 3 | 7 | 1 | 5 | 6 | 8 | 9 |
| Worker | 2 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 4 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 9 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Machine | | | | | | | | |
| 1 | 2 | 4 | 8 | 5 | 6 | 7 | 9 | 3 |
| Worker | 2 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 4 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 5 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 6 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 8 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 9 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |

**Table5. The third Pareto optimal solution**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Worker | | | | | | | | |
| 3 | 7 | 1 | 2 | 4 | 5 | 6 | 8 | 9 |
| Worker | 3 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 9 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Machine | | | | | | | | |
| 5 | 6 | 7 | 9 | 1 | 2 | 4 | 8 | 3 |
| Worker | 3 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 7 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 8 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 9 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |

**Table6. The forth Pareto optimal solution**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Machine | | | | | | | | |
| 2 | 8 | 1 | 4 | 3 | 5 | 6 | 7 | 9 |
| Worker | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 4 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 6 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 8 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 9 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 3 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 7 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

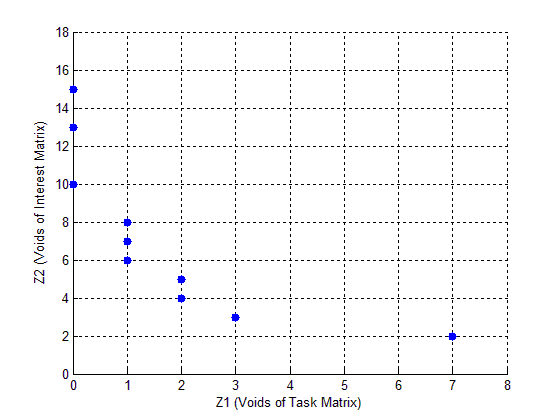
|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Worker | | | | | | | | |
| 1 | 4 | 6 | 2 | 5 | 8 | 9 | 3 | 7 |
| Worker | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 4 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 7 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |

**Table7. The final Pareto optimal solution**

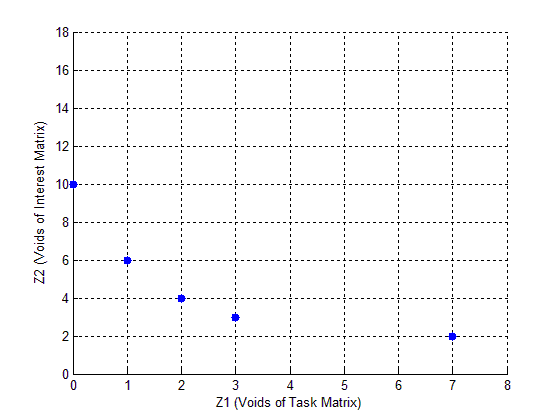
|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Machine | | | | | | | | |
| 4 | 5 | 6 | 9 | 1 | 3 | 7 | 2 | 8 |
| Worker | 2 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 7 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 8 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 9 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 4 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Worker | | | | | | | | |
| 2 | 3 | 7 | 5 | 8 | 9 | 1 | 4 | 6 |
| Worker | 2 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

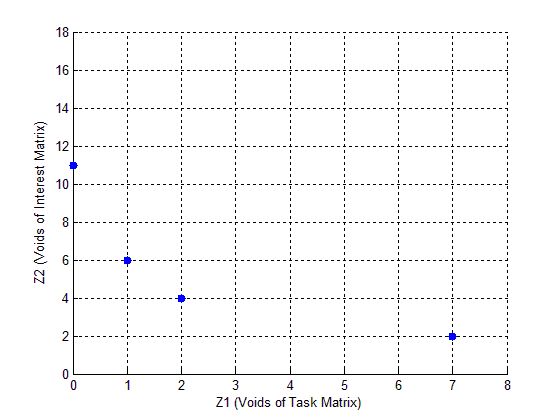
**Figure1. The primal Pareto front solutions**

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**Figure2. The final optimal Pareto front solutions**

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**Figure3. The final Pareto front solution with **

****

**6. Conclusions**

In this paper, we investigated a new concept of being an interactional interest between workers in a manufacturing cell besides the ability to work with its machines and presented a bi-objective mathematical model to carry out this new point of view in cellular manufacturing systems. This bi-objective mathematical model tries to increase the level of interactional interest and the reliability in each cell simultaneously. To find the optimal Pareto front the ε-constraint method was applied and for a small size example with 9 workers, 9 machines and 3 cells reached to the optimal Pareto solutions set. Ε-constraint method is a repetitive algorithm, so for large scale examples is not efficient and to prompt the algorithm. We have to increase that cause diverge the obtained Pareto front from the optimal Pareto front. So implementation an alternative solution approach is so needed. Multi objective evolutionary algorithms (MOEA) are an efficient approach to handle ε-constraint difficulties and can be consider in future related researches.

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