Entropic Economic Order Quantity Model for Items with Imperfect Quality Considering Constant Rate of Deterioration under Fuzzy Inflationary Conditions

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ABSTRACT

It was suggested in 2004 by some researchers that it might be possible to improve production systems performance by applying the first and second laws of thermodynamics to reduce system entropy. Then these laws were used to modify the economic order quantity (EOQ) model to derive an equivalent entropic order quantity (EnOQ). Moreover the political instability or uncertainty of a country (as well as the whole world) leads to a much more unstable situation in the present world economy. Thus, changes in inflation takeplace, and it is needed to consider uncertain inflation rate. In this paper we extend the EnOQ model by considering deteriorating items with imperfect quality and price dependent demand. We also assume fuzzy inflation and discount rates. A mathematical model is developed to determine the number of cycles that maximizes the present value of total revenue in a finite planning horizon. The fuzzified model for inflation and discount rate is formulated and solved by two methods: signed distance and fuzzy numbers ranking. Numerical examples are presented and results are discussed. Results show that the number of cycles decreases in fuzzy inflationary conditions. They also illustrate that defuzzification method results in more cycles than fuzzy method.

1. Introduction

As markets have become more and more competitive, disorder has become a prevailing characteristic of modern productive systems that are operating in complex, dynamic and uncertain environments. Some researchers in the discipline of management science/operational research have applied information theory and entropy approaches to account for disorder when modeling the behavior of productive systems, e.g., ([1],[2] and [3]). The first and the second

laws of thermodynamics were used by some researchers to improve the performance of inventory systems. Jaber et al. [4] proposed an analogy between the behavior of production systems and the behavior of physical systems. Their paper suggested that by applying the first and second laws of thermodynamics to reduce system entropy, improvements could be gained in production system performance. They introduced the concept of entropy cost to account for hidden costs such as the additional managerial cost that is needed to control the improvement process in [5] and increased labor union leverage in [6]. Their results suggested that items should be ordered in larger quantities than indicated by the classical EOQ model. In a subsequent paper, Jaber et al. [7] investigated their earlier model [4] for coordinating orders in a two-tier

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supply chain under the assumptions of a constant rather than an increasing commodity flow and for a finite rather than an infinite planning horizon. Recently, Jaber [8] investigated the model of [4] for the assumption of permissible delay in payments. The results of [4] are interesting but controversial because, as commented above, using smaller batches appeals particularly when control and space are important such as with JIT systems. Intuitively, however, smaller batches may incur lower total entropy costs in several situations.

For instance, when items deteriorate, large batches are likely to suffer more deterioration and therefore have a larger associated entropy cost. This paper examines this idea by extending the analysis of [4] to examine whether the effect of deterioration counterbalances the earlier suggested increases in batch sizes and leads to smaller batches. [9] suggested an Entropic order quantity (EnOQ) model for deteriorating items.

In addition, one of EOQ model's assumptions is that the received items are with perfect quality. However, due to imperfect production process, natural disasters, damage or breakage in transit, or for many other reasons, the lot sizes received may contain some defective items. To capture the real situations better, several researchers studied the effect of imperfect quality on lot sizing policy. Rosenblatt and Lee [10] concluded that the presence of defective products motivates smaller lot sizes. Schwaller[11] presented a procedure that extends EOQ models by adding the assumptions that defectives of a known proportion were present in incoming lots and that fixed and variable inspection costs were incurred in finding and removing the items.

Zhang and Gerchak[12] considered a joint lot sizing and inspection policy Studied under an EOQ model where a random proportion of units are defective and must be replaced by non-defective ones. Salameh and Jaber [13] assumed that the defective items could be sold as a single batch at a discounted price prior to receiving the next shipment, and found that the economic lot size quantity tends to increase as the average percentage of imperfect quality Items increase. Chang [14] proposed an EOQ model with imperfect quality items where the defective rate is presented as a fuzzy number. Chung et al. [15] established a new inventory model with two warehouses and imperfect quality. Jaber et al. [16] applied the concept of entropy cost to extend the classical EOQ model under the assumptions of perfect and imperfect quality[16]. extended the work of [13] by assuming the percentage defective per lot reduces according to a learning curve. Lo et al. [17] developed an integrated production-inventory model with a varying rate of deterioration under Imperfect production processes, partial backordering and inflation.

On the other hand, most of the classical inventory models did not take into account the effects of inflation. This has happened mostly because of the belief that inflation will not influence the cost and price components to any significant degree. But, during the last few decades, due to high inflation in the developing countries the financial situation has been changed and so it is not possible to ignore the effect of inflation any further. The Pioneer research in this direction was Buzacott[18], who developed an EOQ model with inflation subject to different types of pricing policies.

In the same year, Misra[19] also developed an EOQ model incorporating inflationary effects. Padmanabhan and Vrat [20] developed an inventory model under a constant inflation rate for initial stock-dependent consumption rate. Datta and Pal [21] developed a model with linear time-dependent demand rate and shortages to investigate the effects of inflation and time value of money on ordering policy over a finite time horizon. Hariga [22] extended [21] model by relaxing the assumption of equal inventory carrying time during each replenishment cycle and modified their mathematical formulation. Hariga and BenDaya [23] then extended [22] by removing the restriction of equal replenishment cycle and provided two solution procedures with and without shortages. Horowitz [24] introduced inflation uncertainty into a basic EOQ model.

Chern et al. [25] extended the traditional model to allow not only for general partial backlogging rate but also for inflation. They considered that imperfect items are reworked at a cost. Roy et al. [26] established an inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. K. Maity and M. Maiti [27] proposed a numerical approach to a multi-objective optimal inventory control problem for deteriorating multi-items under fuzzy inflation. Mirzazadeh et al. [28] developed an inventory model under stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon. Mirzazadeh [29] presented an inventory model under variable inflationary condition with inflation-proportional demand rate. Sarkar and Moon [30] proposed production inventory model for stochastic demand with the effect of inflation.

Imperfect items with deterioration are shown in industries such as electronic, agriculture and food industries. In the other hand, the major loss due to inflation is caused by its uncertainty instead of its high rate. Uncertainty in future inflation rate rises the inflation rate and in turn high inflation rate increases uncertainty in future inflation rate. So due to the world's current uncertain conditions it is important to consider uncertainty in inflation rate to capture the real world better. Hence, in this paper, a new thermodynamics approach model under fuzzy inflationary conditions, fuzzy time discounting for defective items with constant defective and deterioration rate and time dependent demand has been proposed.

2. Preliminaries

Before presenting fuzzy inventory model, we introduce signed distance defuzzification method which has recently used by some researchers in inventory models (e. g. [31],[32]). The definitions are from [32] and [14].

Definition1. The fuzzy set a_α of $R, 0 \leq \alpha \leq 1$, is called a level α fuzzy point if:

$$\mu_{a_\alpha}(x) = \begin{cases} \alpha, x = a \\ 0, x \neq a \end{cases} \tag{1}$$

Let $F_p(\alpha)$ be the family of all α level fuzzy points.

Definition2. The fuzzy set $[a_\alpha, b_\alpha]$ of R, $0 \leq \alpha \leq 1$, is called a level α fuzzy interval if:

$$\mu_{[a_\alpha b_\alpha]}(x) = \begin{cases} \alpha, a \leq x \leq b \\ 0, otherwise \end{cases}. \tag{2}$$

For each $\alpha \in [0,1]$, let $F_I(\alpha) = \{[a_\alpha, b_\alpha] \forall a < b, a, b \in R\}$.

Definition3. For any a and $0 \in R$, define the signed distance from a to 0 as $a = d_0(a, 0)$.

If $a > 0$ the distance from a to 0 is $a = d_0(a, 0)$. If $a < 0$ the distance from a to 0 is $-a = -d_0(a, 0)$. Hence, $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\widetilde{B}$ defined on R with which the $\alpha - cutB(\alpha) = [B_l(\alpha), B_u(\alpha)]$ exists for every $\alpha \in [0,1]$, and both $B_l(\alpha)$ and $B_u(\alpha)$ are continuous functions on $\alpha \in [0,1]$. Then, for any $\widetilde{B} \in \Omega$, we have:

$$\widetilde{B} = \bigcup_{0 \leq \alpha \leq 1}[B_l(\alpha)_\alpha, B_u(\alpha)_\alpha]. \tag{3}$$

From Definition3, the signed distance of two end points, $B_l(\alpha)$ and $B_u(\alpha)$, of the $\alpha - cutB(\alpha) = [B_l(\alpha), B_u(\alpha)]$ of $\widetilde{B}$ to the origin 0 is $d_0(B_l(\alpha), 0) = B_l(\alpha)$ and, respectively.

Their average, $(B_l(\alpha) + B_u(\alpha))/2$, is taken as the signed distance of $[B_l(\alpha), B_u(\alpha)]$ to 0.

That is, the signed distance of interval $[B_l(\alpha), B_u(\alpha)]$ to 0 is defined as:

$$d_0([B_l(\alpha), B_u(\alpha)], 0) = [d_0(B_l(\alpha), 0) + d_0(B_u(\alpha), 0)]/2 \tag{4}$$
$$= (B_l(\alpha) + B_u(\alpha))/2.$$

In addition, for every $\alpha \in [0,1]$, there is a one-to-one mapping between the α-level fuzzy interval $[B_l(\alpha)_\alpha, B_u(\alpha)_\alpha]$ and the real interval $[B_l(\alpha), B_u(\alpha)]$, that is, the following correspondence is one-to-one mapping:

$$[B_l(\alpha)_\alpha, B_u(\alpha)_\alpha] \leftrightarrow [B_l(\alpha), B_u(\alpha)]. \tag{5}$$

Also, the 1-level fuzzy point $\widetilde{0}_1$ is mapping to the real number 0. Hence, the signed distance of $[B_l(\alpha)_\alpha, B_u(\alpha)_\alpha]$ to $\widetilde{0}_1$ can be defined as:

$$d([B_l(\alpha)_\alpha, B_u(\alpha)_\alpha], \widetilde{0}_1) = d_0([B_l(\alpha), B_u(\alpha)], 0) =$$
$$(B_l(\alpha) + B_u(\alpha))/2.$$

Moreover, for $\widetilde{B} \in \Omega$, since the above function is continuous on $0 \leq \alpha \leq 1$, we can use the integration to obtain the mean value of the signed distance as follows:

$$\int_0^1 d([B_l(\alpha)_\alpha, B_u(\alpha)_\alpha], \widetilde{0}_1) d\alpha = \tag{6}$$
$$\frac{1}{2} \int_0^1 (B_l(\alpha) + B_u(\alpha)) d\alpha$$

Property1. For the trapezoidal fuzzy number $\widetilde{B} = (p, q, r, s)$ the signed distance of $\widetilde{B}$ to $\widetilde{0}_1$ is:

$$d(\widetilde{B}, \widetilde{0}_1) = \frac{1}{2} \int_0^1 [p + (q-p)\alpha + s - (s-r)\alpha] = \tag{7}$$
$$\frac{1}{4}(p + q + r + s).$$

We also need a method for ranking fuzzy numbers. The definition is from [33].

Definition4. Let $M = (m_1 \backslash m_2 \backslash m_3 \backslash m_4)$ and $N = (n_1 \backslash n_2 \backslash n_3 \backslash n_4)$. Then $N < M$ if $n_2 < m_2$ and $N > M$ if $n_2 > m_2$. Assume that $n_2 = m_2$. Then $N < M$ if $n_3 < m_3$ and $N > M$ if $n_3 > m_3$. Assume that $n_2 = m_2$ and $n_3 = m_3$. Then $N < M$ if $n_1 < m_1$ and $N > M$ if $n_1 > m_1$. Assume that $n_2 = m_2$, $n_3 = m_3$ and $n_1 = m_1$. Then $N < M$ if $n_4 < m_4$ and $N > M$ if $n_4 > m_4$. If $n_2 = m_2, n_3 = m_3$, $n_1 = m_1$ and $n_4 = m_4$, then $M = N$ and discard one of them, with their corresponding z_i values.

3. Mathematical Modeling

To derive the entropic model for deteriorating items considering the effect of imperfect items and fuzzy inflationary conditions we first define a system. Then the concepts of commodity flow and entropy costs are introduced and the assumptions are presented. Finally, the mathematics for the proposed model is presented. This paper develops the model proposed in[9] to include imperfect items and fuzzy inflationary conditions. The fuzzified model for inflation and discount rate is formulated and solved by signed distance and fuzzy numbers ranking methods. We use a simple algorithm to find optimal solution.

3.1. Basis of the Model

A physical thermodynamic system is defined by its temperature, volume, pressure and chemical composition. A system is in equilibrium when these variables have the same value at all points. In a similar manner, a production system could be described by its characteristics, for example the price (*P*) that the system ascribes to ascertain commodity (or collection of commodities) that it produces. Reducing the price of a commodity below the market price may increase customers' demand, and produce a commodity flow (sales) from the system to its surroundings. This could be considered to be similar to the flow of heat from a high-temperature reservoir (source) to a low-temperature reservoir (sink) in a thermodynamic system.

It is recommended to see [7] to understand how the first and the second laws of thermodynamics were applied in a one-node commodity flow system .To guarantee a flow of commodity from the inventory system to the market, the following strategies are considered. The first strategy suggests that a firm may provide the same quality product as its competitors at a lower price, and the second strategy suggests that a firm provides a better quality product than its competitors at the same price. Like [4], this paper adopts the first strategy where the suggested commodity flux, or demand rate is of the form:

$$\delta(t) = -k(P(t) - P_0(t)), \tag{8}$$

where *K* (analogous to a thermal capacity) represents the change in the flux for a change in the price of a commodity and is measured in additional units of demand per year per change in unit price e.g. units/year/$. Let *P(t)* be the unit price at time t, and $P_0(t)$ the market equilibrium price at time *t*. [4] assumed an increasing commodity flow(customer's demand) over a specified and short period of time. This strategy assumes a constant equilibrium price, that is, "Eq. (8)" is written as $\delta(t) = -k(P(t) - P_0(t))$, where P_0 is constant for every $t \in [0,T]$. Jaber et al. [4], who also presented the background and further discussion,

assumed a strategy of increasing commodity flow. With this strategy, the equilibrium price $P_0(t) = P_0$ is a constant for every $t \in [0,T]$, and *P(t)* is a monotonically decreasing function over the specified interval, i.e., $P_0'(t) < 0$ and $P_0''(t) > 0$ for every $t \in [0,T]$. This strategy is assumed to increase customer demand, i.e., $\delta(t)$ increases over the interval *[0,T]* as a result of the price discounts $(P(t) - P_0(t) < 0)$. [4] suggested adding a third component, representing the entropy cost, to the order quantity cost function. Noting that when $P < P_0$, the direction of the commodity flow is from the system to the surroundings, the entropy generation rate must satisfy:

$$\dot{S} = \frac{d\sigma(t)}{dt} = \delta(t)\left(-\frac{1}{P_0(t)} + \frac{1}{P(t)}\right) = k\left(\frac{P(t)}{P_0(t)} + \frac{P_0(t)}{P(t)} - 2\right). \tag{9}$$

The expression for $\delta(t)$ is given from (8), $\sigma(t)$ is the total entropy generated by time *t*, and $\dot{S}$ is the rate at which entropy is generated. In [4] the entropy cost has computed by dividing the total commodity flow in a cycle of duration *T* (determined by integrating "Eq. (8)") by the total entropy generated over time *T* (determined by integrating "Eq. (9)") as $\left(\sigma(t) = \int_0^T \dot{S} dt\right)$, to give:

$$E(T) = \frac{Q}{\sigma(t)} = \frac{D(T)}{\sigma(t)} = \frac{\int_0^T -k(P(t) - P_0(t))dt}{k\int_0^T \left(\frac{P(t)}{P_0(t)} + \frac{P_0(t)}{P(t)} - 2\right)dt}. \tag{10}$$

E(T) is measured in an appropriate price unit such as dollars and it is of constant value irrespective of the length of the cycle *T*.

Jaber et al. [4] assumed $Q = \int_0^T -k(P(t) - P_0(t))e^{-\theta.t}dt$ because there is no deterioration, and therefore, $Q = D(T)$, where $D(T)$ is the total demand in a cycle of length *T*. Now by considering the effect of inflation, deterioration and imperfect items we could rewrite the formulae of *Q* as:

$$Q = \frac{\int_0^T -k\left((P(t) - P_0(t))e^{-R.t}e^{-\theta.t}\right)dt}{(1-p)}. \tag{11}$$

3.2. Assumptions and Notation

A lot of size *Q* is delivered instantaneously with a purchasing price of *c* per unit and an ordering cost of *A*. It is assumed that each lot received contains *p*

percentage defectives. The selling price of good-quality items is $P(t)$ per unit. A 100% screening process of the lot is conducted at a rate of b units per unit time and cost of d; items of poor quality are kept in stock and sold prior to the next replenishment (in τ) as a single batch with a lower price of v per unit. The inflation and discount rates are shown by $\tilde{i}$ and $\tilde{r}$, respectively, and are represented by trapezoidal fuzzy numbers so the discounted rate of inflation is shown by $\tilde{R} = \tilde{r} - \tilde{i}$. The constant deterioration rate is θ. Demand is increasing by decrease in firm price over time and shown by $D(T) = -k(P(t) - P_0(t))$.

K is coefficient of elasticity that implicates the change in the flux for a change in the price of a commodity. Let the equilibrium and firm's price functions conform to $P_0(t) = P_0$ and $P(t) = P(0) - \dfrac{a.t}{T}$.

Lead time is negligible and no shortages are allowed. We assume n cycle of length T in finite planning horizon, H, so $H = n.T$ (The inventory level is shown in Fig. 1).We use present value method to find n which maximize total profit during the finite time horizon which length is H.
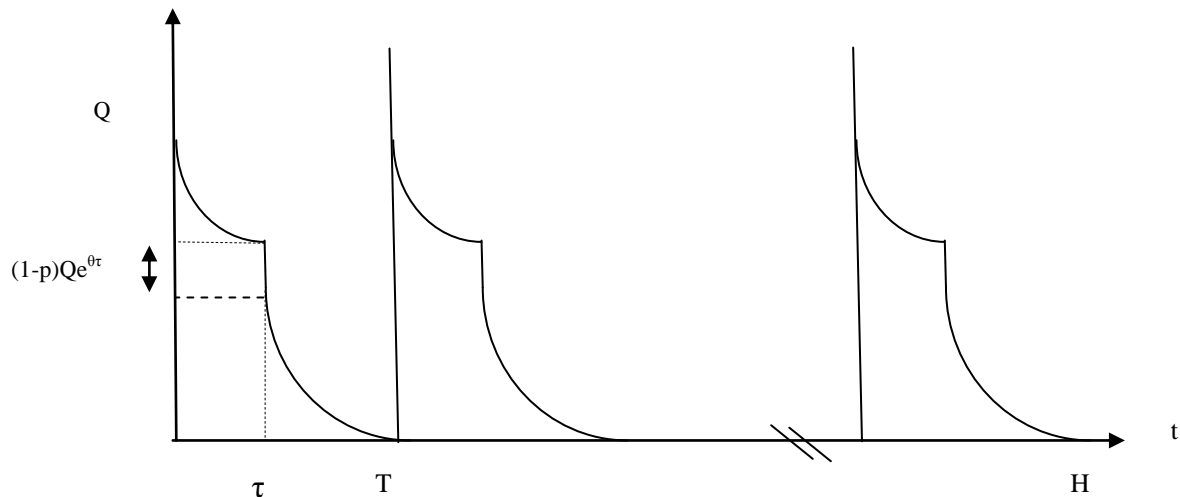

Fig. 1. The inventory level in finite time horizon H

3.3. Inventory Level
The differential equation describing the inventory level $I(t)$ in the interval $(j-1).T < t < j.T \quad (1 \le j \le n)$ is given by:

$$dI(t)/dt + \theta I(t) + \delta(t) = 0. \tag{12}$$

$$Q = \frac{\int_0^T -k(P(t) - P_0(t)).e^{-R.t}.e^{-\theta.t}dt}{(1-p)} = \frac{\int_0^T -k(P(0) - \frac{a.t}{T} - P_0}{(1-p)}$$

$$= -\frac{k}{(-1+p).T.(R+\theta)^2}.(R+\theta).P(0).T + a - (R+\theta).P_0.T$$

$$- e^{T.(R+\theta)}.(R+\theta).P(0).T - e^{T.(R+\theta)}.a + e^{T.(R+\theta)}.(R+\theta).P_0.T$$

$$+ e^{T.(R+\theta)}.\theta.a.T. \tag{13}$$

The inventory level in two intervals of each cycle is derived by solving the differential equation subject to the initial conditions:

$$I_1(t) = -e^{-\theta.t}.\frac{-Q + k.\dfrac{\theta.f.T + a - \theta.a.(j-1).T}{\theta^2.T}}{e^{-\theta.(j-1).T}} +$$

$$k.\frac{\theta.f.T + a - \theta.a.t}{\theta^2.T} \quad (j-1).T \le t \le (j-1).T + \tau \tag{14}$$

subject to the initial conditions $I(t) = Q$ at $(j-1)*T$ and $I(t) = 0$ at $j*T$.

From Eq.(11) we can calculate Q according to our assumptions, so we have:

$$I_2(t) = -e^{-\theta.T}.k.\frac{\theta.f.T + a - \theta.a.T.j}{\theta^2.T.e^{-\theta.T.j}} + k.\frac{\theta.f.T + a - \theta.a.t}{\theta^2.T}$$

$$(j-1).T + \tau \le t \le j.T. \tag{15}$$

Now we could calculate the cost functions and the revenue functions to obtain the final profit function.

3.4. Cost Functions
Present value of holding cost of the inventory for the jth $(1 \le j \le n)$ cycle is given by

$$HC_j = HC(\tau) + HC(T - \tau) =$$

$$\int_{(j-1).T}^{(j-1).T+\tau} h.e^{-R.t}.(-e^{-\theta.t}.\frac{-Q + k.\frac{\theta.f.T + a - \theta.a.(j-1).T}{\theta^2.T}}{e^{-\theta.(j-1).T}}$$

$$+ k.\frac{\theta.f.T + a - \theta.a.t}{\theta^2.T})dt \qquad (16)$$

$$+ \int_{(j-1).T+\tau}^{j.T} h.e^{-R.t}.(-e^{-\theta.t}.k.\frac{\theta.f.T + a - \theta.a.T.j}{\theta^2.T.e^{-\theta.T.j}}$$

$$+ k.\frac{\theta.f.T + a - \theta.a.t}{\theta^2.T})dt.$$

Present value of purchasing cost for the jth $(1 \le j \le n)$ cycle is given by:

$$PC_j = c.Q.e^{-R.T.(j-1)} = -c.\frac{k}{(-1+p).T.(R+\theta)^2}.$$

$$(R+\theta).P(0).T + a - (R+\theta).P_0.T - e^{T.(R+\theta)}. \qquad (17)$$

$$(R+\theta).P(0).T - e^{T.(R+\theta)}.a + e^{T.(R+\theta)}.(R+\theta).P_0.T +$$

$$e^{T.(R+\theta)}.\theta.a.T.\frac{e^{-R.T.(j-1)}}{(-1+p).T.\theta^2}.$$

Present value of ordering cost for the jth $(1 \le j \le n)$ cycle is given by:

$$PO_j = A.e^{-R.(j-1).T}. \qquad (18)$$

Screening cost occurs in the first interval of each cycle. We assume that this cost occurs in the middle of the interval $\frac{\tau}{2}$. So the estimate for the present value of screening cost for the jth $(1 \le j \le n)$ cycle is given by:

$$PS_j = d.Q.e^{-R.\left((j-1)T+\frac{\tau}{2}\right)}. \qquad (19)$$

Present value of entropy cost for the jth $(1 \le j \le n)$ cycle is given by:

$$PE_j = \frac{Q}{\sigma(t)}.e^{-R.(j-1).T}. \qquad (20)$$

3.5. Revenue Functions
Present value of perfect items sales revenue for the jth $(1 \le j \le n)$ cycle is given by:

$$PP_j = D(t).P(t).e^{-R.t} =$$

$$\int_{(j-1).T}^{j.T} -k.\left(\left(P(0) - \frac{a.t}{T} - P_0\right)\left(P(0) - \frac{a.t}{T}\right)e^{-2T.t}\right)dt. \qquad (21)$$

Present value of imperfect items sales revenue for the $j$thM$(1 \le j \le n)$ cycle is given by:

$$PI_j = v.Q.e^{-\tau.\theta}.p.e^{((j-1).T+\tau).R} =$$

$$-\frac{v.k.e^{-\tau.\theta}.p.e^{((j-1).T+\tau).R}}{(-1+p).T.(R+\theta)^2}.(R+\theta).P(0).T + a - \qquad (22)$$

$$(R+\theta).P_0.T - e^{T.(R+\theta)}.(R+\theta).P(0).T - e^{T.(R+\theta)}.a +$$

$$e^{T.(R+\theta)}.(R+\theta).P_0.T + e^{T.(R+\theta)}.\theta.a.T.$$

3.6. Profit Function
Present value of profit in the planning horizon (H) is:

$$PVP(H) = \sum_{j=1}^{n}\left(PP_j + PI_j\right) - \sum_{j=1}^{n}(HC_j +$$

$$PC_j + PS_j + PO_j + PE_j). \qquad (23)$$

3.7. Solution Methodology
Since the inflation and discount rates are fuzzy numbers the objective function becomes fuzzy. So we can't use common methods to maximize it. We use both fuzzy numbers ranking and signed distance defuzzification to find optimal solution. The derived function is complicated so instead of finding optimal T, we find optimal solution for n, which is a discrete variable. Since the planning horizon is finite and known, H, we substitute T by H/n so we have an objective with one variable, n.

3.8.1. Fuzzy Numbers Ranking Methodology
We assume that $\tilde{R}$ is a trapezoidal fuzzy number so $\tilde{Z} = P\tilde{V}P(H)$ becomes a fuzzy number too which is shown by (z_1, z_2, z_3, z_4). To find optimal value for n, we need to order fuzzy numbers for different values of n. We use method in [33] to rank the fuzzy numbers. Then the best value is selected as optimal solution.

3.8. 2. Signed Distance Defuzzification Methodology
We also apply a method of defuzzification, named the signed distance to find an estimation of the present value of total profit in planning horizon and compare the results with fuzzy ranking method's results. The previous researches on fuzzy production/inventory problems (e.g., [34], [35], [36],[31]) often have used the centroid method. But there is some difficulty in using this method; one should first find the membership function of fuzzy total cost by using the extension principle, while the derivations are very complex and cumbersome, especially, for the case where the fuzzy number is located in the denominator. [14]showed that using the decomposition principle with the signed distance method can solve similar problems easier. The concept of signed distance utilized by [32] and [14] has summarized in section 2.

After substituting all fuzzy terms with related signed distances, we have an estimate of the present value of total profit in planning horizon in the fuzzy sense, which is shown by *Z*. The resultant function is very complicated and difficult to solve by using common methods such as the first derivative. So we assign cardinal integers to *n* and estimate $Z = d\left(\widetilde{Z}, \widetilde{0}_1\right)$ and continue until we find a point in which the value of *Z* is less than the previous value obtained for *Z*.

4. Numerical Example and Analysis

An example is chosen to analyze the theoretical results. This example represents an inventory system where the commodity price at the beginning of a cycle is $P(0) = 100\$$, the market price $P_0 = 110\ \$$, the holding cost $h = 0.1\$unit/day$, $k = 4$, and the commodity unit cost is *c=50$*, selling price of imperfect quality items, *v=90$*, the annual screening rate *b=120000*, screening cost *d=15$,percentage of imperfect items p=0.1,planning horizon H=10*. The optimal values of the derived model are obtained for a range of different cost reductions over the cycle *a=5, 10* and *15*, order costs *A = 200*, and different deterioration percentages *θ = 1%, 5%* and *10%*. The results are summarized in Tables 1-7.

Consider column 3 *(θ=0.01)* of table 1, it can be seen that as *a* is increasing the amount of Q^* and $P\widetilde{V}R$ increase too. This profit increase arises from the increase in commodity flow due to competitive pricing (increasing the value of *a*). Now assume *a=5*, as *θ* increases from 0.01 to 0.1 n^* increases and Q^* reduces to counter high deterioration rate by purchasing less commodities each time and having fewer inventory levels.

In table 2 we have smaller lot size and more cycles than in fuzzy model and the present value of profit is also less than which in fuzzy model. In this case first signed distance method converts uncertain model to a deterministic one. Thus by solving the model we saw more cycles than the case in which we keep solve an uncertain model.

Now compare the results in table 3 with results of table 1 and table 2. When *R* is crisp the lot size is less than one in fuzzy and defuzzy models. It is a result of uncertainty, so larger lot sizes and higher inventory levels are recommended to deal with these uncertainties. In uncertain conditions, R can rise up to 0.14 and thus we expect more revenue than in the case in which R is exactly equals to 0.12.

In tables 4, 5, 6 and 7 results for different $\widetilde{R}$ are shown. As inflation rate increases, $\widetilde{R}$ decreases and less cycles in each planning horizon and larger lot sizes in each cycle is advisable in either fuzzy or defuzzy models, but lot sizes are even larger in fuzzy one. High inflation rate and uncertainty makes it more profitable to buy more commodities in present and keep it for future, as we can see in tables 4-7. Tables 8 and 9 shows more results for different rates of R and θ. Increasing R causes more cycles and increasing θ enlarges the lot size and simultaneously lowers profit.

Tab. 1. Results for fuzzy model, $\widetilde{R} = (0.11, 0.12, 0.13, 0.14)$

	Θ	0.01	0.05	0.1
a=5	PVR^*_1	-2.6684e+005	-2.3043e+005	-2.3463e+005
	PVR^*_2	-1.1003e+005	-9.9255e+004	-1.0066e+005
	PVR^*_3	1.4290e+005	1.4900e+005	1.5803e+005
	PVR^*_4	6.1091e+005	6.2608e+005	6.5738e+005
	Q^*	535.6418	290.7511	203.0457
	n^*	1	2	3
a=10	PVR^*_1	-2.9181e+005	-2.5694e+005	-2.5350e+005
	PVR^*_2	-1.2113e+005	-1.1235e+005	-1.1561e+005
	PVR^*_3	1.4756e+005	1.6142e+005	1.5359e+005
	PVR^*_4	6.4150e+005	6.8733e+005	6.7526e+005
	Q^*	644.1935	350.8217	406.0914
	n^*	1	2	2
a=15	PVR^*_1	-3.1744e+005	-2.8574e+005	-2.8263e+005
	PVR^*_2	-1.3284e+005	-1.2736e+005	-1.3203e+005
	PVR^*_3	1.5178e+005	1.7284e+005	1.6281e+005
	PVR^*_4	6.7205e+005	7.4947e+005	7.3438e+005
	Q^*	752.7453	410.8922	477.4170
	n^*	1	2	2

Tab. 2. Results for defuzzy model, $\widetilde{R} = (0.11, 0.12, 0.13, 0.14)$

	θ	0.01	0.05	0.1
a=5	PVR^*	7.7952e+003	1.4227e+004	1.6074e+004
	Q^*	172.2476	88.4725	78.3228
	n^*	3	6	7

Tab. 3. Results for crisp mode, R=0.12

	θ	0.01	0.05	0.1
a=5	PVR^*	2.5639e+004	2.0904e+004	1.9606e+004
	Q^*	42.4897	52.1386	53.5642
	n^*	12	10	10

Tab. 4. Results for fuzzy model, $\widetilde{R} = (0.02, 0.03, 0.05, 0.06)$

	θ	0.01	0.05	0.1
a=5	PVP1(H)	-1.0247e+007	-2.6915e+006	-1.9062e+006
	PVP2(H)	-2.2987e+006	-1.2664e+006	-1.1679e+006
	PVP3(H)	7.0237e+006	6.1480e+006	6.0389e+006
	PVP4(H)	5.3417e+007	4.7010e+007	4.6297e+007
	Q*	535.6418	669.5315	900.8251
	n*	1	1	1

Tab. 5. Results for defuzzy model, $\widetilde{R} = (0.02, 0.03, 0.05, 0.06)$

	θ	0.01	0.05	0.1
a=5	PVR^*	2.3218e+004	6.2476e+004	6.3275e+004
	Q^*	535.6418	669.5315	334.7657
	n^*	1	1	2

Tab. 6. Results for fuzzy model, $\widetilde{R} = (0.05, 0.1, 0.12, 0.15)$

	θ	0.01	0.05	0.1
a=5	PVP1(H)	-3.3350e+006	-8.7173e+005	-6.6822e+005
	PVP2(H)	-2.4619e+005	-2.1592e+005	-2.1701e+005
	PVP3(H)	4.5498e+005	4.2136e+005	4.5530e+005
	PVP4(H)	2.7898e+007	2.6340e+007	2.7729e+007
	Q*	532.9365	669.5315	334.7657
	n*	1	1	2

Tab. 7. Results for defuzzy model, $\widetilde{R} = (0.05, 0.1, 0.12, 0.15)$

	θ	0.01	0.05	0.1
a=5	PVR^*	7.1789e+004	9.9640e+004	1.1055e+005
	Q^*	260.6931	107.1284	92.5723
	n^*	2	5	6

5. Summary and Conclusions

In this paper we has extended the work of [4] by considering items with imperfect quality, not necessarily defective, where, upon the arrival of order lot, 100% screening process is performed and the items of imperfect quality are sold as a single batch and lower price, prior to receiving the next replenishment. Items are subject to deterioration in fuzzy inflationary conditions.

A mathematical model has developed to determine the number of cycles in finite planning horizon which maximizes the present value of total revenue in planning horizon. Numerical examples has presented and results have indicated that as a increases the amount of Q^* and $P\widetilde{V}R$ increase too. This profit increase arises from the increase in commodity flow due to competitive pricing (increasing the value of a).

Also as θ increases from *0.01* to *0.1*, n^* increases and Q^* reduces in order to deal with high deterioration rate.

We also showed that when R is crisp the lot size is less than one in fuzzy/defuzzy models. It is a result of uncertainty, so larger lot sizes and higher inventory levels are recommended to tolerate these uncertainties. In the other hand, as inflation rate increases, $\widetilde{R}$ decreases and less cycles in each planning horizon and larger lot sizes in each cycle is advisable in either fuzzy or defuzzy models. Smaller lot size, more cycles and less present value of profit are obtained in defuzzy method in comparison with fuzzy method In uncertain conditions, R can rise up to 0.14 and thus we expect more revenue than in the case in which R is exactly equals to 0.12. High inflation rate and uncertainty makes it more profitable to buy more commodities in present and keep it for future. In this case, two factors (high inflation and deteriorating rate) play opposite roles, high inflation tries to increase inventory levels and purchases more commodities, in the same time deterioration rate tries to reduce inventory levels and buys less. So a trade of between these factors determines the best policy for planning horizon.

Tab. 8. Results for fuzzy model, $\widetilde{R} = (0.16, 0.18, 0.19, 0.21)$

θ		0.01	0.05
a=20	PVR*1	1.0e+04 *0.7224	1.0e+04 *0.7139
	PVR*2	1.0e+04 *0.8352	1.0e+04 *0.8084
	PVR*3	1.0e+04 *0.9041	1.0e+04 *0.8686
	PVR*4	1.0e+04 *1.0618	1.0e+04 *1.0096
Q*		197.2592476	200.7559348
n*		4	4

Tab. 9. Results for fuzzy model, $\widetilde{R} = (0.24, 0.25, 0.28, 0.3)$

θ		0.01	0.05
a=20	PVR*1	1.0e+04 *1.4358	1.0e+04 *1.3658
	PVR*2	1.0e+04 *1.5623	1.0e+04 *1.4854
	PVR*3	1.0e+04 *1.9813	1.0e+04 * 1.8826
	PVR*4	1.0e+04 *2.2981	1.0e+04 *2.1837
Q*		154.6138546	158.2529453
n*		5	5

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