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RESEARCH PAPER

An Aggregated Newsvendor Model for Multi-Item Perishable Inventories with Uncertain Demand

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ABSTRACT
In industrial and commercial settings, inventory systems often involve managing multiple products with diverse demand patterns, making the direct application of the single-item newsvendor model inefficient. To address this complexity, this study proposes an adaptation of the newsvendor model through demand aggregation, where related items are grouped into a product family. By aggregating demand and financial parameters, the traditional newsvendor approach can be extended to multi-item systems, simplifying the inventory management process. This method was tested in two different case studies—a coffee roaster company and a meatball producer—demonstrating its validity and applicability. The aggregated newsvendor model was found to enhance inventory accuracy and efficiency, reducing random error and improving operational performance. This approach offers a valuable extension of the newsvendor model, with potential for broader application across various industries.

KEYWORDS: Probabilistic inventory system; Uncertain demand; Newsvendor model; expected profit.

1. Introduction

An inventory system comprises various processes and procedures designed to monitor and control inventory (Waters, 2008). It's crucial for businesses of any size to implement an efficient inventory system, as it enhances efficiency, lowers costs, and boosts customer service (Muller, 2019). Maintaining the optimal inventory levels allows businesses to minimize holding costs, swiftly and effortlessly meet customer demands, cut down expenses related to stockouts and backorders, and lessen the risk of obsolescence, particularly for perishable items (Alfares & Ghaithan, 2019).

In many industries, product demand can be probabilistic, meaning it encompasses a spectrum of potential demand values, each with corresponding probabilities (Zhang et al., 2016). This type of demand can arise from factors such as product seasonality (Spiliotis et al., 2021), market trends (Zhang et al., 2016), economic conditions like recessions (Alaminos et al., 2022), and unforeseen events like natural disasters (Garrido & Aguirre, 2020) or policy uncertainty (Huang & Abedinia, 2021).

An inventory system designed for probabilistic demand must account for the unpredictability of demand, encompassing the spectrum of potential demand values and their respective probabilities (Puga & Tancrez, 2017). Several inventory systems suitable for probabilistic demand include the base stock system, which maintains a consistent inventory quantity based on anticipated demand and the desired service level (Olsson, 2019); the safety stock system (Darom et al., 2018); the reorder point system, which triggers new inventory orders based on current stock levels and expected demand (Efrilianda & Isnanto, 2018) and the Just-in-time (JIT) system, which minimizes inventory by ordering only as needed (Omar & Zulkipli, 2014). The optimal probabilistic inventory system depends on the specific circumstances of the system.

The Newsvendor model is a mathematical framework designed to determine the optimal inventory level for a perishable product with unpredictable demand (Yamini, 2021). Named after the dilemma faced by newsboys who needed to decide the number of newspapers to purchase each day to maximize their profits (Khouja, 1999), the model assumes that the product is sold in a

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single period, demand is uncertain, the product price and holding cost are constant, and the cost of lost sales is known. The Newsvendor model is applied to manage orders for perishable items like fresh fruits, vegetables, seafood, and cut flowers (Baron, 2010), or other products with relatively short shelf lives such as newspapers and magazines (Wei & Chen, 2021). At the conclusion of the sales period, any remaining inventory has little to no value. This widely recognized approach is particularly valuable in environments where demand cannot be predicted with certainty, necessitating decisions based on statistical probabilities. By utilizing the Newsvendor method, businesses can determine the optimal order quantity that minimizes the risk of either overstocking or understocking, thus balancing potential costs associated with surplus inventory or lost sales.

Traditionally, the newsvendor model is designed to handle single-item inventory management (Khouja, 1999). This model focuses on determining the ideal stock level for one specific product, ensuring that the business can meet customer demand without incurring excessive costs from excess inventory. The simplicity and effectiveness of the single-item newsvendor model make it a popular choice for businesses dealing with straightforward inventory situations (Huber et al., 2019). However, this single-item focus also presents limitations, particularly in industries where inventory systems are more complex and involve multiple products.

In many industrial and commercial settings, inventory systems are inherently multi-item (Nenes et al., 2010), involving numerous products with varying demand patterns and characteristics. Directly applying the single-item newsvendor model to these multi-item systems is impractical and inefficient. Multi-item inventory management requires a more sophisticated approach to account for the diverse nature of the products and their interactions.

2. Literature Review

The literature on the multi-product newsvendor problem (MPNP) has seen significant advancements, particularly in developing mathematical models and solution methods that address various constraints and product interactions. The foundational work in this area focuses on single and multiple constraints within multi-item newsvendor problems, examining the influence of substitute and complementary products on stocking decisions and expected profits. These studies provide a comprehensive understanding of how product interdependencies affect inventory management, particularly in stylized settings with limited products, and suggest several directions for future research in expanding these models (Turken et al., 2012).

Further contributions to the MPNP literature explore the incorporation of risk measures into the traditional newsvendor framework. Specifically, the integration of Value at Risk (VaR) within a multi-product context has been investigated, addressing the limitations of the classical newsvendor model that focuses solely on maximizing expected profit. By considering downside risk constraints, these models aim to mitigate the risk of falling short of a target profit or incurring unacceptable losses due to demand variability. This approach introduces a risk-averse perspective, highlighting the trade-offs between profit maximization and risk management (Özler et al., 2009).

Another significant development in the MPNP literature is the exploration of robust models that account for uncertainty in demand and substitution rates. The Robust Multi-product Newsvendor Model with Substitution (R-MNMS) is a notable example, where the objective is to determine optimal order quantities that maximize worst-case total profit under stochastic conditions. This model addresses the uncertainty by using cardinality-constrained uncertainty sets, offering a robust solution to the complexities of product substitution and demand variability (Zhang et al., 2021).

Additionally, research has expanded into retail contexts, where budget-sensitive shoppers influence demand. In such settings, the newsvendor problem extends beyond determining order quantities to include pricing decisions. The demand for each product is stochastic and cross-elastic, depending on the demands of other products. By transforming the objective function into a form suitable for integer programming and employing numeric optimization techniques like the Nelder-Mead search, this research contributes to a more practical and applicable framework for retailers managing multiple products with interdependent demands (Murray et al., 2012).

3. Methodology

While the traditional newsvendor method is designed for single-item inventory management, its principles can be adapted for multi-item systems through demand aggregation. By treating a group of related items as a product family and using aggregated parameters (Kwak, 2015), industries can extend the newsvendor model's benefits to more complex inventory scenarios. This adaptation facilitates a broader and more practical application of the newsvendor method, making it valuable in various industrial contexts.

As in forecasting, computing the inventory based on aggregate demand is essential because it provides a holistic view of inventory needs, reducing the likelihood of errors that can occur when calculating inventory levels for each item individually. When ordering quantities are computed one by one for each item, each calculation can introduce errors due to variations in demand, lead times, and other factors. These errors can compound across multiple items, leading to significant inaccuracies and inefficiencies in inventory management (Dekker et al., 2004). By considering the aggregate demand for all items, the overall demand variability is smoothed out, enabling more accurate and efficient inventory computations. This approach minimizes the cumulative error, ensuring that inventory levels are optimized to meet the overall demand while reducing the risk of stockouts or overstocking.

Building on this concept, we propose applying the newsvendor method by first aggregating the demands of multiple items into a single family demand. This approach is feasible when items can be categorized as a family (Ashraf & Hasan, 2015). Aggregating the demands simplifies the complex multi-item problem into a more manageable single-item framework (Dekker et al., 2004), enabling the use of the newsvendor model in multi-item inventory systems.

In this aggregated framework, the parameters for the newsvendor model are derived from the combined data of the item family. These include the aggregated demand mean and standard deviation, which offer a unified view of demand patterns across all items in the family. Additionally, aggregated sales value, cost, and residual value reflect the collective financial implications of managing the inventory for the family. By using these aggregated parameters, the newsvendor method can be effectively applied to multi-item systems, ensuring more accurate and efficient inventory management.

Let there be an inventory system where n items and m periods were considered. Notations related to the problem are shown as follows:

d_{it} = demand of item i at period t (t = 1, …, m),
s_i = selling price of item i (i = 1, …, n),
r_i = residual value of item i when unsold,
u_i = unit cost of item i,
a_i = value of item i in the aggregation,
p_i = proportion of aggregate demand of item i,
D_t = aggregate demand at period t,
μ = mean of aggregate demand,
σ = standard deviation of aggregate demand,
S = aggregate selling price,
R = aggregate residual value,
U = aggregate unit cost,
C_s = shortage cost,
C_o = overage cost,
L = service level,
Q^* = optimal aggregate ordering quantity,
q_i^* = optimal ordering quantity of item i.

The proposed procedure is as follows: using aggregation values a_i, we firstly combined individual demands d_{it} to get the aggregate demand D_t as outlined in Eq. (1). Please remind that a_i could be, e.g., processing time, weight, or selling price. When selling price is used for aggregation, then $a_i = s_i$.

$$D_t = \sum_{i=1}^{n} a_i \cdot d_{it} \;\; , \forall t \tag{1}$$

Then, we calculated the mean and standard deviation of aggregate demand D_t over all periods t using Eq. (2) and (3).

$$\mu = \frac{1}{n}\sum_{t=1}^{m} D_t \tag{2}$$

$$\sigma = \sqrt{\frac{1}{m}\sum_{t=1}^{m}\left(D_t - \mu\right)^2} \tag{3}$$

After that, we calculated the aggregate selling price, residual value and unit cost in Eq. (5) to (7), using the proportion of each item's aggregate demand calculated in Eq. (4).

$$p_i = \frac{\sum_{t=1}^{m} a_i \cdot d_{it}}{\sum_{t=1}^{m} D_t} \;, \forall i \tag{4}$$

$$S = \sum_{i=1}^{n} p_i \cdot s_i \tag{5}$$

$$U = \sum_{i=1}^{n} p_i \cdot u_i \tag{6}$$

$$R = \sum_{i=1}^{n} p_i \cdot r_i \tag{7}$$

Next, we calculated the costs of shortage and overage respectively in Eq. (8) and (9), followed by the service level in Eq. (10). Using the standard normal table, we found the corresponding Z^* to the service level as shown in Expression (11). The optimal ordering quantity was then calculated using Eq. (12).

$$C_s = S - U \quad (8)$$

$$C_o = U - R \quad (9)$$

$$L = \frac{C_s}{C_s + C_o} \quad (10)$$

$$P(z < Z^*) < L \quad (11)$$

$$Q^* = \mu + Z^* \sigma \quad (12)$$

Based on Q^*, we calculated the optimal item ordering quantity as shown in Eq. (10).

$$q_i^* = \frac{p_i \cdot Q^*}{a_i}, \forall i \quad (13)$$

Finally, we compared the total profit resulting from the newsvendor method and from the current inventory system.

The proposed procedure was implemented in two different cases, showing that the method is valid and applicable. The first case was a coffee roaster company, while the second was a meat ball producer. Through these cases, the newsvendor method can be effectively applied to multi-item systems, ensuring more accurate and efficient inventory management.

Data was gathered through observations and interviews with key stakeholders. Then, the newsvendor model was used to calculate the data, comparing the anticipated profit from the Newsvendor model with the existing company inventory system, and determining the percentage increase in profit.

4. Results

Case 1: The coffee roaster

Custom Coffee Garage Roastery is a company engaged in the coffee roasting industry. The company produces eight types of coffee products. Its production activities consist of roasting, sorting, and packaging processes. Roasting is performed three times a week, while sorting and packaging are done every workday, which is six days a week. The company only sells finished products in the form of roast beans to various coffee shops. Roasted beans have a shelf life of 7 days after the packaging process. Beyond this resting period, the company does not want to take the risk of any change in taste quality, which would result in leftover products. Leftover products are collected and sold to collectors at a price below the selling price.

Table 1 shows the individual demands of each item at each period in units (d_{it}). These demands were aggregated using selling price (thus, in Eq. (1), $a_i = s_i$), resulting D_t at the last column of the table. At the bottom of the table, item's selling price (s_i), unit cost (u_i) and residual value (r_i) are shown, together with the computation of p_i.

Tab. 1. Item and aggregate coffee demands

Week	Item demands (d_{it})								D_t
(t)	i 1	2	3	4	5	6	7	8	
1	20	5	2	1	0	1	1	0	4,700
2	22	12	1	0	1	1	0	1	5,950
3	25	15	1	0	2	0	0	0	6,605
4	25	14	0	1	0	1	1	1	6,685
5	20	15	1	0	1	2	1	0	6,500
6	24	10	0	1	2	0	1	0	5,840
7	22	15	1	0	1	1	0	2	6,750
8	20	18	1	1	0	2	0	1	7,040
9	22	18	1	0	1	1	2	1	7,550
10	25	19	1	1	0	2	1	1	8,105
11	24	20	0	2	1	1	2	0	8,160
12	25	20	1	2	2	1	1	0	8,545
13	30	20	1	0	1	0	2	2	8,910
14	32	21	0	1	1	2	1	1	9,340
15	30	20	2	1	0	2	1	0	8,910
16	30	22	1	2	2	1	0	1	9,530
17	32	20	1	2	2	1	0	1	9,420
18	36	22	1	1	2	0	2	2	10,540
19	34	24	1	2	0	2	0	2	10,390
20	32	24	1	0	3	1	2	0	10,140
21	36	25	3	2	1	1	0	1	11,080
22	38	24	2	1	3	2	1	0	11,410

Week (t)	Item demands (d_{it}) i 1	2	3	4	5	6	7	8	D_t
23	40	24	1	1	3	2	2	0	11,660
24	38	26	1	2	1	2	2	0	11,510
25	40	25	1	2	2	2	2	0	11,840
26	41	26	2	3	0	2	1	0	11,885
27	42	27	2	1	1	3	1	1	12,450
28	40	28	2	0	0	3	2	2	12,380
29	42	28	1	2	3	0	2	1	12,630
30	46	27	3	2	2	3	0	0	13,210
31	41	29	1	2	1	2	2	1	12,685
32	44	28	2	1	2	2	2	1	13,140
33	45	30	0	2	3	2	2	1	13,625
34	48	32	1	2	2	4	2	0	14,620
35	50	31	1	1	2	2	3	2	14,690
36	52	34	2	0	2	4	1	1	15,220
37	47	32	3	2	1	2	1	1	14,235
38	49	35	2	3	1	4	1	0	15,285
39	52	34	1	2	4	3	1	3	16,260
40	46	36	1	4	3	2	1	1	15,350
41	48	40	4	1	1	2	2	1	16,060
42	50	42	2	3	2	4	1	2	17,450
43	63	38	2	4	2	2	1	0	17,575
44	60	35	2	2	4	1	1	2	16,920
s_i	125	180	260	260	260	260	260	260	$\sum_t D_t =$ 492,780
u_i	90	127	192	192	192	192	192	192	
r_i	5	8	15	15	15	15	15	15	
$\sum_t s_i d_{it}$	203500	196200	15600	16380	17680	20280	13520	9620	
p_i	41.3%	39.8%	3.2%	3.3%	3.6%	4.1%	2.7%	2.0%	

Note: s_i, r_i, u_i and D_t are in thousand rupiahs.

From Table 1, we calculated the other variables using Eq. (2) to (12). The results are shown in Table 2, showing that the aggregate demand mean (μ) and standard deviation (σ) are 11,200 and 3,502.4, respectively. The aggregate selling price, unit cost and residual value were 172.398, 123.998, and 8.083, respectively, resulting the corresponding shortage and overage costs of 48.4 and 115.915. These further resulted a service level of 0.294, which brought Z^* at -0.542, and Q^* at 9,301.699. The service level of 0.294 indicated that 29.4% of customer demand was expected to be met, while the remaining 70.6% experienced stockouts. This relatively low service level was a result of balancing the costs of shortage and overage, due to holding excess inventory was significantly more expensive than facing stockouts. Therefore, the newsvendor model optimized the total costs by accepting a lower service level, indicating a strategic decision to reduce the overage cost at the expense of higher stockout occurrences.

Tab. 2. Calculation results for case 1

μ	11,200
σ	3,502.4
S	172.398
U	123.998
R	8.083
C_s	48.4
C_o	115.915
L	0.294
Z^*	–0.542
Q^*	9,301.699

The Z^* score and the service level resulting from the calculations are shown in Figure 1.

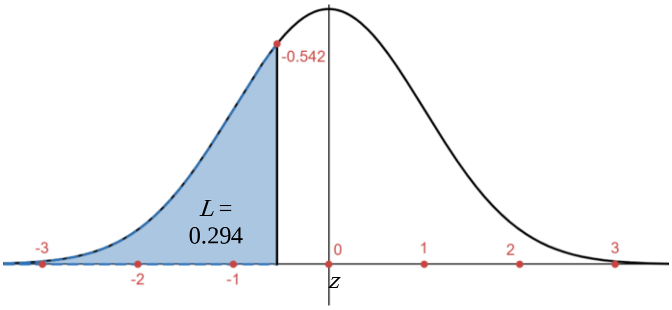

Fig. 1. Service level and optimal coffee quantity

Q^* was then disaggregated using Eq. (13), resulting item optimal quantity shown in Table 3.

Tab. 3. Coffee optimal quantity

Item optimal quantity (q_i*)								
i	1	2	3	4	5	6	7	8
	31	21	1	1	1	1	1	1

Case 2: The meatball producer

A meatball producer manufactured 15 items of meatball in 2023, including beef meatballs, chicken meatballs, and fish meatballs. Each of these products has different varieties. The company has a distinct raw material supplier for each product it produces. The meat used to make meatball products can only be stored in the freezer for one month, which sets the ordering period for raw materials from the supplier for production needs. Beef meatballs have a higher demand compared to chicken and fish meatballs. The company's production system is make-to-stock, ordering raw materials and determining production quantities based on estimates. The demand for beef meatballs fluctuates from period to period, meaning its demand is probabilistic.

Table 4 shows item demands from April to September 2023. The demands were aggregated on meat component (a_i) in kg per piece, which produced D_t at the bottom of the table. At the right columns of the table, item's selling price (s_i) and unit cost (u_i) are shown, along with the calculation of p_i. In Case 2, residual value is zero for all items.

Tab. 4. Demand, meat usage and profit of meatball items

Item (i)	Month (t) Apr	May	Jun	Jul	Aug	Sep	a_i (kg/pc)	s_i	u_i	p_i
1	761700	772700	753800	754125	761800	742900	0.0025	300	205	27.6%
2	356020	372500	367000	354750	356670	359050	0.0035	500	295	18.4%
3	10520	10255	10930	11082	11097	11270	0.003	400	250	0.5%
4	2260	2030	2090	2115	2030	2010	0.0032	420	266	0.1%
5	46010	48305	46350	46835	43655	46712	0.02	2100	1610	13.5%
6	26935	28055	26675	27670	26820	28822	0.05	4300	3930	20.0%
7	10055	11090	10880	10200	9940	10510	0.013	1200	1030	2.0%
8	4450	4665	4422	4525	4535	4525	0.0033	470	278	0.2%
9	945	982	915	940	930	820	0.02	2100	1610	0.3%
10	92575	94650	92600	93557	92122	94041	0.012	1100	950	16.3%
11	1380	1380	1302	1670	1320	1310	0.01	900	790	0.2%
12	630	650	645	625	625	615	0.049	4100	3840	0.5%
13	555	655	610	640	650	630	0.0032	420	266	0.0%
14	1690	1680	1615	1645	1695	1600	0.015	1400	1190	0.4%
15	1045	1070	1055	1050	1010	1045	0.004	780	358	0.1%
D_t	6807.238	7033.842	6829.783	6854.416	6750.352	6899.393	-	-	-	-

Using Equations (2) to (12), we computed the other variables from Table 4. The outcomes are displayed in Table 5, which reveal an aggregate demand mean (μ) of 6862.504 and a standard deviation (σ) of 97.4535. The aggregate selling price and unit cost, and residual value were 1558, 1321 and 0, respectively. This resulted in shortage and overage costs of 3622 and 16869, respectively. Consequently, the optimal service level was 0.152, with Z^* at -1.027 and Q^* at 6772.164. This service level meant that 15.2% of customer demand is expected to be fulfilled, while 84.8% will face stockouts. The relatively low service level arises from balancing the costs associated with shortages and overages, given that holding excess inventory is much costlier than experiencing stockouts. Therefore, the newsvendor model optimizes total costs by accepting a lower service level, indicating a strategic choice to reduce overage costs even if it means higher stockout rates.

Tab. 5. Calculation results for case 2

μ	6862.504
σ	97.4535
S	1558
U	1321
R	0
C_s	3622
C_o	16869
L	0.152
Z^*	–1.027
Q^*	6772.164

The Z^* score and the corresponding service level from the calculations are displayed in Figure 2.

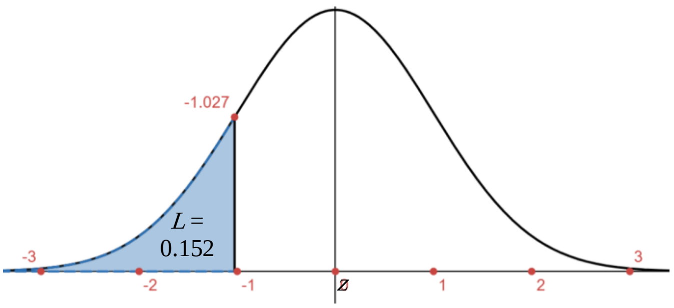

Fig. 2. Service level and optimal meatball quantity

Q^* was subsequently broken down using Eq. (13), leading to the optimal item quantities displayed in Table 6.

Tab. 6. Meatball optimal quantity

i	q_i^*
1	747861
2	356246
3	10716
4	2062
5	45702
6	27134
7	10308
8	4461
9	910
10	92030
11	1375
12	623
13	615
14	1632
15	1032

Case 3: The cosmetics industry

Rice groats, the main ingredient used in the production of body scrub in the cosmetics industry, have a limited shelf life of only one month, meaning they cannot be carried over to the next period. Extended storage may lead to deterioration. Therefore, it was crucial to manage the procurement of rice groats effectively, particularly considering the uncertain demand and the short shelf life of the ingredient, in order to ensure efficient production and maximize benefits.

Table 7 presents the demand for each item during each period, measured in units (d_{it}). These demands were aggregated based on the usage of rice groats, leading to the total demand (D_t) shown in the last column. The bottom row of the table displays each item's selling price (s_i), unit cost (u_i), and residual value (r_i), along with the calculation of p_i.

Tab. 7. Item and aggregate demands of rice groats

Month (t)	Item demands (d_{it}) i 1	2	3	4	D_t
Oct-20	15330	3940	2982	2901	2112.852
Nov-20	14548	1959	1999	2457	1760.892
Dec-20	14468	1928	1918	2459	1744.932
Jan-21	14588	2529	3182	3281	1980.720
Feb-21	14794	2213	2178	2928	1857.492
Mar-21	14428	1742	1945	1958	1686.132
Apr-21	17201	2531	3942	2941	2235.660
May-21	14329	1928	1982	2392	1733.004
Jun-21	15091	2058	2620	3284	1936.452
Jul-21	14448	1793	1748	2427	1714.944
Aug-21	15721	2223	2187	2628	1911.756
Sep-21	14552	1761	1989	2464	1744.344
Oct-21	15275	2042	2213	3094	1900.416
Nov-21	14491	1941	1442	2325	1696.716
Dec-21	16020	3192	2531	2851	2065.896
Jan-22	16024	2284	2928	4942	2198.952
Feb-22	14655	2127	2058	2624	1802.976
Mar-22	16582	2343	2941	3842	2159.472
Apr-22	16091	2176	3192	3249	2075.472
May-22	14580	1983	1988	2454	1764.420
Jun-22	17842	2545	2827	4529	2330.412
Jul-22	16515	3942	2628	3952	2271.108
Aug-22	18021	2482	2374	2482	2130.156
Sep-22	14589	2020	2034	2589	1783.488
Oct-22	17421	2248	2325	4021	2185.260
Nov-22	15929	3284	3851	2942	2184.504
Dec-22	16524	2239	2485	5821	2273.796
Jan-23	14827	2429	2852	4203	2042.124
Feb-23	15246	2249	2248	4921	2071.776
Mar-23	14543	1871	1920	2459	1746.612
a_i	0.084	0.084	0.084	0.084	-
s_i	10000	10000	10000	10000	
u_i	5500	5500	5500	5500	
r_i	5000	5000	5000	5000	
p_i	66.1%	9.9%	10.4%	13.6%	

Based on the data in Table 7, we calculated the remaining variables using Equations (2) to (12). The results are presented in Table 8, revealing that the mean (μ) and standard deviation (σ) of aggregate demand are 1970.091 and 207.182, respectively. The aggregate selling price, unit cost, and residual value were found to be 10000, 5500, and 5000, respectively, leading to shortage and overage costs of 4500 and 500. Consequently, this resulted in a service level of 0.9, which corresponded to a Z^* value of 1.282 and a Q^* value of 2235.61. A service level of 0.9 indicated that 90% of customer demand can be met, with the remaining 10% experiencing stockouts. This high service level comes from the high cost of shortage, leading the model to avoid shortages by serving most customers and allowing high level of overage due to its cheaper cost.

Tab. 8. Calculation results for case 3

μ	1970.091
σ	207.182
S	10000
U	5500
R	5000
C_s	4500
C_o	500

L	0.9
Z^*	1.282
Q^*	2235.61

Figure 3 displays the Z^* score and the corresponding service level based on the calculations.

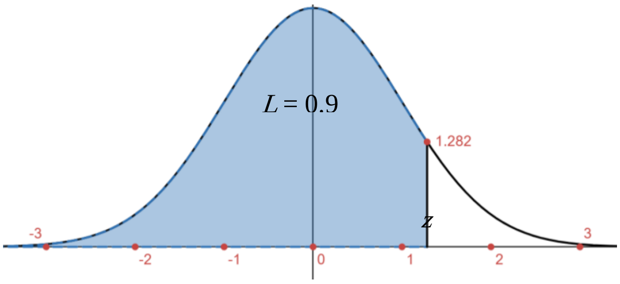

Fig 3. Service level and optimal rice groats quantity

Q^* was then decomposed using Eq. (13), resulting in the optimal item quantities shown in Table 6.

Tab. 9. Body scrub optimal quantity

Item optimal quantity (q_i^*)				
i	1	2	3	4
	17577	2648	2781	3609

Individual-item newsvendor calculation

Technically, at each case provided in this paper, we could use the newsvendor method to calculate individual ordering quantities for each item without aggregation. This means, the newsvendor needed to be conducted repeatedly for each item. This was conducted by using Eq. (2), (3), and (8) to (12) for each item at each case. The results were then compared to those conducted with aggregation in Table 3, 6 and 9. The differences between aggregated and individual calculation for each case, notated as *PD* (proportional difference), were calculated using Eq. (14). We can see that *PD* was calculated by considering quantity proportion of an item compared to all items quantity.

$$PD = \sum_{i=1}^{n} p_i \cdot |q_i - \hat{q}_i|, \tag{14}$$

where q_i and $\hat{q}_i$ respectively represent aggregate and individual ordering quantity of item *i*.
Calculations of *PD* for Case 1 to 3 are shown in Table 10.

Tab. 10. Aggregate vs individual quantities

Item	Case 1		Case 2		Case 3	
	q_i	$\hat{q}_i$	q_i	$\hat{q}_i$	q_i	$\hat{q}_i$
1	30.73	30.77	746784.7	753044.6	17576.61	16911.27
2	20.57	20.67	355733.3	359374.4	2647.88	3057.99
3	1.13	0.85	10700.6	10734.8	2780.53	3209.01
4	1.19	0.82	2058.7	2057.5	3609.33	4384.27
5	1.28	0.90	45635.7	45204.5		
6	1.47	1.14	27095.1	26346.1		
7	0.98	0.72	10293.5	9948.0		
8	0.70	0.37	4454.4	4500.8		
9			908.6	882.2		
10			91897.4	92179.5		
11			1373.3	1231.1		
12			622.5	611.4		
13			614.2	610.7		
14			1630.0	1612.3		
15			1030.6	1047.9		
PD	5.7%		1.3%		7.7%	

5. Discussion

The proposed newsvendor model with aggregation was successfully applied in the three cases presented, demonstrating its effectiveness across different inventory systems. In each case, the model provided a robust framework for managing inventory, leading to optimal ordering decisions that aligned with the unique requirements of multi-item inventory systems. The success of these implementations highlights the versatility and reliability of the proposed procedure in addressing complex inventory challenges.

One of the key contributions introduced by the proposed model is its ability to extend the traditional newsvendor model to multi-item inventory systems. Traditionally, the newsvendor model is used for single-item inventory decisions, but through aggregation (Kashkoush & ElMaraghy, 2014), the proposed model now allows for the simultaneous management of multiple items. This enhancement broadens the applicability of the newsvendor approach, making it a powerful tool for industries dealing with diverse product portfolios.

The proposed newsvendor model with aggregation performed as anticipated, producing optimal ordering decisions with a single standard deviation. As widely understood, standard deviation is a widely used measure of the random error associated with a large number of observations. In the context of inventory management, it quantifies the variability in demand and helps businesses understand the level of uncertainty they face. With the proposed model's aggregation approach, the newsvendor produced a single standard deviation for all items, streamlining the error measurement process. In contrast, using individual newsvendor models for each item would result in separate standard deviations for each item, which can complicate the analysis and increase the potential for error.

The aggregation process contributed to a more accurate representation of overall demand, minimizing the risk of stockouts or overstocking. By reducing random error, the model enhanced the efficiency and cost-effectiveness of inventory management, thereby improving the overall performance of the system.

In this context, our proposed approach generated a single standard deviation for each case, whereas, without aggregation, the newsvendor model would produce n standard deviations, necessitating n repetitions for each case. This demonstrates that our model offers two significant benefits simultaneously: reduced random error and faster calculations. However, the proposed method does require aggregation before, and disaggregation after, the application of the newsvendor model. These additional steps are well compensated by the dual advantages of lower errors and increased computational efficiency.

Therefore, we argue that our proposed model is capable of reducing random error in multi-item inventory systems. By consolidating the standard deviation into a single value through aggregation, the model simplifies the decision-making process and enhances accuracy. This reduction in random error is a significant benefit for businesses managing multiple products, as it leads to more reliable inventory decisions and improved overall system performance.

6. Concluding Remarks

In conclusion, the proposed newsvendor model with aggregation has proven to be a successful extension of the traditional newsvendor approach, particularly in its application to multi-item inventory systems. By enabling the simultaneous management of multiple items and reducing random error through the consolidation of standard deviation, the model enhances the accuracy and efficiency of inventory decisions. This advancement not only improves operational performance but also demonstrates the model's potential for broader application in various industries.

For future research, the model could be enhanced by integrating dynamic demand forecasting, incorporating price-dependent demand, and accounting for multiple suppliers and product substitution. These additions would make the model more adaptable to changing market conditions and more robust across different industries. Exploring its application in various sectors could reveal its versatility and limitations. Additionally, investigating different aggregation strategies could optimize error reduction, improving the model’s accuracy and efficiency in specific contexts.

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Appendix

Tab. A1. Individual item calculation for case 1

Item	1	2	3	4	5	6	7	8
C_s	35	53	68	68	68	68	68	68
C_o	85	119	177	177	177	177	177	177
L	29.2%	30.8%	27.8%	27.8%	27.8%	27.8%	27.8%	27.8%
Z^*	-0.549	-0.501	-0.590	-0.590	-0.590	-0.590	-0.590	-0.590
μ	37.00	24.77	1.36	1.43	1.55	1.77	1.18	0.84
σ	11.36	8.19	0.87	1.04	1.09	1.08	0.79	0.81
Q^*	30.77	20.67	0.85	0.82	0.90	1.14	0.72	0.37

Tab. A2. Individual item calculation for case 2

Item	C_s	C_o	L	Z^*	μ	σ	Q^*
1	95	205	31.7%	-0.477	757837.5	10047.2	753,044.6
2	205	295	41.0%	-0.228	360998.3	7136.7	359,374.4
3	150	250	37.5%	-0.319	10859.0	389.9	10,734.8
4	154	266	36.7%	-0.341	2089.2	92.9	2,057.5
5	490	1610	23.3%	-0.728	46311.2	1520.4	45,204.5
6	370	3930	8.6%	-1.366	27496.2	842.2	26,346.1
7	170	1030	14.2%	-1.073	10445.8	464.0	9,948.0

Item	C_s	C_o	L	Z^*	μ	σ	Q^*
8	192	278	40.9%	-0.231	4520.3	84.6	4,500.8
9	490	1610	23.3%	-0.728	922.0	54.7	882.2
10	150	950	13.6%	-1.097	93257.5	982.9	92,179.5
11	110	790	12.2%	-1.164	1393.7	139.7	1,231.1
12	260	3840	6.3%	-1.527	631.7	13.3	611.4
13	154	266	36.7%	-0.341	623.3	37.1	610.7
14	210	1190	15.0%	-1.036	1654.2	40.4	1,612.3
15	422	358	54.1%	0.103	1045.8	19.9	1,047.9

Tab. A3. Individual item calculation for case 3

Item	1	2	3	4
C_s	4500	4500	4500	4500
C_o	500	500	500	500
L	0.900	0.900	0.900	0.900
Z^*	1.282	1.282	1.282	1.282
μ	15489.10	2333.40	2450.30	3180.67
σ	1109.72	565.40	592.03	939.18
Q^*	16911.27	3057.99	3209.01	4384.27

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