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RESEARCH PAPER

Development and Optimization of Maintenance Using the Monte Carlo Method

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ABSTRACT
Maintenance plan efficacy traditionally prioritizes long-term predicted maintenance cost rates, emphasizing performance-centric approaches. However, such criteria often neglect the fluctuation in maintenance costs over renewal cycles, posing challenges from a risk management perspective. This study challenges conventional solutions by integrating both performance and robustness considerations to offer more suitable maintenance options.
The study evaluates two representative maintenance approaches: a block replacement strategy and a periodic inspection and replacement strategy. It introduces novel metrics to assess these approaches, including long-term expected maintenance cost rate as a performance metric and variance of maintenance cost per renewal cycle as a robustness metric.
Mathematical models based on the homogeneous Gamma degradation process and probability theory are employed to quantify these strategies. Comparative analysis reveals that while higher-performing strategies may demonstrate cost efficiency over the long term, they also entail greater risk due to potential cost variability across renewal cycles.
The study underscores the necessity for a comprehensive evaluation that balances performance and resilience in maintenance decision-making. By leveraging the Monte Carlo Method, this research offers a critical appraisal of maintenance strategies, aiming to enhance decision-making frameworks with insights that integrate performance and robustness considerations.

KEYWORDS: Maintenance Planning; Robustness; Block Replacement Strategy; Periodic Inspection and Replacement Strategy; Monte Carlo Method.

1. Introduction

Over time, maintenance strategies have evolved significantly. Initially, maintenance was reactive, focusing on fixing equipment only when it broke down (breakdown maintenance) [1]. Subsequently, time-based maintenance (TBM) emerged, where maintenance tasks are scheduled at predetermined intervals regardless of equipment condition [1]. More recently, condition-based maintenance (CBM) has gained prominence [2]. CBM relies on real-time monitoring and diagnostics to schedule maintenance activities based on the actual condition of the equipment [2].

A common metric used to assess maintenance efficacy is the long-term predicted maintenance cost rate [3]. This metric quantifies the average maintenance cost per unit time over an extended period. However, traditional approaches often overlook the variability in maintenance costs that occur from cycle to cycle [3]. This variability can pose challenges for budgeting and risk management, particularly when maintenance costs fluctuate unpredictably [4].

While CBM techniques have demonstrated potential cost savings and improved performance compared to TBM strategies [5], their robustness in managing cost variability remains

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underexplored [6]. Robustness here refers to the ability of a maintenance strategy to maintain stable cost outcomes across different renewal cycles, thereby reducing financial uncertainty and enhancing overall resilience [6, 7].

Despite the economic advantages of CBM, the existing literature lacks a comprehensive evaluation framework that considers both economic performance and robustness in maintenance strategies [8]. Most studies focus primarily on minimizing long-term predicted costs without adequately addressing the variability and risk associated with maintenance expenditures over time.

This study seeks to bridge this gap by introducing a novel criterion termed maintenance cost per renewal cycle (MCPRC). Unlike traditional metrics, which assess long-term cost averages, the MCPRC evaluates the variability of maintenance costs within each renewal cycle [9]. This criterion provides a more holistic assessment of maintenance strategy effectiveness by integrating both cost efficiency and resilience considerations.

Moreover, the study systematically compares two representative maintenance strategies — periodic inspection and replacement (PIR) versus quantile-based inspection and replacement (QIR) — using the MCPRC criterion [10]. These strategies are evaluated within the context of a single-unit degrading system modelled using a homogeneous Gamma process, where failure occurs upon reaching a critical degradation threshold [11].

By developing and analysing cost models based on this new criterion, the study aims to provide actionable insights into the comparative performance of PIR and QIR strategies [12]. Specifically, it examines how these strategies manage cost variability and maintain cost-effective operations over time.

The subsequent sections of this paper are structured as follows: Section 2 provides a brief overview of the deterioration and failure model used in the study. Section 3 outlines the assumptions and cost models for both the PIR and QIR strategies. Section 4 discusses the inadequacies of traditional long-term predicted maintenance cost rates and introduces the MCPRC criterion as a more relevant alternative. Section 5 presents the comparative analysis of PIR and QIR strategies using the MCPRC criterion. Section 6 provides Managerial Insights. Section 7 presents a Comparison of This Study with Previous Research. Finally, Section 8 concludes the study and outlines avenues for future research in this critical area of maintenance management.

2. Degradation and Failure Model

Consider a hypothetical situation in which we have a single-unit degrading system. This system might consist of either a single component or a collection of linked components, all of which are prone to deterioration from a maintenance standpoint. The deterioration of this system develops naturally over time and might result in unanticipated breakdowns. These failures may be related to numerous physical degradation processes such as cumulative wear, fracture propagation, erosion, corrosion, fatigue, and other similar causes [13].

Alternatively, this degradation process might also be a fake portrayal of how the system's health and performance deteriorate as it matures and experiences frequent use [14], [15]. In such instances, the system's functioning steadily deteriorates, leading to a drop in its overall performance and efficiency.

To adequately predict the deterioration of these systems, Hameed and Proschan [16] advise adopting time-dependent stochastic processes. This method enables a more thorough and detailed description of the system's behaviour across time. By applying time-dependent stochastic processes, we may acquire insights into the system's deterioration trends and, subsequently, create more accurate forecasts about its failure time [17].

Let's describe X_t as a scalar random variable that reflects the cumulative deterioration of the system at every given period $t \geq 0$. Initially, when no maintenance activities have been conducted on the system, we define X_0 as 0, signifying the system's pristine condition when it was new.

Moreover, assuming that the degradation increment between two-time points, t and s (where $t \leq s$), denoted as $X_s - X_t$, is independent of the degradation levels before t, we can employ any monotonic stochastic process from the Lévy family [18] to model the evolution of the system's degradation. The Lévy family of stochastic processes gives a large variety of alternatives to represent the degrading behaviour of the system properly.

By applying this technique, we may better understand how the system's deterioration advances over time, allowing us to make educated judgments about maintenance interventions and anticipate the system's failure time more accurately. This extensive deterioration modeling enables us to monitor the system's health condition and take proactive changes to maximize its performance and lifetime.

In this research study, we intentionally chose to select the well-established homogeneous Gamma process, defined by a shape parameter α and a scale parameter β, as our degradation modeling framework. This option has been widely defended and validated by a variety of practical applications spanning numerous fields. Notably, the Gamma approach has proved its usefulness in capturing degradation processes in corrosion damage mechanisms [19], deterioration of carbon-film resistors [20], SiC MOSFET threshold voltage degradation [21], fatigue crack propagation [22], and performance loss in actuators [23]. These real-world applications offer empirical proof of the applicability and usefulness of the Gamma process in simulating varied degradation processes.

Furthermore, the implementation of the Gamma process provides various benefits, notably in terms of mathematical formulation and analysis. The Gamma distribution, derived from the Gamma process, is well-known for its versatility in capturing a broad variety of deterioration patterns. It offers a robust probabilistic framework that permits the characterization of deterioration increments across time. In our particular case, for each given time interval $t < s$, the degradation increment $X_s - X_t$ may be described as a random variable that follows a Gamma distribution. The parameters α and β of the Gamma distribution influence the form and size of the deterioration process, respectively.

The Gamma approach has received recognition and respect among professionals in the area of deterioration modelling [24]. Its successful implementation in different real contexts, along with its mathematical tractability, makes it a perfect alternative for our research aims. By using the Gamma process, we can effectively capture the degradation dynamics, estimate degradation rates, and provide valid predictions on the future behavior of the system under study.

To recap, the choice to adopt the homogeneous Gamma process, defined by shape parameter α and scale parameter β, is anchored in its established track record in practical applications across different areas, as well as its support by domain experts. The use of the Gamma process not only aids the mathematical formulation and study of degradation processes but also allows us to describe the degradation increment $X_s - X_t$ as a random variable following a Gamma distribution with a well-defined probability density function:

$$f_{\alpha.(s-t),\beta}(x) = \frac{\beta^{\alpha.(s-t)} x^{\alpha.(s-t)-1} e^{-\beta x}}{\Gamma(\alpha.(s-t))} . 1_{\{x \geq 0\}}, \qquad (1)$$

and survival function:

$$\bar{F}_{\alpha.(s-t),\beta}(x) = \frac{\Gamma(\alpha.(s-t), \beta x)}{\Gamma(\alpha.(s-t))}, \qquad (2)$$

Where $\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} e^{-z}\, dz$ and $\Gamma(\alpha, x) = \int_x^\infty z^{\alpha-1} e^{-z}\, dz$ represent the complete and upper incomplete Gamma functions, respectively.

In the context of the degradation process, we apply a threshold-type model to characterize the failure of the system. This model enables us to assess when a system is regarded to have failed based on economic or safety factors. For instance, a system may be regarded to have failed if it no longer fulfils the needed product quality requirements or if there is a significant risk of dangerous failures, even if it is technically still operating.

To quantify this failure situation, we construct a critical threshold, abbreviated as L. As the deterioration of the system continues over time, the system is regarded to have failed as soon as its degradation level is above this specified threshold. This threshold acts as a limit that separates an acceptable operating situation from an unsatisfactory one.

To further describe the system's failure time, we add the random variable τ_L, which reflects the time at which the system encounters failure. In other words, τ_L specifies the random failure time of the system. It is vital to note that this failure time is dependent on the degradation level reaching or surpassing the critical threshold L.

Now, let's look into the mathematical formula for τ_L. To denote the indicator function, which produces a value of 1 if the input is true and 0 otherwise, we use the notation $1\{\cdot\}$. In this scenario, we may describe the failure time as follows:

$$\tau_L = \inf\{\mathrm{t} \in \mathbb{R}^+ | \mathrm{X_t} \geq \mathrm{L}\} \qquad (3)$$

where X_t represents the accumulated degradation of the system at time t, and inf denotes the infimum or the smallest value in a set. This equation signifies that the failure time occurs at the earliest time t for which the degradation level X_t surpasses or equals the critical threshold L.

To model a range of degradation behaviours, spanning from nearly deterministic to highly erratic, we introduce a pair of parameters, (α, β). These parameters provide a means to adjust the degradation model accordingly. The average degradation rate is characterized by the ratio α/β, while the variance is determined by the ratio α/β^2.

By manipulating these parameters, we can capture different degradation patterns and determine the average rate and variability of the degradation process.

In cases where degradation data is available, these parameters (α, β) can be estimated using conventional statistical techniques such as maximum likelihood estimation or moments estimation [25]. These techniques allow us to derive the most suitable values for the parameters based on the observed degradation data, enabling us to accurately model the degradation behavior and estimate the failure time of the system.

The density function of τ_L at time $t \geq 0$ is given by [6] :

$$f_{\tau L}(t) = \frac{\alpha}{\Gamma(\alpha t)} \int_{L\beta}^{\infty} (\ln(z) - \psi(\alpha t)) z^{\alpha t-1} e^{-z} dz, \quad (4)$$

Where $\psi(\nu) = \frac{\partial}{\partial \nu} \ln(\Gamma(\nu))$ is known as the digamma function.

3. Maintenance Strategies and Cost Models

In this specific section, our major purpose is to look into the examples of Condition-Based Maintenance (CBM) methods, especially the Periodic Inspection and Replacement strategy (PIR) and the quantile-based inspection and replacement strategy (QIR). Through this extensive description, we hope to give a complete knowledge of these techniques, including their underlying concepts, decision criteria, and the step-by-step procedure involved in formulating their Maintenance Cost per Renewal Cycle (MCPRC).

To begin, it is vital to identify the assumptions connected to the system being maintained. These assumptions serve as the basis for applying CBM methods successfully. By explicitly defining the fundamental assumptions, we create the framework for understanding the upcoming choice criteria and MCPRC formulation process.

Moving further, we give complete choice criteria for both the PIR and QIR techniques. These decision criteria operate as guidance for making educated decisions about maintenance activities depending on the system's status. The choice criteria take into consideration numerous elements such as the system's degradation trends, criticality, and reliability objectives. By considering these parameters, maintenance personnel may decide when inspections and replacements should be conducted, maximizing the overall maintenance approach.

Finally, we present a thorough explanation of the step-by-step method required in developing the MCPRC for both the PIR and QIR strategies. The MCPRC is a critical statistic that helps assess the cost-effectiveness of CBM techniques across a renewal cycle. It takes into consideration elements like as inspection expenses, replacement costs, and possible savings resulting from preventing catastrophic failures. By understanding this process, practitioners may examine the financial consequences of applying these methods and make educated judgments about their adoption.

In summary, this section fully examines the exemplars of CBM methods, especially the PIR and QIR. By explaining the assumptions, offering extensive decision criteria, and exhibiting the MCPRC formulation process, we want to provide readers with a full grasp of these strategies and their practical application in maintenance management.

3.1. Maintenance assumptions

Taking into consideration the extensive analysis presented in Section II about the single-unit system, we assume that the deterioration level of the system stays concealed and its failure condition is not immediately obvious. In other words, the system cannot independently convey its deterioration degree or operational/failure state. Instead, this information can only be gathered via inspection efforts. It's crucial to note that the phrase "inspection" comprises more than simply data gathering; it entails extracting significant characteristics from the obtained data, building deterioration indicators, and maybe additional duties [26]. Essentially, this activity comprises all the essential processes preceding the Maintenance Decision Making process in a predictive maintenance program [2]. However, performing inspections incurs money and demands time. Nevertheless, when considering the lifetime of a system, the time needed for an examination is low. Therefore, we assume that each inspection operation is quick, perfect, non-destructive, and incurs a constant cost indicated as C_i, where $C_i >$ 0.

The system under examination provides two maintenance options: Preventive Replacement (PR) and Corrective Replacement (CR). A replacement may be carried out rapidly and entails either physically replacing the component or executing a thorough repair or overhaul that returns the system to a state equal to being brand new. However, the expenses associated with PR and CR operations may not be equal. Corrective replacements, being unexpected and possibly causing environmental harm, often incur greater expenses compared to preventative replacements. Additionally, even when performing the same sort of maintenance activity, the expenses incurred by the system may differ. This is because executing maintenance on a more deteriorated system is likely to be more involved and, thus, more expensive. Let $C_p(X_t)$ and $C_c(X_t)$ reflect the costs of preventative and corrective replacements at time t, respectively. These costs are both rising functions of the deterioration level X_t and meet the connection $0 < C_i < C_p(X_t) < C_c(X_t)$.

Furthermore, because a replacement may only be conducted at discrete periods (particularly, during inspection times in the PIR and QIR schemes), there is downtime for the system once a failure occurs. This downtime incurs an extra cost, which accumulates from the moment of failure until the next replacement time, at a constant cost rate represented as C_d, where $C_d > 0$.

3.2. Maintenance strategies

3.2.1. Periodic inspection and replacement policy (PIR):

The Periodic Inspection and Replacement (PIR) strategy is acknowledged as one of the simplest Condition-Based Maintenance (CBM) techniques. Its strategy entails maintaining a static inspection period, and both Preventive Replacement (PR) and Corrective Replacement (CR) procedures are synchronized with the inspection periods. The decision-making process within the PIR policy involves several steps:

Step 1: Regular inspections: The system receives inspections at defined intervals of time, designated as δ, independent of its present status or age. These inspection periods are expressed as $T_k = k\delta$, where k takes on values of 1, 2, and so on.

Step 2: Decision depending on deterioration level: At each inspection time T_k, the observed deterioration level X_{T_k} is examined to make a judgment about the necessary maintenance action. The choice alternatives are as follows:

• If X_{T_k} is larger than or equal to the failure threshold L, it signals that the system has failed and needs corrective replacement with a new one at time T_k.

• If X_{T_k} falls within the range of M to L (inclusive), the system is considered to be running but sufficiently degraded, necessitating preventive replacement with a new one at time T_k.

• If X_{T_k} is less than the threshold M, the system is deemed to be in a healthy state, and no maintenance action is taken at T_k.

Step 3: Scheduling the next inspection: Regardless of the kind of intervention made during the current inspection, the next examination for the system is scheduled at $T_{k+1} = T_k + \delta$. In other words, the inter-inspection time stays constant. The success of the PIR policy is controlled by two main variables: the inspection period δ and the preventative replacement (PR) level M. The inspection period reflects the time interval between subsequent inspections, regulating the frequency at which the system's status is checked. The PR threshold M determines the degradation level at which the system is regarded as sufficiently deteriorated to require preventative replacement. Adjusting these factors enables maintenance practitioners to adjust the PIR policy to the unique features and needs of the system under consideration.

3.2.2. Quantile-based inspection and replacement policy (QIR):

Diverging from the Periodic Inspection and Replacement (PIR) policy, the Quantile Inspection and Replacement (QIR) policy adopts a new technique to analyse the system's status. In the QIR policy, the inspection schedule is decided based on a quantile schedule given by a parameter α, where $0 < \alpha < 1$.

Rather than sticking to predefined inspection intervals as in the PIR policy, the QIR policy adds flexibility by matching inspections with specified quantiles of the system's degradation distribution. These quantiles are specified by the parameter α, reflecting a proportion between 0 and 1 that describes the intended percentile of the deterioration distribution at which an inspection should occur.

For example, if α = 0.5, the QIR strategy targets the median of the degradation distribution. This implies that inspections are scheduled when the system's degradation level reaches the point where 50% of the degradation distribution is below it. Similarly, if α = 0.9, inspections are triggered when the system's deterioration level crosses the threshold where only 10% of the degradation distribution stays below it.

By leveraging quantile schedules, the QIR policy provides for a more dynamic and adaptable inspection technique. The inspection intervals are chosen by the system's degradation behavior, ensuring that inspections occur at points of relevance within the degradation distribution. This technique allows more focused maintenance operations and may possibly decrease wasteful inspections in times of low deterioration.

$$T_{k+1} = T_k + \Delta T_{k+1}\ , \quad \Delta T_{k+1} = \delta(X_{T_k}) = \inf\{t \geq 0, R(t \mid X_{T_k}) \geq \alpha\}, k = 1, 2, \ldots \quad (5)$$

In the given context, $X_{T0} = X_0 = 0$ denotes the initial degradation level of the system at time T0, which is assumed to be zero. This implies that at the start of the system's life, there is no degradation present.

The term R (t | $X_{T_k} = x_k$) represents the system's conditional reliability at time t, given the degradation level xk observed at the inspection time T_k. This conditional reliability provides an understanding of the system's probability of functioning properly up to time t, given its degradation level at the inspection time T_k.

When we have $X_{T_k} = x_k$, we can calculate the conditional reliability as follows:

$$R(t \ / \ X_{Tk} = x_0) = \ 1 - \ \bar{F}_{\alpha.(t-T_k),b}(L - x_k), \quad (6)$$

Here, $\bar{F}_{\alpha.(t-T_k),b}(L - x_k)$ is obtained from equation (2), which refers to the survival function of the distribution governing the degradation process. This function calculates the probability that the system's degradation level, starting from xk at inspection time T_k, remains below a certain threshold L over the time interval [T_k, T_{k+1}].

By subtracting this survival probability from 1, we obtain conditional reliability, representing the probability that the system remains functional up to time t, given its degradation level xk at inspection time T_k.

3.3.Maintenance cost per renewal cycle

In our analysis of the PIR and QIR strategies, we propose the utilization of the Maintenance Cost per Renewal Cycle (MCPRC) as a metric to assess their robustness. The MCPRC is calculated based on the length of a renewal cycle, denoted as S, and the overall maintenance cost incurred during that cycle, represented by C(S). The MCPRC is defined as the ratio of the maintenance cost to the length of the renewal cycle:

$$K = \frac{C(S)}{S}. \quad (7)$$

The value of K is a random variable, and we aim to evaluate it using its mean value, denoted as μ = E(K), and its standard deviation, expressed as:

$$\sigma = \sqrt{E(K^2) - E^2(K)} = \sqrt{E(K^2) - \mu^2}. \quad (8)$$

Here, $E(K^2)$ represents the expected value of K squared. Evaluating the mean and standard deviation of the MCPRC provides insights into the average cost per renewal cycle and the variability around this average.

It's vital to note that when the value of σ (the standard deviation) grows, the resilience of the maintenance solutions lowers. Higher σ suggests a broader band of MCPRC values, signaling more uncertainty and unpredictability in the maintenance costs each renewal cycle.

In the remaining portions of our research, we will present analytical equations to determine σ for both the PIR and QIR techniques. These expressions will enable us to statistically examine the resilience of each method by determining the amount of uncertainty and unpredictability in their relative maintenance costs every renewal cycle.

3.3.1. Standard formulation of the MCPRC for the PIR pol

Let's suppose a system that conducts regular inspections to evaluate its status and decide whether any maintenance activities are necessary. For simplicity and without loss of generality, we assume that the system undergoes either preventative or corrective replacement at the k-th inspection time, where k might take values from 1, 2, and so on.

Each inspection cycle has a period of $S_k = k\Delta T$, where ΔT indicates the time gap between successive inspections. As the number of inspections rises with each cycle, the system incurs inspection expenses proportionate to the number of inspections completed. Therefore, during the k-th inspection occasion, the system accrues a total of k inspection costs.

In addition to the inspection costs, there are additional charges related to the system's behavior throughout the renewal cycle. If the system stays functioning and does not need replacement by the end of the cycle, it incurs a preventative replacement (PR) cost. This cost accounts for the proactive replacement of particular components or subsystems to avoid probable failures or performance deterioration.

On the other side, if the system fails during the renewal cycle, it pays the expenses of k inspections, a corrective replacement (CR) cost, and the downtime resulting from the failure. The CR cost covers the fees associated in repairing the failed components, while the downtime cost reflects the losses experienced due to the system being out of action.

Considering these criteria, we can compute the Mean Cost Per Renewal Cycle (MCPRC) for the preventative inspection and replacement (PIR) program. The MCPRC indicates the average cost incurred by the system across a whole renewal cycle. To compute it, we total up the expenses associated with each feasible possibility for the system's behavior throughout the cycle.

The Mean Cost Per Renewal Cycle (MCPRC) for the PIR policy can be stated as:

$$K^{PIR} = \sum_{k=1}^{\infty} \Big(\frac{C_p\,(X_{S_k}) + kC_i}{S_k} 1_{\left\{X_{S_{k-1}} < M \le X_{S_k} < L\right\}} + \frac{C_c\,(X_{S_k}) + kC_i}{S_k} 1_{\left\{X_{S_{k-1}} < M < L \le X_{S_k}\right\}} + \frac{C_d W_{d,PIR}}{S_k} \Big) \quad , \tag{9}$$

Within the context of the PIR (preventive inspection and replacement) strategy, $W_{d,\text{ PIR}}$ refers to the system downtime that occurs during the time span [S_{k-1}, S_k]. This time span corresponds to the interval between the (k-1)-th inspection and the k-th inspection, encompassing a complete renewal cycle.

System downtime refers to the period when the system is not operational due to maintenance activities or failures. In the case of PIR, downtime can occur for two reasons: preventive replacements and corrective replacements.

During the time span [S_{k-1}, S_k], if the system undergoes a preventive replacement, it means that certain components or subsystems are proactively replaced to prevent potential failures or performance degradation. This replacement activity may require the system to be taken offline temporarily, resulting in downtime. The duration of this downtime associated with preventive replacement is denoted as $W_{d,\text{ PIR}}$.

Alternatively, if the system experiences a failure or requires a corrective replacement during the time span [S_{k-1}, S_k], it will also incur downtime. In this case, the system is out of operation while the failed components are replaced and the necessary repairs are carried out. The duration of this downtime associated with the corrective replacement is also denoted as $W_{d,\text{ PIR}}$.

Therefore, $W_{d,\text{ PIR}}$ captures the cumulative downtime occurring within a renewal cycle under the PIR strategy. It includes both the downtime resulting from preventive replacements and the downtime resulting from corrective replacements. By quantifying the duration of downtime, we can evaluate the impact of system maintenance activities on its overall availability and operational efficiency.

Here, $W_{d,\text{ PIR}}$ is articulated as:

$$W_{d,PIR} = (S_k - \tau_L).\,1_{\{S_{K-1} < \tau_L \le S_K\}} = \int_{S_{K-1}}^{S_K} 1_{\left\{X_{S_{k-1}} < M < L \le X_t\right\}}\, dt \tag{10}$$

The mean MCPRC of the PIR strategy $\mu^{PIR} = E[K^{PIR}]$ is thus computed as

$$\mu^{PIR} = \sum_{k=1}^{\infty} \frac{1}{S_k} \int_0^M \Big(\int_{M-x}^{L-x} (C_p\,(x+z) + k\,C_i) \times f_{\alpha\Delta T,\beta}\,(z) dz + \int_{L-x}^{\infty} (C_c\,(x+z) + k\,C_i\,) f_{\alpha\Delta T,\beta}\,(z) \quad dz + C_d \int_{S_{k-1}}^{S_k} \bar{F}_{\alpha(t-S_{k-1}),,\beta}\,\, (L-x) \quad dt \Big)\, f_{\alpha S_{k-1},\beta}\,(x) dx \tag{11}$$

where $f_{\alpha(.),\beta}$ and $\bar{F}_{\alpha(.),\beta}$ are derived from (1), (2) respectively. The associated mean of square $E[(K^{PIR})^2]$ is given by

$$E[(K^{PIR})^2] = \sum_{k=1}^{\infty}\{ \int_0^M (\int_{M-x}^{L-x} \left(C_p\,(x+z) + k\,C_i\right)^2 \times f_{\alpha\Delta T,\beta}\,(z)dz + \int_{L-x}^{\infty} (C_c\,(x+z) + k\,C_i)^2 \times f_{\alpha\Delta T,\beta}\,(z)\ \ dz \ ) f_{\alpha S_{k-1},\beta}\,(x)dx + 2C_d \int_{S_{k-1}}^{S_k} \left(\int_0^M (\int_{L-x}^{\infty} \left(\int_{x+z}^{\infty} (C_c\,(y) + k\,C_i)\ f_{\alpha S_k,\beta}\,(y)dy \right) f_{\alpha.(t-S_{k-1}),\beta}\,(z)dz\,)\ f_{\alpha S_{k-1},\beta}\,(x)dx \right) dt + {C_d}^2 \int_{S_{k-1}}^{S_k} (S_k - t)^2\ \ f_{\tau_L}\,(t)\ dt \ \} \ /S_k \quad , \tag{12}$$

To derive the formula for the standard deviation σ^{PIR} (standard deviation of the Mean Cost Per Renewal Cycle) within the framework of the PIR (preventive inspection and replacement) strategy, we need to incorporate certain equations and factors.

$f_{\alpha(\cdot),\beta}$ represents the cumulative distribution function (CDF) of the time to failure for the system. It provides information about the probability that the system will fail within a given time interval. α is a parameter associated with the failure distribution.

β represents the fraction of the failure cost compared to the cost of a new system. It quantifies the relative expense of failure to the replacement cost.

$f_{\tau L}$ represents the probability density function (PDF) of the time between preventive replacements. It describes the likelihood of needing a preventive replacement at a specific time interval. τ_L is a parameter associated with the preventive replacement distribution.

Equations (1) and (4) are used to deduce these probability functions, taking into account the system's failure behaviour and the occurrence of preventive replacements.

By incorporating equations (11) and (12) into equation (8), we can derive the formula for σ^{PIR}, the standard deviation of the MCPRC. Equation (8) represents the mean cost per renewal cycle within the PIR strategy.

The standard deviation σ^{PIR} provides a measure of the variability or dispersion of the MCPRC values. It allows us to assess the level of uncertainty associated with the average cost estimation for a renewal cycle under the PIR strategy.

By considering the failure characteristics, the cost components, and the probability distributions associated with preventive replacements, we can calculate σ^{PIR}. This information helps in evaluating the potential cost fluctuations and risks involved in implementing the PIR strategy for system maintenance and replacement.

3.3.2. Standard formulation of the MCPRC for the QIR policy:

The equation for the Mean Cost Per Renewal Cycle (MCPRC) of the QIR (Quality Improvement and Replacement) policy, assuming the standard deviation, may be determined using a similar approach as the one used for the PIR (Preventive Inspection and Replacement) strategy. In the purpose of this study, we assume that the system is changed either preventively or correctively at the k-th inspection time, where k spans from 1 to an arbitrary number.

To articulate the MCPRC of the QIR policy over a renewal cycle, we use the symbol K^{QIR} to represent it. The expression for K^{QIR} can be stated as follows:

$$K^{QIR} = \frac{1}{\sum_{k=1}^{\infty} T_k.\,1_{\{X_{T_{k-1}} < M \le X_{T_k}\}}} . \sum_{k=1}^{\infty} (\left(C_p\left(X_{T_k}\right) + kC_i\right).\,1_{\{X_{T_{k-1}} < M \le X_{T_k} < L\}} + \left(C_c\left(X_{T_k}\right) + kC_i\right).\,1_{\{X_{T_{k-1}} < M < L \le X_{T_k}\}} + C_d W_{d,QIR} \ ) \quad , \tag{13}$$

where the downtime of the system over a renewal cycle under the QIR policy is obtained by

$$W_{d,QIR} = (T_k - \tau_L).\,1_{\{T_{K-1} < \tau_L \le T_K\}} = \int_{T_{K-1}}^{T_K} 1_{\{X_{T_{k-1}} < M < L \le X_t\}}\ dt \tag{14}$$

In addition, the determination of T_k is iteratively calculated based on equation (5). The analytical computation of $\mu^{QIR} = E[K^{QIR}]$ and $E[(K^{QIR})^2]$ from equation (13) poses significant challenges, primarily due to the dynamic nature of the inspection schedule. Consequently, our attention is directed toward acquiring the standard deviation σ^{QIR} of the Mean Cost Per Renewal Cycle (MCPRC) for the QIR policy by employing a Monte Carlo simulation methodology.

4. Maintenance Strategies Assessment

The evaluation of strategy success demands the application of frequently adopted criteria in the literature, known as the long-term projected maintenance cost rate. This criteria has attracted substantial interest and has been thoroughly researched to measure the efficiency of different tactics [3]. Its mathematical model, derived from the standard renewal-reward theorem, offers a rigorous framework for analyzing and comparing the long-term maintenance costs associated with alternative techniques. this condition may be quantitatively stated as [4]:

$$C_\infty = \lim_{t\to\infty} \frac{E[C(t)]}{t} = \frac{E[C(S)]}{E[S]} \qquad (15)$$

The period of a renewal cycle is represented as S, and $C(S)$ reflects the cumulative maintenance cost incurred throughout this cycle. Equation (15) focuses entirely on the mean values of the renewal cycle and its related maintenance cost, neglecting the fluctuation in maintenance costs from one cycle to another. Consequently, depending merely on the long-term predicted maintenance cost rate may not effectively evaluate maintenance solutions in terms of both performance and robustness.

To alleviate this constraint, a suggested cost criteria involves combining the long-term predicted maintenance cost rate C_∞ with the standard deviation of the MCPRC (Maintenance Cost per Renewal Cycle). The combined criteria may be represented as:

$$\varphi = C_\infty + \lambda.\sigma; \qquad \lambda \geq 0. \qquad (16)$$

The mathematical equations for σ under the PIR (Proportional Interval Replacement) method and the QIR (Quadratic Interval Replacement) approach have been supplied in Section III-C. The process for establishing their long-term projected maintenance cost rates is comparable, and its specifics may be found in [6]. The coefficient λ in equation (16) shows the proportional weight given to cost variability, compared to the mean, in affecting decision-making. When $\lambda < 1$, decision-makers emphasize the performance of maintenance measures. Conversely, when $\lambda > 1$, they trend towards preferring the robustness of the techniques. Thus, the suggested cost criteria show more suited than the long-term projected maintenance cost rate for assessing maintenance techniques from both performance and robustness viewpoints. Furthermore, the former reverts to the latter when $\lambda = 0$. By utilizing these cost criteria, the optimization of the PIR strategy and the QIR method includes calculating their optimum decision parameters that minimize φ in equation (16).

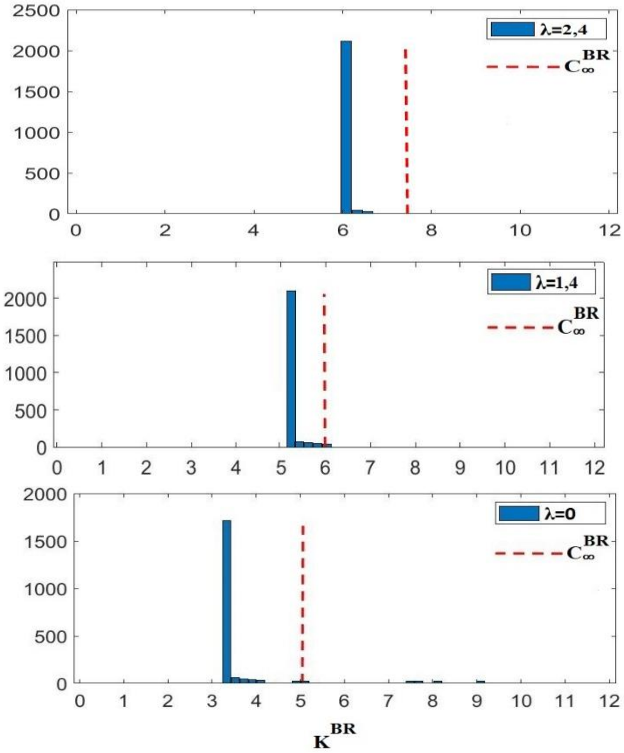
To demonstrate the benefits of the new criteria, let's use a system characterized by the parameters $\alpha = 0.1$, $\beta = 0.1$, and $L = 29$. For simplicity, we assume the CR (Corrective Repair) cost stays constant, whereas the PR (Preventive Replacement) cost is represented as a quadratic function of the deterioration degree. The maintenance cost values are chosen as follows: $C_i = 5$, $C_d = 34$, and $C_c = 98$. The link between maintenance costs is described as :

$$C_p(X_t) = C_0 + \frac{C_c + C_0}{2}\left(\frac{X_t - M_s}{L - M_s}\right)^2 \quad 1_{\{M_s < X_t < L\}}, \qquad (17)$$

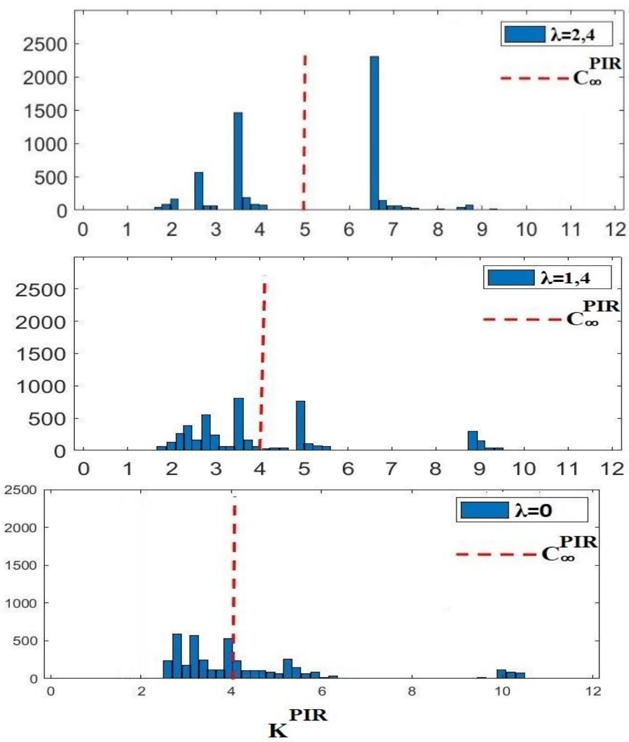
where $C_0 = 48$ indicates the fundamental cost of PR and $M_s = 14$ marks the system threshold.

Applying the PIR method and the QIR strategy to the investigated system leads to their optimum configurations, long-term predicted cost rates, and the accompanying standard deviations of MCPRC for different values of λ, as provided in Table 1. The table indicates that, for both techniques, as λ grows, the standard deviation of MCPRC reduces. This is a consequence of introducing maintenance cost variability into the optimization process.

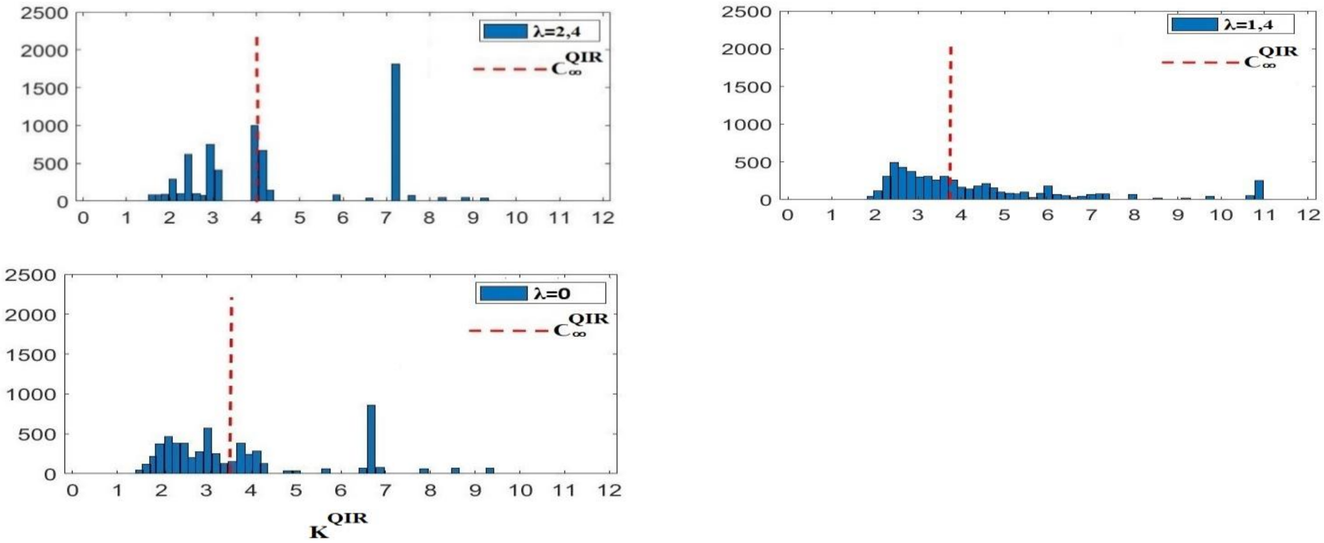
To offer a more visual and intuitive understanding of the distribution of MCPRC values, histograms (visual representations) of the MCPRC for both the PIR and QIR techniques have been generated. Figure 1 depicts these histograms, constructed using Monte-Carlo simulation based on the maintenance cost variables indicated previously. The histograms give a supplementary viewpoint to the information offered in Table 1, allowing for a better assessment of cost changes across various situations.



(a) BR strategy



(b) PIR strategy



(c) QIR strategy

Fig. 1. MCPRC Histogram of BR,PIR and QIR strategies.

Tab. 1. Optimal configurations φ of the BR, PIR, and QIR strategies

Strategies	Relative weight	Optimal decision variables	Optimal configurations of φ
BR	$\lambda = 1.4$	$T_{opt} = 9.70$	$\varphi_{opt}^{BR} = 11.738$
PIR	$\lambda = 1.4$	$\Delta T_{opt} = 6.19$ $M_{opt} = 12.8$	$\varphi_{opt}^{PIR} = 9.864$
QIR	$\lambda = 1.4$	$\alpha_{opt} = 0.55$ $M_{opt} = 18.3$	$\varphi_{opt}^{QIR} = 9.628$

5. Maintenance Strategies Comparisons

In this part, a detailed comparative study is undertaken to assess the performance and resilience of the PIR and QIR maintenance techniques in diverse circumstances. The idea is to discover which approach is better fit for given aims or scenarios. By studying the changes in optimum decision variables for various techniques, insights are acquired into the ideal settings for different combinations of maintenance costs and system features. This study aids educated decision-making when choosing maintenance techniques based on unique demands and conditions. The paper gives a systematic way to analyze and pick solutions, including aspects such as maintenance costs, system features, and the relative weight parameter, λ. By assessing the efficacy and resilience of the PIR and QIR techniques across varied situations, informed judgments may be made to optimize maintenance efforts and successfully manage complex systems.

5.1. Sensitivity to the maintenance costs

It seems that after studying the influence of CR (Corrective Repair) and PR (Preventive Repair) expenses, it has been shown that these costs do not substantially alter the differences between the PIR (Periodic Inspection and Repair) and QIR (Condition-based Inspection and Repair) methods. As a consequence, you have taken the choice to maintain the CR cost unchanged at $C_c = 98$.

Furthermore, you have provided the PR cost function according to equation (17), where $C_0 = 48$. This equation enables you to determine the PR cost depending on specified characteristics and situations.

To analyze the implications of maintenance expenditures, especially on the inspection side, you have completed the first case study. In this research, you are introducing modifications in the inspection cost (C_i) and examining a range from 1 to 45, with an increment of 1 for each step. It is vital to remember that throughout this study, the system downtime cost rate (C_d) stays constant at a set value of 19.

Moving on to the second case study, you are studying differences in the system downtime cost rate (C_d). Here, you are evaluating a range from 10 to 50, with increments of 1 for each step. However, in this study, you are keeping the inspection cost (C_i) constant at a set value of $C_i = 7$.

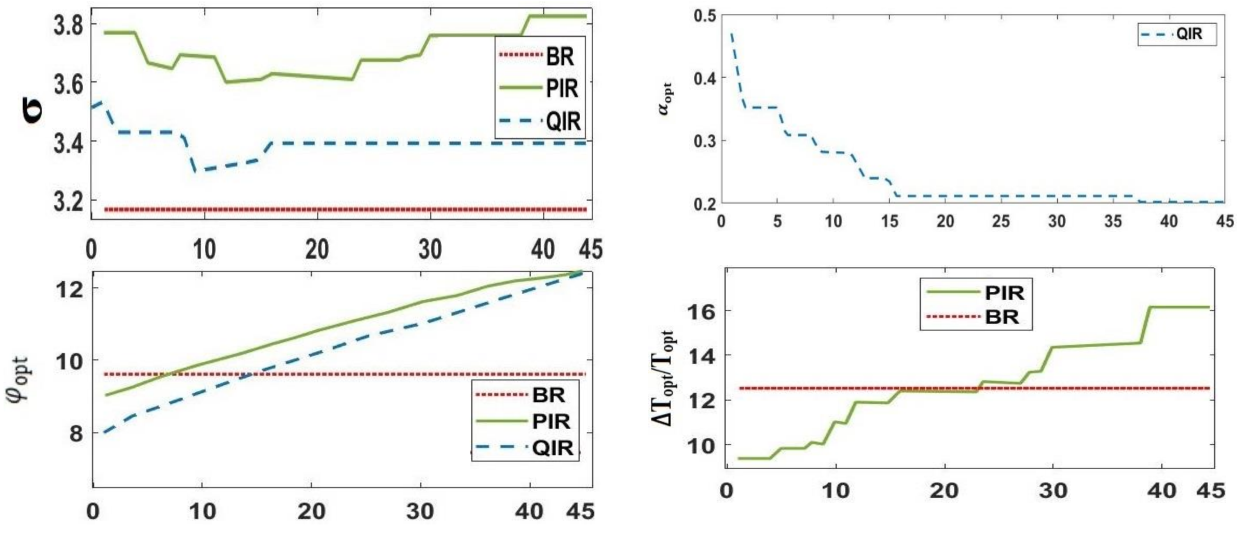
In all of these case examples, you have provided the relative weight parameter λ a precise value of 1.4. This metric plays a key role in defining the relative relevance of maintenance costs and system performance.

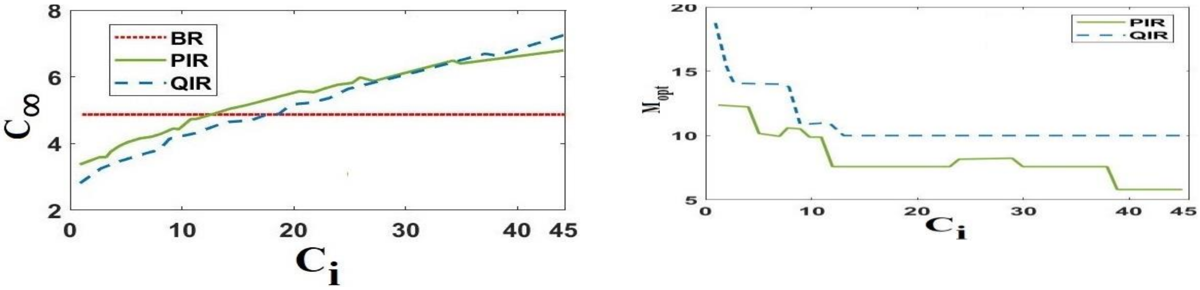
Additionally, to offer a full overview of the system, you have established the following system characteristics: $\alpha = \beta = 0.1$, $L = 29$, and $M_s = 14$. These factors indicate particular elements of the system, such as failure rates, maintenance efficacy, system size, and repairable components.

Here's a more detailed explanation of Case Study 1 and Case Study 2:

Case Study 1:

In case study 1, the focus is on understanding how the inspection cost (C_i) affects the QIR and PIR strategies. The results are presented in Figure 2, which provides a comprehensive visualization of the performance and cost implications of these strategies.





(a) Cost functions : σ, ϕ_{opt} and C_∞ (b) Optimal decision variables : α_{opt}, T_{opt} , ΔT_{opt} and M_{opt}

Fig. 2. Varied inspection cost

In Figure 2a, the graph represents the changes in the optimal value of the cost function ϕ as the inspection cost (C_i) varies. It also illustrates how these variations in C_i impact the long-run expected cost rate $C\infty$ for both the QIR and PIR strategies. Additionally, the graph displays the standard deviation of MCPRC (Mean Cost per Repair Cycle) as C_i changes. This visual representation allows for a clear understanding of how changes in the inspection cost influence different aspects of the cost function and performance measures for both strategies.

The analysis reveals that the QIR strategy outperforms the PIR strategy in terms of cost-effectiveness. Furthermore, it demonstrates that the QIR strategy is more reliable and consistent, as indicated by its consistently smaller standard deviation of the MCPRC. This lower variability suggests that the QIR strategy yields more predictable outcomes compared to the PIR strategy.

These findings align with expectations, as the QIR strategy is known for its adaptability. By adjusting the inspection intervals based on the current degradation level of the system, the QIR strategy effectively manages system downtime.

Interestingly, the standard deviation (σ) remains relatively stable for both strategies, indicating that changes in the inspection cost (C_i) have a limited impact on the robustness of the QIR and PIR strategies. In other words, variations in inspection costs do not strongly affect the reliability and consistency of these maintenance strategies.

The analysis demonstrates that the QIR strategy is more cost-effective than the PIR strategy up to a certain value of C_i, which in this case is $C_i = 27$. Beyond this threshold, the PIR strategy becomes more cost-effective. The examination of the optimal values presented in Figure 2b provides further insights into the strategies' behaviors:

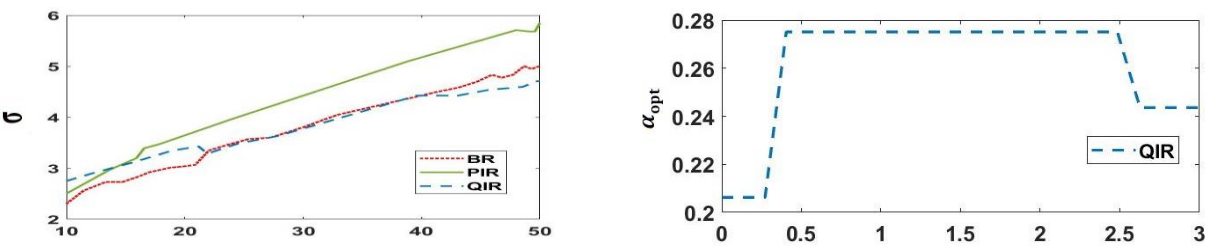
For the QIR strategy, the optimal value of α (α_{opt}) decreases as C_i increases. This suggests that the QIR strategy adjusts α_{opt} to lower values, reducing the number of inspections as the inspection cost rises.

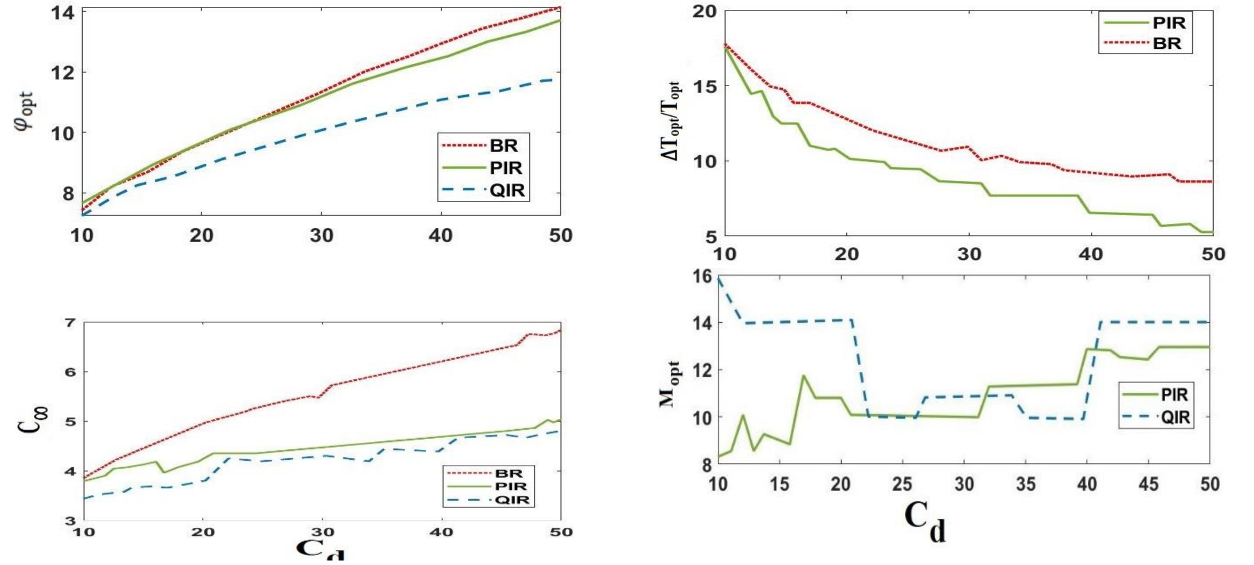
In contrast, for the PIR strategy, the optimal value of ΔT (ΔT_{opt}) increases as C_i increases. This implies that the PIR strategy increases ΔT_{opt}, leading to fewer inspections as the inspection cost becomes higher.

These adjustments in α_{opt} and ΔT_{opt} reflect the strategies responses to changes in inspection costs. Each strategy aims to optimize its performance and cost-effectiveness based on the cost implications associated with inspections.

Case Study 2:

In case study 2, the focus shifts to understanding the impact of the system downtime cost rate (C_d) on the robustness and adaptability of the QIR and PIR strategies. The results are presented in Figure 3, providing insights into the strategies' performance and decision-making.





(a) Cost functions : σ, ϕ_{opt} and C_∞ (b) Optimal decision variables : α_{opt}, T_{opt} , ΔT_{opt} and M_{opt}

Fig. 3. Varied system downtime cost rate

Figure 3a showcases the optimal cost function ϕ_{opt} for both the QIR and PIR strategies, along with the associated long-run expected maintenance cost rate C_∞ and the standard deviation of MCPRC (σ) to the system downtime cost rate (C_d). This visualization allows for a comprehensive understanding of the strategies' behavior under varying Cd values.

Figure 3b presents the evolutions of the optimal decision variables, providing further insights into the strategies' adaptations.

Similar to case study 1, the QIR strategy demonstrates higher profitability and reliability compared to the PIR strategy. The system downtime cost rate (C_d) plays a significant role in determining the robustness of both strategies. As C_d varies, both strategies exhibit predictable changes in their behavior. They become more stringent in controlling system downtime as C_d increases.

The QIR strategy adjusts a certain parameter (α_{opt}) to optimize its performance, while the PIR strategy modifies another parameter (ΔT_{opt}) to adapt to changing conditions. Additionally, both strategies fine-tune the parameter M_{opt} to further optimize their performance. Overall, the QIR strategy outperforms the PIR strategy in terms of adaptability, and both strategies effectively adjust their settings to minimize system downtime as the system downtime cost rate changes.

These findings provide valuable insights into the behavior and performance of the QIR and PIR strategies under varying inspection costs (C_i) and system downtime cost rates (C_d), helping decision-makers make informed choices regarding maintenance strategies.

Here's a more extensive explanation of Case Study 1 and Case Study 2:

Case Study 1:

In case study 1, the emphasis is on understanding how the inspection cost (C_i) influences the QIR and PIR techniques. The findings are provided in Figure 2, which gives a thorough depiction of the performance and cost consequences of different solutions.

In Figure 2a, the graph depicts the variations in the optimum value of the cost function ϕ as the inspection cost (C_i) fluctuates. It also indicates how these differences in Ci affect the long-run projected cost rate C∞ for both the QIR and PIR techniques. Additionally, the graph depicts the standard deviation of MCPRC (Mean Cost per Repair Cycle) as Ci fluctuates. This graphic depiction clearly explains how changes in the inspection cost affect various components of the cost function and performance measurements for both techniques.

The research demonstrates that the QIR method beats the PIR technique regarding cost-effectiveness. Furthermore, it reveals that the QIR method is more trustworthy and consistent, as evidenced by its constantly decreased standard deviation of the MCPRC. This decreased variability shows that the QIR technique provides more predictable results compared to the PIR strategy.

These results match with predictions, given the QIR method is recognized for its versatility. By modifying the inspection intervals depending on the present degradation level of the system, the QIR technique efficiently controls system downtime.

Interestingly, the standard deviation (σ) stays rather steady for both techniques, demonstrating that changes in the inspection cost (C_i) have little influence on the robustness of the QIR and PIR procedures. In other words, fluctuations in inspection costs do not dramatically influence the dependability and consistency of these maintenance procedures.

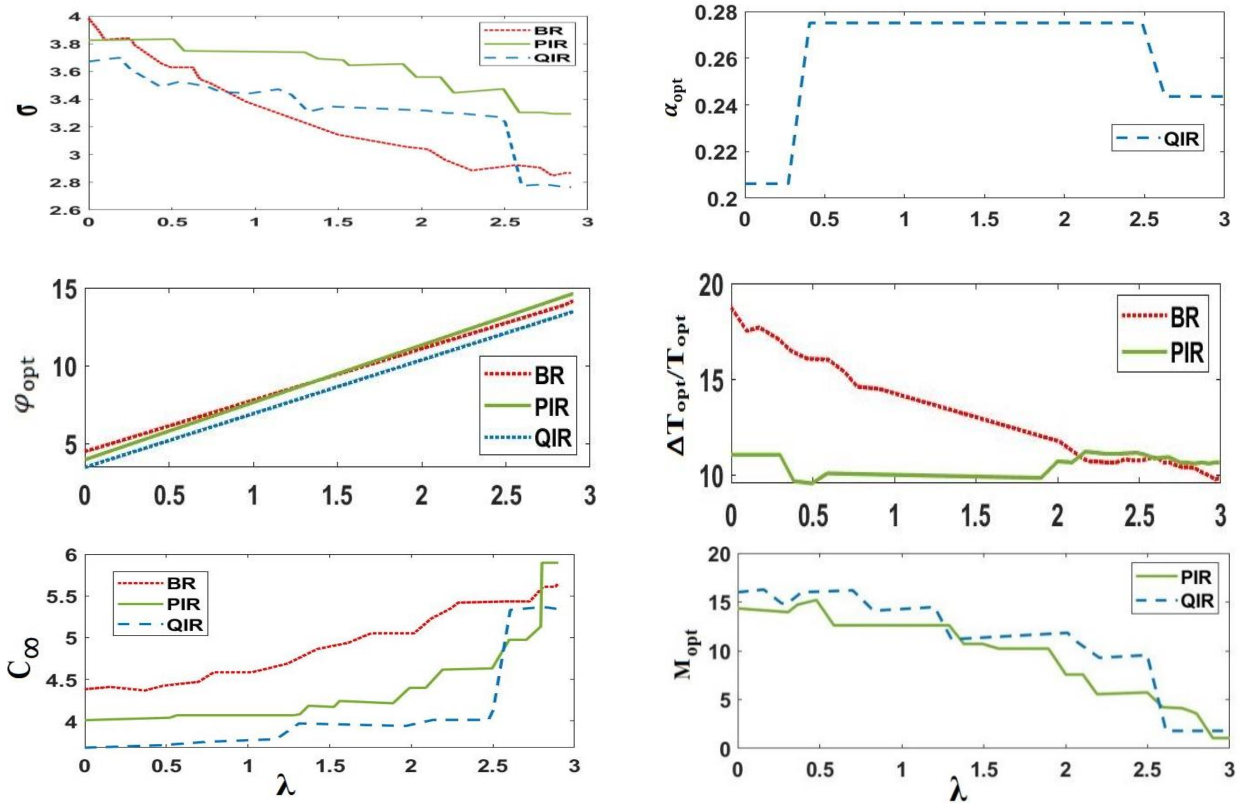
The study reveals that the QIR technique is more cost-effective than the PIR strategy up to a specific value of C_i, which in this instance is $C_i = 27$. Beyond this level, the PIR method becomes more cost-effective. The evaluation of the optimum values shown in Figure 2b gives more insights into the strategies' behaviors:

For the QIR technique, the ideal value of (α_{opt}) drops as C_i grows. This shows that the QIR method adjusts α_{opt} to lower values, lowering the frequency of inspections as the inspection cost grows.

In contrast, with the PIR method, the optimum value of (ΔT_{opt}) grows as C_i increases. This means that the PIR method increases ΔT_{opt}, leading to fewer inspections as the inspection cost grows larger.

These revisions in α_{opt} and ΔT_{opt} indicate the strategies' reactions to changes in inspection costs. Each method seeks to maximize its performance and cost-effectiveness depending on the financial implications connected with inspections.

Case Study 2: In case study 2, the attention changes to studying the influence of the system downtime cost rate (C_d) on the robustness and flexibility of the QIR and PIR techniques. The findings are displayed in Figure 4, offering insights into the strategies' performance and decision-making.



(a) Cost functions : σ, ϕ_{opt} and C M_{opt}

(b) Optimal decision variables : α_{opt}, T_{opt} , ΔT_{opt} and

Fig. 4. Varied relative weight of the cost variability

Figure 4a highlights the ideal cost function ϕ_{opt} for both the QIR and PIR techniques, together with the related long-run projected maintenance cost rate C_∞ and the standard deviation of MCPRC (σ) to the system downtime cost rate (C_d). This image offers a full understanding of the strategies' behavior under varied Cd levels.

Figure 4b displays the evolutions of the optimum choice variables, offering deeper insights into the strategies' adaptations.

Similar to case study 1, the QIR method offers improved profitability and dependability compared to the PIR technique. The system downtime cost rate (C_d) has a key role in determining the resilience of both techniques. As C_d fluctuates, both techniques demonstrate predictable variations in their behavior. They become more strict in managing system downtime as Cd grows.

The QIR strategy changes a particular parameter (α_{opt}) to improve its performance, whereas the PIR strategy alters another parameter (ΔT_{opt}) to adapt to changing circumstances. Additionally, both techniques fine-tune the parameter Mopt to further enhance their performance. Overall, the QIR method beats the PIR strategy in terms of flexibility, and both strategies efficiently update their settings to reduce system downtime when the system downtime cost rate varies.

These results give useful insights into the behavior and performance of the QIR and PIR techniques under changing inspection costs (Ci) and system downtime cost rates (C_d), allowing decision-makers to make educated decisions about maintenance strategies.

5.2. Sensitivity to the relative weight of the cost variability

In this subsection, we delve into the influence of the relative weight parameter λ on the selection of a maintenance strategy. This parameter represents the balance between financial viability and risk tolerance for decision-makers when choosing a maintenance strategy. To conduct this investigation, we set the system characteristics as follows: α = β = 0.1, L = 29, and Ms = 14. Additionally, we fixed the maintenance costs at specific values: C_i = 7, C_d = 19, C_c = 98, and C_0 = 48. By keeping these system characteristics and maintenance cost parameters constant, we can vary λ to understand its effect on the choice and performance of maintenance strategies. This allows us to assess the trade-off between financial considerations and risk management in decision-making.

Regardless of the relative weight parameter λ, it is consistently observed that the QIR strategy outperforms the PIR strategy and exhibits higher robustness. This implies that the QIR strategy is a superior choice, regardless of the specific value of λ.

However, as λ increases, a trade-off emerges between robustness and cost performance. On one hand, the robustness, represented by the standard deviation (σ) of costs, improves as λ increases. This means there is less variation in costs, resulting in more predictable outcomes and reduced risk.

On the other hand, as λ increases, the cost performance metrics, such as the long-run expected cost rate C_∞ and the optimal cost function φ_{opt}, worsen. This indicates that the overall cost-effectiveness of the maintenance strategy decreases as the weight of risk tolerance (represented by λ) becomes more significant.

To adapt to this trade-off, the PIR strategy adjusts a single parameter, M_opt, to control cost variability. By optimizing M_{opt}, the PIR strategy aims to minimize fluctuations in costs and maintain a certain level of stability.

In contrast, the QIR strategy takes a more comprehensive approach by simultaneously optimizing two parameters, α_{opt} and M_{opt}. This allows the QIR strategy to achieve the best balance between performance and robustness. By fine-tuning both α_{opt} and M_{opt}, the QIR strategy optimizes the inspection and maintenance intervals to achieve an overall superior outcome.

In essence, the QIR strategy consistently maintains its superiority over the PIR strategy, regardless of the value of λ. However, increasing λ improves the robustness of the strategies, reducing cost variability and increasing predictability. Nonetheless, this improvement in robustness may come at the expense of cost performance metrics, such as C_∞ and φ_{opt}.

The PIR strategy focuses primarily on controlling cost variability, while the QIR strategy takes a more holistic approach, optimizing multiple parameters for better performance and robustness.

6. Managerial Insights

This section aims to distill our research findings into actionable insights that can guide strategic decision-making in maintenance practices. Here are the key points we have emphasized:

- Strategic Selection of Maintenance Techniques: We elaborate on the conditions under which the QIR strategy demonstrates superiority over the traditional PIR strategy. By detailing scenarios where each strategy excels, managers can better align their choice of maintenance technique with specific operational goals and constraints.
- Balancing Cost Efficiency and System Resilience: We delve into the trade-off between cost-effectiveness and robustness inherent in maintenance strategies. This discussion helps managers understand how to balance the need to minimize costs with the imperative to maintain system reliability and performance over the long term.
- Impact of Parameter Variations on Strategy Performance: We provide clear insights into how variations in critical parameters such as inspection costs (Ci), system downtime rates (Cd), and the

relative weight parameter (λ) influence the overall effectiveness of maintenance strategies. This guidance enables managers to make informed decisions when adjusting these parameters to optimize maintenance outcomes.

- Implementation Strategies and Practical Considerations: We offer practical advice on implementing QIR and PIR strategies within diverse operational contexts. This includes considerations for adapting strategies to changing system conditions, managing financial constraints, and leveraging predictive maintenance techniques to enhance operational efficiency and minimize downtime.
- Case Studies and Real-World Applications: Drawing on our case studies and numerical results, we illustrate real-world applications of QIR and PIR strategies in different industrial settings. These examples provide concrete examples of how these strategies can be effectively deployed to achieve significant improvements in maintenance performance.

These additions aim to bridge the gap between theoretical findings and practical applications, equipping decision-makers with actionable insights to optimize maintenance strategies in their respective industries.

7. Comparison of This Study with Previous Research:

In this section, we present a comparative analysis that highlights the distinctive contributions of our study within the context of existing literature on maintenance strategies. The table 2 provided below outlines key differences between our work and other reviewed papers, focusing on methodologies employed, evaluation metrics utilized, main contributions, key findings, and the practical implications for Condition-Based Maintenance (CBM) strategies.

Tab. 2. Maintenance strategies optimization of the BR, PIR, and QIR strategies

Strategies	Relative weight	Optimal decision variables	Long-run expected cost rate	Standard deviation of MCPRC
BR	$\lambda = 2.4$	$T_{opt} = 8.20$	$C_{\infty}^{BR} = 6.713$	$\sigma_{\infty}^{BR} = 3.721$
	$\lambda = 1.4$	$T_{opt} = 9.70$	$C_{\infty}^{BR} = 5.986$	$\sigma_{\infty}^{BR} = 4.109$
	$\lambda = 0$	$T_{opt} = 15.90$	$C_{\infty}^{BR} = 5.054$	$\sigma_{\infty}^{BR} = 5.682$
PIR	$\lambda = 2.4$	$\Delta T_{opt} = 8.29$ $M_{opt} = 4.80$	$C_{\infty}^{PIR} = 4.982$	$\sigma_{\infty}^{PIR} = 4.409$
	$\lambda = 1.4$	$\Delta T_{opt} = 6.19$ $M_{opt} = 12.8$	$C_{\infty}^{PIR} = 4.103$	$\sigma_{\infty}^{PIR} = 4.115$
	$\lambda = 0$	$\Delta T_{opt} = 5.50$ $M_{opt} = 16.50$	$C_{\infty}^{PIR} = 4.025$	$\sigma_{\infty}^{PIR} = 5.009$
QIR	$\lambda = 2.4$	$\alpha_{opt} = 0.37$ $M_{opt} = 8.80$	$C_{\infty}^{PIR} = 4.063$	$\sigma_{\infty}^{PIR} = 4.020$
	$\lambda = 1.4$	$\alpha_{opt} = 0.55$ $M_{opt} = 18.3$	$C_{\infty}^{PIR} = 3.832$	$\sigma_{\infty}^{PIR} = 4.140$
	$\lambda = 0$	$\alpha_{opt} = 0.34$ $M_{opt} = 15.60$	$C_{\infty}^{PIR} = 3.667$	$\sigma_{\infty}^{PIR} = 4.487$

This comparative analysis serves to underscore the unique insights and advancements offered by our study, particularly in optimizing maintenance strategies through the application of composite cost criteria and Condition-Based Maintenance (CBM) principles. By contrasting our approach with existing literature, we aim to elucidate the specific areas where our research contributes novel perspectives and practical guidance for industry practitioners and decision-makers.

Key Differences:

- Focus : This study focuses on optimizing maintenance strategies (PIR and QIR), whereas other papers explore a wider range of maintenance approaches.
- Methodology: We utilize composite cost criteria and CBM principles, contrasting with studies that rely primarily on empirical data and statistical models.
- Evaluation Metrics: Our study evaluates based on long-run expected cost rate and variability in Mean Cost per Repair Cycle (MCPRC), while other studies often use metrics such as MTBF and direct repair

costs.

- Main Contribution : Our research contributes by analyzing the trade-offs between performance and resilience in maintenance strategies, providing practical insights for industry decision-makers.
- Key Findings: We find that the QIR method consistently outperforms PIR in terms of cost-effectiveness and resilience. In contrast, other studies report varying outcomes regarding the optimal maintenance strategy to adopt.
- Impact : Our findings have practical implications for implementing CBM strategies in real-world scenarios, contrasting with theoretical frameworks that lack direct applicability in industrial settings.

This table 2 provides a clear numerical example of how to compare your study with other reviewed papers, highlighting specific differences in focus, methodology, evaluation metrics, main contributions, key findings, and impact. Adjust the specifics based on your actual study and the papers you have reviewed.

8. Conclusion and Perspectives

This study undertook a comprehensive analysis of two primary maintenance strategies: the Periodic Inspection and Replacement (PIR) and the quantile-based Inspection and Replacement (QIR) approaches. We utilized a composite cost criterion combining long-term predicted maintenance cost rates and the standard deviation of the Mean Cost per Repair Cycle (MCPRC) to evaluate these strategies under Condition-Based Maintenance (CBM) principles, emphasizing cost-effectiveness and adaptability to cost fluctuations.

Our findings underscored a fundamental trade-off inherent in maintenance strategies: the inverse relationship between performance and resilience. Strategies prioritizing performance enhancements often compromise on robustness, and vice versa, posing a significant dilemma for decision-makers aiming to optimize maintenance practices.

Numerical analyses consistently favored the QIR method over PIR across various performance metrics. The QIR strategy demonstrated superior cost-effectiveness and performance, suggesting its potential to minimize long-term maintenance costs while maintaining high system performance levels. Moreover, our analysis highlighted the substantial impact of downtime-related maintenance expenses on the resilience of maintenance plans, with QIR proving more adept at managing disruptions compared to PIR.

In terms of contributions, this research provided a detailed performance evaluation of PIR and QIR strategies, offering decision-makers insights into their relative strengths and weaknesses. Additionally, it elucidated the inherent trade-off between performance gains and robustness, clarifying the implications of maintenance strategy choices for system reliability and operational continuity. The practical implications of these findings are significant for optimizing maintenance planning in industries where system uptime and maintenance costs significantly impact operational efficiency and profitability.

9. Acknowledgements

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10. Competing Interests

No potential conflict of interest was reported by the author.

11. Data Availability Statements

The code that supports the findings of this study are available upon request to the corresponding author.

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