International Journal of Industrial Engineering & Production Research March 2022 Vol. 33, No. 1: 1-19
DOI: 10.22068/ijiepr.33.1.8

RESEARCH PAPER

Determination of the Safety Stock for Intermittent Demand: the Grey and Hybrid Theories

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Received 3 October 2021; Revised 11 January 2022; Accepted 22 January 2022;
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ABSTRACT
This study seeks to introduce the influential factors in controlling and dealing with uncertainty in intermittent demand. Hybrid forecasting and Grey Theory have been used simultaneously due to their potential in facing complex nature and insufficient data. Different modeling and unbiased weighting results have been used to estimate the safety stock(SS) by theoretical and experimental methods. In other words, this work deals with the less studied feature of various modeling errors and their effect on SS determination and recommends its use to address the uncertainty of intermittent demand as a criterion for introducing a superior model in the field of inventory.

KEYWORDS: *Intermittent demand; Safety stock; Grey forecasting; Hybrid modeling; Croston method.*

1. Introduction

Uncertainty in intermittent demand has increased the risk of managing their inventory. So, most of the uncertainty in the demand occurrence, the amount of this, and the time interval between two consecutive non-zero demands can be sought. The chaotic dispersion of zero-periods among non-zero disparate demands has highlighted their time series. In the face of this uncertainty, using safety stock (SS) has been accepted as an efficient strategy [1]. This study seeks a reliable estimate for this value by considering the influential factors in the hybrid forecasting of intermittent demand.

Although it is common to examine accuracy criteria when comparing procedural forecasting models, using them without considering other error characteristics can lead to incorrect decisions. According to conventional theoretical formulas, the proper estimation of forecast error and its normality affect the adjustment of SS level. Hybrid models are used to achieve the former. Using them is to receive and model all the structures and patterns in intermittent data.

Managers need to achieve accurate forecasts to plan in all areas [2][3][4][5][6][7]. They also need decision-making methods in situations of uncertainty [8][9]. Inventory management always requires review and research on spare parts and safety stock, so different methods have been used for it [10][11][12]. The high capability of gray theory in the face of systems uncertainty has been demonstrated in research [13][14].

Insufficient data and lack of transparency make this type of demand system similar to grey systems. Grey System Theory shows acceptable performance in dealing with these features [15]. The Grey forecasting model is at the core of this theory. However, since this potential has not been well used in this type, the performance of the hybrid model's different structures is examined by several weighting methods in this paper, and the best model is reported according to the obtained results. In this study, this method's performance and behavior in estimating intermittent demand uncertainty in different structures of the hybrid model are also investigated. The second influential factor is investigated by measuring the deviation from different models' normal error distribution. Besides, error bias is also considered as another influential factor.

Given that the hybrid forecasting model in intermittent demand has not yet become a common procedure, it cannot be expected that these models' error characteristics will be

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comprehensively investigated, and definitive results will be available. Depending on the complexity and way of producing the results, examining their statistical distribution can be done differently. This research investigates the effect of varying modeling on their output.

This study also examines the behavioral similarities between in-sample and out-of-sample errors of hybrid models with different structures. This similarity improves managers' decision-making in inventory control of this demanding type. In theoretical calculations, the normality of forecast error is expressed as a basic assumption. This hypothesis has not been questioned in many studies, while in the face of real-world data has been frequently violated [16]. The present study also verifies this hypothesis's correctness. A lack of normality can be affected by factors and does not depend only on the characteristics of the time series.

In cases where the assumption of normality is questioned, an experimental approach is recommended to obtain more reliable results. In this view, SS is estimated without considering the statistical distribution of the forecast error [17] [18]. Finally, the results of these two modeling techniques are compared with different inspection structures and their performance.

It should be noted that the motivation behind this study deals with intermittent demand uncertainty effectively by considering the factors affecting the improvement of SS estimation. The main research idea is taken from Croston's method, whose assumptions such as the independence of variables have been questioned in several studies.[19][20][21]. However, this hypothesis is still maintained even in Croston's improved method. Research into the need to consider this hypothesis went so far that Shenstone and Hyndman (2005)[21] stated that it could be ignored. Therefore, in this research, the mentioned hypothesis is examined, and in two cases, modeling is performed, and its acceptance is subject to obtaining acceptable results.

This paper is organized as follows: First, a brief description of the concepts used in section 2 is presented. Then the proposed hybrid model, its sub-models, and the weighting methods used are described in sections 3, 4, respectively. Explanations related to reliability storage are expressed in both theoretical and empirical perspectives in section 5. The following three sections include the comparative approach used throughout the article, the data set, and a brief explanation of the models' implementation. Expressing the results and discussing the findings in section 9, including the Common Assessment, examining the statistical distribution, and evaluating the amount of SS calculated in two perspectives for different modeling, are described in detail in this section. In the last part, the conclusion is briefly stated.

2. Background Review

Supply chain management requires a variety of strategies. Selecting the right method to estimate demand accurately is effective [22]. Meanwhile, commodities with intermittent demand, such as electronics distributors in the supply chain, have become more critical, and researchers are always looking for accurate approximations [23]. This information leads to designing an agile, flexible production chain.

Hybrid models have penetrated the forecasting literature due to their better performance than individuals by applying accuracy assessment criteria [24][25]. Combined models with Grey Theory components have found a special place in the field of energy, including oil and electricity [26][27]. If these models' elements are selected intelligently, they can cover the shortcomings of individual models and show preferable overall performance. Hybrid research on intermittent demand has recently received attention [28].

The hybrid model is selected according to the demand type and the research objectives. Choosing a parallel hybrid is inevitable due to applying the forecasting results in inventory control management. Due to the complexity and ambiguity of the intermittent demand production processes, the individual model selection can lead to undesirable modeling of the behavioral patterns. Therefore, the hybrid model is smart because of the complication and confusion in this demanding type.

Nevertheless, it is common to examine the accuracy criteria of predictive results in selecting the superior model. But applying this approach without considering other error characteristics can lead managers to be misled into making decisions. Failure to evaluate the statistical distribution errors or check their lack of normality can adversely affect determining the SS levels adequately [29]. Few studies have been done on error characteristics, such as examining the symmetry of statistical distribution by comparing its mean and mean absolute value. Its effect on demand forecast (not intermittent demand) has been investigated in a handful of studies [30][31]. But, examining the normality of statistical distribution has an effective degree in controlling the product's risk.

Yet, research has been done on the relationship between forecast accuracy and system inventory

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reduction or retail advertising optimization [32][33]. In the high-risk fields of these types of goods and the simultaneous effect of the hybrid and Grey Theory, no study has been conducted that leads to controlling or reducing demand uncertainty. The efforts of Phillips (1979) can be mentioned as the first related research in this field [34]. He also realized the importance of error distribution. Nevertheless, many researchers have compared the proposed model error reduction.

Croston's method is one of the base and standard models for predicting intermittent demand [35]. This and its improvements have performed forecasting correctly [36]. The time series is broken down into two variables in the model, the interval between two consecutive non-zero demand $\boldsymbol{X_i(t)}$ and the non-zero demand value$\boldsymbol{Z_i(t)}$. Then, SES is executed on two series, and the results are divided. Thus the final forecast is obtained as a demand rate. We can refer to the mentioned article [37].

Addressing the challenge of intermittent demand can be summarized in the following five categories. Each of these methods focuses on a specific aspect of intermittent demand.1- Croston's variant includes improved methods based on the replacement of demand interval with demand probability [38] and its improvements. 2- Time series: [39] 3- Machine learning and neural network [40] [41] [20] 4- Temporal aggregation [42] 5- Bootstrap [43].

Although most intermittent research has been done on spare parts, anticipating this type of demand in retail and supply chain studies has also recently come to the attention of researchers [44][45].

Preventing chaos and dealing with all kinds of uncertainties along the supply chain requires accurate determination of SS [46]. The scope of the SS research is not limited to its value. Rather issues such as its location in the chain have been investigated, encountering this uncertainty [47]. SS estimation using different forecasting models' results has been the central issue in various studies [17].

Placement models are considered the factors influential on production [48], supply chain design, and management with different methods by different types of system costs and service levels [49] [50]. Whatever the method, these calculations can lead to more competitiveness and flexibility, improve customer satisfaction, gain more market share, and ultimately achieve a competitive advantage [1].

3. The Proposed Hybrid Model Structure

Applying the results obtained from the correct selection of the hybrid model components leads to gaining a competitive advantage for the superior model. Therefore, considering that the present study is based on Croston's method, it has used the SES model. It is selected as the first component of the hybrid members. The exponential smoothing group is considered one of the best known and oldest methods in point forecast and has provided good performance in the supply chain [51] [52].

Given the applicability of grey models to capture hidden rules and structures in small data and their relevance to the intermittent time-series characteristics, the grey forecasting model GM (1,1) is chosen. As the second component, this set of models has performed well in the face of system uncertainty and opacity [15].

The third component must be chosen smartly. This component should not overlap with the previous elements in estimating the production processes so that the modeling does not incur additional costs. At the same time, this model should potentially identify intermittent structure patterns. By reviewing the hybrid literature, ARMA models' family has provided useful results alongside the grey models and other models. According to the predictive purpose, these models have been present almost as a permanent member of the hybrid models any attitude, whether series or parallel hybrid [53] [54].

Considering each three base model in the proposed hybrid model is reasonable, and since the present work, for the first time, studies this set of base models by considering this set of weighting methods. The three base models' useful hybridization is selected according to the accuracy evaluation criteria, error characteristics, and the SS values of different structures. The selection of a valuable and efficient model is based on evidence from the models' implementation.

3.1. The grey forecasting model GM(1,1)

Mastering the theory of fuzzy systems and control theory, Deng (1982) first introduced Grey System Theory [55]. It includes five subsets of planning control, system analysis, forecasting, decision making, and modeling.

Grey Model: GM (1,1)

To execute the GM (1,1) [56] is considered as the primary data sequence $\boldsymbol{x^{(0)}}$.

(1)

These data are subjected to the function of the

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AGO (Accumulating Generation Operation) operator to specify the pattern of the internal order of the data and are converted to the sequence $\boldsymbol{x^{(1)}(k)}$.

$$x^{(0)}(k) = \left(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right),\ k=1,2,\dots,n\ ,\ n \geq 4\ ,\ x^{(0)}(k) \geq 0 \tag{1}$$

$$x^{(1)}(k) = \left(x^{(1)}(k)\right) = \left(\sum_{k=1}^{1} x^{(0)}(k), \sum_{k=1}^{2} x^{(0)}(k)\ , \dots, \sum_{k=1}^{n} x^{(0)}(k)\right) \tag{2}$$

Then we form the sequence $\boldsymbol{z^{(1)}(k)}$, which can be called the average sequence of the series $\boldsymbol{x^{(1)}}$. Researchers usually consider the value of α to be 0.5.

$$z^{(1)}(k) = (1-\alpha)x^{(1)}(k-1) + \alpha x^{(1)}(k)\ ,\ k = 2\ , \dots, n \quad , \alpha \epsilon (0,1) \tag{3}$$

The grey differential equation is as follows.

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{4}$$

Parameters a and b are the development and grey input coefficients, respectively. The white differential equation is produced by these two values as follows:

$$\frac{d\, x^{(1)}(k)}{dt} + ax^{(1)}(k) = b \tag{5}$$

It is possible to identify and forecast the system's behavior using these equations. Using the least squares method, the forecast values can be obtained as follows:

$$\boldsymbol{\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-a(k-1)} + \frac{b}{a}} \tag{6}$$

Hence, the value $\boldsymbol{\hat{x}^{(0)}(k)}$can be calculated using the IAGO (Inverse accumulated generating operation):

$$\begin{aligned} &\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \\ &\hat{x}^{(0)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-a(k-1)}.(1-e^{a}), \\ &k = 2,3, \dots \end{aligned} \tag{7}$$

3.2. Autoregressive moving average model (ARMA)

The ARIMA (p, d, q) model is used to model stationary adapted data series. The absence of this feature, which is checked with the unit root test, results in d-order difference disposal for non-stationery data. According to the following Eq. 8 (Alsaleh, & Abdul-Rahim, 2018) [57]

$$Y_t^* = (1-B)^d Y_t \tag{8}$$

$\boldsymbol{Y_t}$is the original data and $\boldsymbol{Y_t^*}$is the stationary data, d represents the different order. The variable B is introduced as the following matrix.

$$B = \begin{pmatrix} -(y_1^1 + y_1^1)/_2 & \dots & 1 \\ \vdots & \dots & \vdots \\ -(y_{m-1}^1 + y_m^1)/_2 & \dots & 1 \end{pmatrix} \tag{9}$$

The extended ARIMA model is an autoregressive (AR) model, including the moving average MA process. This is shown in the following equations, respectively. The basis of the ARIMA model is $\boldsymbol{\mu_t}$, which is fully represented in Eq. (10).

$$\begin{aligned} Y_t^* &= C + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \dots + \alpha_p Y_{t-p} + \mu_t \\ Y_t^* &= \mu_t + \beta_1 \mu_{t-1} + \beta_2 \mu_{t-2} + \dots + \beta_q \mu_{t-q} \\ Y_t^* &= C + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \dots + \alpha_p Y_{t-p} + \mu_t + \beta_1 \mu_{t-1} + \beta_2 \mu_{t-2} + \dots + \beta_q \mu_{t-q} \end{aligned} \tag{10}$$

P represents the order of AR, and q represents the order of MA. $\boldsymbol{\alpha_k}$ and $\boldsymbol{\beta_k}$ represent regression coefficients. $\boldsymbol{Y_t \mu_t}$The autocorrelation coefficient:

$$\hat{p} = \frac{\sum_{t=1}^{n-k}(y_t - \bar{Y})(y_{i+k} - \bar{Y})}{\sum_1^n (y_i - \bar{y})^2}, \forall 0 < k < n \tag{11}$$

According to the Cramer method, solve the equation to estimate the partial autocorrelation coefficient.

$$\hat{q}_k = \frac{\widehat{D}_k}{\widehat{D}}, \forall 0 < k < n Where$$

$$\widehat{D} = \begin{pmatrix} 1 & \hat{p}_1 & \dots & \hat{p}_{k-1} \\ \hat{p}_1 & 1 & \ddots & \hat{p}_{k-2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{p}_{k-1} & \hat{p}_{k-2} & \dots & 1 \end{pmatrix} \tag{12}$$

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$$\widehat{D}_k = \begin{pmatrix} 1 & \hat{p}_1 & \cdots & \hat{p}_{k-2} & \hat{p}_1 \\ \hat{p}_1 & 1 & \cdots & \hat{p}_{k-3} & \hat{p}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{p}_{k-2} & \hat{p}_{1-3} & \cdots & 1 & 1 \\ \hat{p}_{k-1} & \hat{p}_{k-2} & \cdots & \hat{p}_1 & \hat{p}_k \end{pmatrix}$$

3.3. Single exponential smoothing (SES)

Exponential smoothing is one of the oldest and well-known forecasting methods. More than five decades have passed since its widespread use in various scientific fields. The simplicity, transparency, and capability to adapt to different conditions are the most important reasons for its application in various scientific fields. This feature has made the exponential smoothing based on Croston's method. One of this model's most important features is to provide acceptable results for an extensive range of data generation processes. The appropriate model is selected according to the different data patterns. The smoothing coefficient plays a significant role in prediction accuracy.

Nevertheless, no global procedure has been introduced for it yet. Researchers have attributed values to this parameter based on their research background and field using theoretical or trial and error methods. Since the data used in this study do not show any trend/seasonality, the Single Exponential Smoothing (SES) method is proposed from the family of smoothing models.

$$Y_t = \alpha d_{t-1} + (1 - \alpha) Y_{t-1} \qquad 0 \leq \alpha \leq 1 \tag{13}$$

4. Weighting Methods

This research has chosen the method of the parallel combination according to its characteristics.

$$\hat{f}_{hybrid} = w_{GM} \cdot \hat{f}_{GM,t} + w_{ARMA} \cdot \hat{f}_{ARMA,t} + w_{SES} \cdot \hat{f}_{SES,t}, \quad t = 1, \dots, m \tag{14}$$

Vectors $\mathbf{w_i}$ and $\hat{\mathbf{f}}_{\mathbf{i,t}}$ show weights and single models' forecasting. Vector $\hat{\mathbf{f}}_{\mathbf{hybrid}}$ is the final hybrid forecasting. What is essential in the hybrid method is determining the models' weight. This article uses conventional weighting methods that have yielded promising results.

4.1. Covariance matrix-based weighting

The basis for calculating weights in this method is the forecast error covariance. The forecast error is unbiased, and the error variance reaches its lowest value [58]. The weight of this method is shown in W_S.

$$w_S = \frac{S^{-1} I}{I S^{-1} I} \tag{15}$$

S: The covariance matrix is a forecast error. At the same time, I is a column vector with one whose dimension is equal to the number of sub-models in the combination model.

4.2. Constrains of independence of forecast

In the previous case, the assumption of independence was not considered for the base models. By confirming this assumption, new weights are obtained, which are called $\boldsymbol{W_{IS}}$. In this case, the other factors are zero except for the S matrix's original diameter elements.

4.3. Regression-based weighting

The basis of this method is based on the least-squares. The regression coefficients, which are the base models' weight, are calculated to zero the regression error.

$$y_t = c_0 + w_t \hat{y}_t + \varepsilon_t \tag{16}$$

$\boldsymbol{y_t}$ is the original time series, $\boldsymbol{\hat{y}_t}$ is the output of the basic forecasting models, $\boldsymbol{\varepsilon_t}$ is the regression error, $\boldsymbol{C_0}$ is the intercept parameter, and $\boldsymbol{W_t}$ is the regression coefficients that have the same weights in the composition of the base models. The combined forecast of this weighting model is also unbiased [59].

4.4. Constrains of the intercept parameter

If we consider the constraint C_0=0, the resulting weights are $\boldsymbol{W_{wc}}$, otherwise, considering the intercept in the regression line, the resulting weights are $\boldsymbol{W_c}$. The hybrid forecast of this weighting model is also unbiased [59].

5. Safety Stock (SS)

5.1. Theoretical safety stock

The accuracy estimation of SS is closely related to the forecast error. The SS is calculated based on quantitative characteristics and statistical error distribution. Choosing the forecasting process is a crucial factor affecting the SS optimal adjustment. The proper forecasting model is unbiased, and its error has the least dispersion rate and the slightest deviation from the normal distribution. Little research has been done on the error characteristics in the group hybrid models and the individual class. The most significant

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focus has been on improving the accuracy of the models. In applying their inventory control decisions results, paying attention to other error characteristics is necessary. The SS calculation formula is defined as follows:

$$SS = K_{\alpha}\hat{\sigma}_L \,, \hat{\sigma}_L = \sqrt{L}\hat{\sigma}_L \quad (17)$$

Where $\boldsymbol{K_{\alpha}}$ represents the confidence factor in achieving the desired level of target service. This value is given to the standard normal distribution table. $\boldsymbol{\sigma_L}$ defines the standard deviation of forecast errors for the respective lead time. In the SS calculation, different estimates for $\boldsymbol{\sigma_L}$ are expressed. It is usually estimated at RMSE (Root Mean Square Error). It is acceptable when the forecasts are unbiased. In this case, only lead time is paid equal to one. Therefore, the following formula is used directly:

$$SS = k\hat{\sigma}_1 \quad (18)$$

This study compares different aspects of the error for hybrid and base and examines the behavioral similarities in-sample, out-of-sample data. The out-of-sample error is estimated using the in-sample error in the SS calculation. The precision and quality of this estimate affect the applicability of the control inventory and managers' decisions. The values calculated according to what has been said are called theoretical SS.

5.2. Empirical approach

The SS theoretical calculations are based on the assumption of correctness of the error normality. If the assumption accuracy is ambiguous, an empirical approach is suggested. This approach has been used in a small number of studies [17][18]. Its performance in intermittent demand has not yet been studied. In this case, the formula for calculating SS in the non-parametric experimental approach is transformed as follows:

$$SS = Q_L(SL) \quad (19)$$

Where $\boldsymbol{Q_L}$ is the forecast error quantile at the probability defined by service level. The desired percentile is obtained directly using the data error time series. Linearly interpolated is usually used to calculate these values.

6. The Comparison Approach

This paper divides the data into two in-sample and out-of-sample categories with a 35-35% ratio. Different percentages have also been used for this classification [60]. A comparison criterion alone cannot show all the information in the forecast error. The introduction of different indicators has not stopped yet [61]. Each indicator identifies and introduces one dimension of error. Dependent non-scale criteria are used here. The two MAE and ME performance indicators are used to evaluate the models' accuracy and bias, respectively. These indicators are also common in both hybrid and grey demand areas. AE is a relationship between average absolute error and average demand is shown in a specified range.

$$E_t = \frac{y_t - \hat{y}_t}{\sum_{i=1}^{n} y_i} \, AE_t = \frac{|y_t - \hat{y}_t|}{\sum_{i=1}^{n} y_i} \quad (20)$$

E indicates a systematic error. It shows that the values are underestimated or overestimated on average. In other words, it represents the model bias. The error and its absolute magnitude are calculated for each time series. Then they are divided by demand values in the study period. Finally, the two indicators' average is calculated during the whole set of time series and is reported as MAE and ME.

7. Data Collection

In this research, automotive sector data was used for intermittent demand modeling. In the face of real-world uncertainty, this data was collected for spare parts demand during twenty-four weeks [41]. The advantage of this set is that they are not simulated. Also, trends and seasonality have not been reported in this collection. This data covers all four demand groups. Two ADI and CV2 indices have been used [36]. These two indicators measure dispersion and disorder types in the demand structure. Table (1) illustrates other features of this dataset.

Tab. 1. The characteristics of the time series data used

Series	1	2	3	4	5	6	7	8
Length	75	76	69	73	76	75	61	65
ADI	1.19	1.23	1.61	1.35	1.38	1.34	1.3	1.3
CV	0.71	0.57	0.44	0.57	0.62	0.96	0.63	0.65
CV2	0.5	0.32	0.19	0.32	0.35	0.92	0.4	0.42
Mean	22.23	3.07	0.87	3.89	2.61	12.01	13.34	4.23
Series	9	10	11	12	13	14	15	16

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Length	84	73	84	74	72	74	83	82
ADI	1.91	1.92	1.87	1.85	1.5	2.85	1.38	1.41
CV	0.65	0.44	0.68	0.57	0.52	0.33	0.52	0.74
CV2	0.42	0.19	0.46	0.32	0.27	0.11	0.27	0.55
Mean	4.92	0.9	1.62	2.85	28.44	0.42	2.18	124.22
Series	17	18	19	20	21	22	23	24
Length	76	74	68	414	83	81	83	81
ADI	1.52	1.18	3.78	1.87	1.36	1.35	1.22	1.27
CV	0.70	0.83	0.42	0.77	0.87	0.57	0.60	0.57
CV2	0.49	0.69	0.18	0.6	0.76	0.32	0.36	0.32
Mean	6	3.74	1.24	14.02	12.55	128.68	104.25	30.98

8. Implementing the Models

This study aims to provide an efficient hybrid model with components tailored to the intermittent demand structure. To this end, several hybrid structures with different combinations were introduced and evaluated. The names of the hybrid models are abbreviated and taken from the beginning of the individual model names. For example, the GAS model includes SES, ARMA, and GM(1,1). Each time series, according to Croston's idea, is broken into two variables $\boldsymbol{Z_i(t)}, \boldsymbol{X_i(t)}$ and a variety of hybrid models are implemented and evaluated for the following two modes. The first case: The two named variables are considered independently, and hybrid models are identified with the Cr. Two independent variables are not considered in the second case, and the interrelationships are allowed. In this case, hybrid models for the variable $\boldsymbol{R_i(t)} = \frac{\boldsymbol{Z_i(t)}}{\boldsymbol{X_i(t)}}$ are implemented and evaluated. These models are marked with Ra.

Eviews software is used to run two SES and ARMA models. The value of α corresponding to each time series data is estimated in implementing the SES model. Each time series' stationary is first investigated in implementing the ARMA model. Then p in the AR (p) process and q in the MA (q) process are determined. Because one of the grey model features is easy to run without special software. Therefore, the calculations of this method can be easily performed by Excel.

9. Result and Discussion

9.1. Common evaluation results

As typical in this area, the accuracy and bias of forecasts are assessed using the MAE and ME indicators. In hybrid forecasting models, in the Cr modeling category, the best performance belongs to the hybrid C. GS with ME=0.222 and MAE=0.506 showed the appropriate performance among its various structures. GS performance's superiority is also evident compared to other sub-models in other weighting methods in this modeling category.

The improvement percentage was employed using the MAE indicator for providing detailed documentation. The comparison of the performance of GS structure in hybrid model C with other structures showed that the highest improvement percentage for GAS structure in hybrid model S is 0.224. It also indicated that the lowest in all sub-models in the two hybrid models, C and WC, was calculated with the first average of 0.05797124 and the second 0.078618650, respectively. These results show the proper performance of these two weightings compared to another hybrid in this category. The lowest improvement percentage calculated in all hybrid models belongs to the GS structure in these comparisons. These results prove that the GS structure is more appropriate and valuable than other structures in different weighting methods.

In the Ra modeling category, the superiority belongs to the hybrid C with GAS structure with ME=0.198, MAE = 0.492. This superiority of the sub-models is also evident in other weighting methods. The highest behavioral similarity or the lowest improvement percentage was calculated as the average among all structures of weighting methods separately. The lowest value was reported in the two-hybrid models C and WC, with 0.06752830933 and 0.097294199725.

Similar results obtained in the two modeling classes confirm the usefulness and superiority of the C and WC weighting models. In this group of comparisons, the highest improvement percentage belongs to hybrid S with the AS structure as 0.28231945, which shows the poor performance of this hybrid model in identifying and modeling behavioral patterns in intermittent demand.

The most significant behavioral similarity between GS and GAS is in the C weighting method. This procedure has also been observed in the Cr modeling category. Because the Ra modeling category's best performance belongs to Hyb(C) and GAS structure, its improvement percentage was compared with that of the Cr

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group.

As expected, the lowest improvement percentage belongs to hybrid C with a GS structure of 0.02710079, and then in the same hybrid model with GAS structure, the lowest value is 0.03542368. As a result, the similarity of the mentioned structures' performance in the C weighting method can be observed. The highest improvement percentage belongs to the hybrid S with GAS structure with a value of 0.245407785. Due to the model's poor performance, this result was not far off the mark.

The accuracy and bias evaluation criteria are similar in implementing the Ra modeling category's base models. The MAE indicator of all three models is very close to 0.560. Also, for the ME indicator, this value is 0.230. The base models in this category do not have a significant preference over each other in this respect. In the Cr modeling category, the GM model has the best performance. It has the lowest value of two ME indicators as 0.2450, MAE indicators as 0.560.

In the same modeling group, the improvement percentage in using the GM model against the SES is 0.043 and for the ARMA is 0.052. To compare the base and hybrid models in this group, we use the most appropriate implementation of hybrid models, namely the GS structure in the hybrid C, to calculate the improvement percentage. These values are 0.144, 0.136, and 0.972 for ARMA and SES, GM models, respectively. Tables (2-3) express the evaluation criteria of modeling with different sub-models and weights.

Tab. 2. MAE(ME) performance of various hybrid for Ra modeling

Model (Ra)	MAE (ME)	Hyb (S)	Hyb (IS)	Hyb (WC)	Hyb (C)
GAS	Out Of Sample	0.5601 (0.2459)	0.5605 (0.2523)	0.5400 (0.2341)	0.4923 (0.1984)
GA		0.5610 (0.2618)	0.5688 (0.2850)	0.5405 (0.2425)	0.5478 (0.2345)
GS		0.5827 (.3174)	0.5713 (0.2757)	0.5491 (0.2569)	0.5113 (0.2180)
AS		0.6859 (0.3209)	0.5697 (0.2339)	0.5519 (0.2391)	0.5259 (0.2142)
GAS	In Sample	0.5434 (0.2392)	0.5402 (0.2455)	0.5128 (0.2191)	0.4876 (0.1940)
GA		0.5402 (0.2477)	0.5530 (0.2782)	0.5219 (0.2250)	0.5191 (0.2245)
GS		0.5637 (0.2926)	0.5545 (0.2651)	0.5407 (0.2313)	0.5160 (0.2114)
AS		0.6789 (0.3158)	0.5585 (0.2235)	0.5283 (0.2202)	0.5153 (0.2085)

Tab. 3. MAE(ME) performance of various hybrid for Cr modeling

Model (Cr)	MAE (ME)	Hyb (S)	Hyb (IS)	Hyb (WC)	Hyb (C)
GAS	Out Of Sample	0.6524 (0.2993)	0.5618 (0.2372)	0.5493 (0.2481)	0.5103 (0.2332)
GA		0.6092 (0.2680)	0.6007 (0.2523)	0.5535 (0.2500)	0.5535 (0.2759)
GS		0.5602 (0.2451)	0.5518 (0.2379)	0.5368 (0.2371)	0.5060 (0.2224)
AS		0.5843 (0.2528)	0.5857 (0.2548)	0.5576 (0.2451)	0.5497 (0.2234)
GAS	In Sample	0.6018 (0.2808)	0.5398 (0.2155)	0.5371 (0.2128)	0.5111 (0.1997)
GA		0.5908 (0.2528)	0.5927 (0.2311)	0.5458 (0.2480)	0.5412 (0.2509)
GS		0.5462 (0.2320)	0.5385 (0.2071)	0.5351 (0.2191)	0.5005 (0.1973)
AS		0.5651 (0.2521)	0.5635 (0.2250)	0.5380 (0.2189)	0.5260 (0.2236)

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The discussions above show the hybrid models' superiority over the base models by considering the conventional evaluation criteria. This paper compares the indicators to introduce a superior model in the sample data according to what is expected in the forecasting literature.

9.2. Investigation of the statistical error distribution

Theoretical calculations of the SS are based on the assumption that the forecast error is normal. These calculations' accuracy depends on the statistical distribution normality of errors, while this assumption's validity is not usually questioned. In this paper, the Shapiro-Wilk statistical test was employed to test the forecast error normality based on the error's series characteristics.

First, the normality percentage of base and hybrid models was compared. The highest normality percentage belongs to the hybrid model C with a value of 0.667in both Ra and Cr modeling. Compared to the base models, this value differs slightly from ARMA only in the Cr modeling by 0.63. By peer-to-peer comparison of the base models' performance in the two Ra and Cr modeling, the Cr category results' superiority is evident. This is the first superiority of the Cr category over the Ra one in evaluating the computational results in this research. It should be noted in this section; all comparisons are made in the in-sample category.

The highest normality percentage belongs to the hybrid C and has appeared in modeling Ra and Cr. The mean and Standard Deviation (Sd) of the percentages obtained for all structures in this weighting method in the two groups Ra and Cr are equal (0.614, 0.034) and (0.625, 0.029), respectively. Achieving high percentages in all structures and, at the same time, their low dispersion simultaneously leads to gaining a competitive advantage for this weighting method. The results of the Shapiro-wilk test for all hybrid performances are announced in Table (4).

Tab. 4. Shapiro–Wilk results of various hybrid for Ra (Cr) modeling

Model Ra (Cr)	Shapiro Wilk	Hyb (S)	Hyb (IS)	Hyb (WC)	Hyb (C)
GAS	Out Of Sample	0.291 (0.291)	0.291 (0.291)	0.291 (0.291)	0.291 (0.4167)
GA		0.291 (0.291)	0.291 (0.334)	0.291 (0.250)	0.291 (0.291)
GS		0.250 (0.291)	0.208 (0.250)	0.250 (0.334)	0.291 (0.375)
AS		0.291 (0.250)	0.291 (0.334)	0.291 (0.250)	0.291 (0.334)
GAS	In Sample	0.541 (0.625)	0.5 (0.5834)	0.625 (0.625)	0.667 (0.667)
GA		0.541 (0.584)	0.541 (0.541)	0.541 (0.541)	0.584 (0.584)
GS		0.458 (0.541)	0.416 (0.458)	0.458 (0.541)	0.625 (0.625)
AS		0.541 (0.625)	0.541 (0.583)	0.541 (0.625)	0.625 (0.584)

Since the GAS structure in the Ra group's C weighting method has obtained the best evaluation results. Therefore, the following review in this group is wise. In this study, the time series of intermittent demand is divided into four main categories [41], and their error percentage is calculated separately. As expected, the highest percentage is 0.8 and belongs to the Smooth group, and the lowest belongs to Erratic and is equal to 0.5. This value is 0.62 for the intermittent demand category and 0.75 for the lumpy category. This procedure has been seen with similar percentages and the same prioritization of demand groups in other hybrid methods.

In SS theoretical calculations, the out-of-sample variance is estimated with the in-sample variance. The similarity of the two values is essential to ensure the accuracy of this approximation. It should be noted that this method is not the best and only method of estimating error variance. But it is widely used in SS calculations. The similarity of out-of-sample versus in-sample relative variance during the time series was used to investigate this issue. Relative out-of-sample variance over the in-sample variance measured

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across all the time series. The closer this value to one is, the more accurate the estimates. Since the models' accuracy and bias, the hybrid C was selected as the superior model, the mentioned evaluation was also examined in this hybrid. In the Ra category, the highest similarity between out of sample and in sample belongs to GAS structure with 1.067 and in Cr belongs to GS structure with 1.134. Interestingly, this structural superiority is also seen in the C measure regarding accuracy and bias.

9.3. Investigating and comparing SS

This section compares the performance of two theoretical approaches and Empirical Percentile (EP) in calculating SS. The effect of different modeling factors in determining its level is also examined. As discussed, the superiority of hybrid models over the base in terms of applicability in calculating SS efficiency was demonstrated. This superiority is reflected in the precision, bias, and similarity between in-sample and out-of-sample data and the lower deviation of the forecast error from the normal distribution.

Because hybrid models are compatible with a more appropriate estimation condition, their results can be trusted in a theoretical approach than base models. Therefore, the following comparisons are performed on hybrid models. The value stated in the tables results from calculating the one step ahead average safety stock level in both theoretical and EP approaches for the service level of 85%, 90%, and 95%. Since the C weighting method has the lowest error variance and bias and has shown better results in investigating the deviation from the normal distribution, comparisons in this method in two modeling classes in Cr and Ra are rational. These results are shown for structures with different sub-models separately in two computational approaches in Tables (5-6).

Tab. 5. Theoretical SS values, using the Hyb(C)

Theo Ra(Cr)	GAS	GA	GS	AS
85	16.3718 (16.1438)	17.0587 (16.32682701)	16.5783 (16.4581)	16.4514 (17.6923)
90	20.9723 (20.6803)	21.7351 (20.9146654)	21.2368 (21.0829)	21.0743 (22.6638)
95	27.0135 (26.6373)	28.29269 (26.93926457)	27.3543 (27.1559)	27.1448 (29.1923)

Tab. 6. EP SS values, using the Hyb(C)

EP Ra(Cr)	GAS	GA	GS	AS
85	22.4807 (22.9482)	24.2653 (23.1622)	22.4088 (23.2631)	21.8441 (23.2925)
90	29.4787 (29.4522)	32.3838 (30.7120)	30.3365 (29.7258)	30.8626 (32.9307)
95	37.9623 (37.2598)	40.8535 (38.5771)	39.1312 (38.3236)	38.7343 (41.5491)

The results show that theoretical SS lower values are generally reported than the SS empirical percentile in Ra and Cr. The main reason can be found in the lack of error normality. In other words, the hypothesis of normality has been used in the calculations, while the error distribution does not correspond to the normal distribution and deviates from it.

Since the Ra has provided better results by considering the accuracy criteria, it can be argued that in structures with the same sub-models, the theoretical SS results in this group will have higher reliability. In general, the most significant quantitative similarity between EP SS and Theo SS in Ra and Cr modeling categories is seen in GAS and GS structures in weighting category C, respectively, compared to other weighting methods.

In both modeling categories in the superior structure introduced in two computational approaches in other weighting methods, i.e., WC, S, IS, SS values are reported in Tables (7-8). It should be noted that the structure in the two Ra and Cr are GAS and GS, respectively. As mentioned earlier, in theoretical and EP calculations, the out-of-sample behavior of the data with the in-sample category is estimated. Therefore, these two categories should have behavioral similarities with each other. The results of examining this similarity are shown in Table (9). This table shows the in-sample category's EP SS values versus out-of-sample,

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and the similarity index is defined. It makes sense to do the calculations in the superior structure in two modelings. The calculated similarity value for SS EP in the two groups in the sample and out of the sample is very close to one. This issue is due to similar behaviors and structures in both categories. This similarity is generally seen more in the Ra and Cr in all hybrid model C structures than in other weighting models.

Tab. 7. SS values in Ra modeling in its superior structure.

Ra EP (Theo)	Method (WC)	Method (S)	Method (IS)
85	22.6571 (16.6808)	27.2409 (17.3416)	25.6613 (16.6873)
90	30.7774 (21.3682)	35.57722 (22.2146888)	34.0527 (21.3765)
95	38.4357 (27.5234)	42.7054 (28.6137)	42.9504 (27.5341)

Tab. 8. SS values in Cr modeling in its superior structure.

Cr EP (Theo)	Method (WC)	Method (S)	Method (IS)
85	23.0088 (16.4698)	22.9347 (16.4391)	22.5880 (16.5699)
90	30.6865 (21.0978)	30.7945 (21.0586)	31.3065 (21.2261)
95	38.5520 (27.1751)	38.9005 (27.1246)	39.8960 (27.3404)

Tab. 9. Similarity of IN/OUT in SS (EP) in Ra-Cr modeling

SS(EP)	85	90	95
Ra(GAS)	0.9242	0.9866	0.9431
Cr(GS)	0.9608	1.01322	0.9479

As the results show, the theoretical SS estimate is lower than the EP SS. These comparisons should be made in the superior. hybrid model. Otherwise, the increase in the SS value is due to the rise in incorrect variance estimation. Also, the high error dispersion results from the wrong selection of sub-models or improper hybridization of base models. The degree of reliability of the values obtained from EP using the Hyb(C) in both models is higher than other methods. The reason for this can be sought in the superiority of this weighting method.

So far, the hybrid model C's preference and validity in the two Ra and Cr modeling categories have been presented by considering the criteria required to determine the SS values. The SS values calculated using other weighting methods estimate more than the hybrid model C, although this increase is insignificant. Hence, the calculated SS values are more efficient in this group. It is also reasonable to use the results as a benchmark.

The improvement percentage is used for providing more transparent documentation comparing the results of the two computational approaches. Tables (10-12) compiled the percentage improvement of the EP SS values versus the theoretical SS in WC, S, and IS weighting models peer-to-peer structures and modeling for two categories. On average, during the same service level, the Ra's highest improvement percentage belongs to S weighting with the GA structure of 0.6411.

Tab. 10. Calculates the improvement percentage for Hyb(WC)

Ra(Cr)	GAS	GA	GS	AS
85	0.3582 (0.4040)	0.3988 (0.4385)	0.3616 (0.3970)	0.3266 (0.3267)
90	0.4403 (0.4743)	0.4583 (0.4867)	0.4464 (0.4544)	0.4649 (0.4663)
95	0.3964 (0.4427)	0.3901 (0.4520)	0.4219 (0.4186)	0.4355 (0.4493)

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Tab. 11. Calculates the improvement percentage for Hyb(S)

Ra(Cr)	GAS	GA	GS	AS
85	0.5708	0.6618	0.5898	0.2856
	(0.3584)	(0.4210)	(0.3951)	(0.2518)
90	0.6015	0.6803	0.6226	0.4380
	(0.4505)	(0.4545)	(0.4623)	(0.4177)
95	0.4924	0.5812	0.5633	0.4035
	(0.4294)	(0.4310)	(0.4341)	(0.3831)

Tab. 12. Calculates the improvement percentage for Hyb(IS)

Ra(Cr)	GAS	GA	GS	AS
85	0.5377	0.5942	0.5853	0.3393
	(0.3217)	(0.4417)	(0.3631)	(0.2977)
90	0.5929	0.6329	0.6312	0.4438
	(0.4406)	(0.5096)	(0.4749)	(0.4296)
95	0.5598	0.5831	0.5864	0.4367
	(0.4340)	(0.4760)	(0.4592)	(0.4227)

The same study was performed in the Cr, and the highest percentage belonged to the IS weighting with GA sub-models as 0.4758. Achieving such results has not been far from expected, considering the criteria for assessing the error's accuracy and normality.

The improvement percentage considering the value of SS for the best structure in Ra class is calculated with GAS sub-models in C weighting model in EP calculations against the same structure in Cr and Ra categories in other weighting methods in theoretical calculations. Table (13) shows the results. On average, the highest improvement percentage belongs to the Cr modeling category and S weighting method with 0.41473.

Tab. 13. Calculates the improvement percentage in different perspectives.

Ra(Cr)	Method(WC)	Method(S)	Method(IS)
85	0.3476	0.2963	0.3471
	(0.3797)	(0.3928)	(0.3468)
90	0.3795	0.3269	0.3790
	(0.4124)	(0.4258)	(0.3787)
95	0.3792	0.3267	0.3787
	(0.4121)	(0.4255)	(0.3784)

Finally, the SS value for the base models in both computational perspectives is expressed separately in the various modeling in Tables (14-15) to complete and compare the results.

Tab. 14. SS value of basic models for Ra modeling in two perspectives

Ra EP (Theo)	GM	SES	ARMA
85	27.7458	22.46242	23.14945
	(16.5656)	(14.1045)	(17.1021)
90	35.7806	30.85052	31.2388
	(21.2205)	(16.7712)	(21.9078)
95	44.0751	39.9868	39.9156
	(27.3332)	(21.4839)	(28.2185)

Tab. 15. SS value of base models for Cr modeling in two perspectives.

Cr EP (Theo)	GM	SES	ARMA
85	22.1755	21.7834	22.2433
	(14.4550)	(14.0692)	(17.1108)
90	28.9202	30.8722	30.6577
	(18.5169)	(16.7291)	(21.9190)
95	37.0360	39.9525	39.0700
	(23.8508)	(21.4301)	(28.2329)

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10. Conclusion

Managers' decisions in the field of SS require highly accurate forecasting models and the normality of statistical error distribution has a significant impact on determining the SS level. In this study, by examining the compelling features in estimating the theoretical SS value, it was shown that the results obtained from the proposed hybrid models are more reliable than the base models. Moreover, compared with hybrid models, the C weighting method has been reported more applicability because EP estimation requires less testing and validation of hypotheses. Its implementation using the output of hybrid models, especially the weighting model C, provides more reliability to managers' decisions. In other words, this paper deals with this lesser but useful aspect of the forecast error in controlling the inventory of goods with intermittent demand, which has been extensively studied. There is also helpful evidence showing a higher percentage of hybrid models' normal forecast error than base models. The same deviation from the normal distribution creates a competitive advantage for management decisions in inventory control using hybrid models. Another aspect of this work is the production of a straightforward document that shows that the superiority of hybrid models over basic models depends not only on the weighting method but also the choice of sub-models plays a decisive role in the usefulness and validity of hybrid models.

The grey model's presence in the superior composition structure in the two Cr and Ra modeling shows its high potential to receive behavioral patterns and adapt to intermittent demand modeling conditions. It is a potential that has not been well exploited yet.

Finally, by designing the following charts, an attempt has been made to examine the results from another perspective. In this way, it is possible to visually observe the trends and compare them with each other. In the first two figures for the two types of modeling, the MAE value calculated for Out of Sample is evaluated to all structures in a variety of combinations. These charts show the superior structure and hybrid (Figuer1-2). In order to obtain an overview of SS at the 0.95 level, for two models in both theories, considering the superior hybrid model, Figure 3 is designed, which shows the similarity of values in each of the two theories. SS values at the 0.95 level for other hybrid models are shown in Figure 4. To achieve the effect of hybrid type in determining SS at 0.95 level for Ra modeling, attention is drawn to Figure 5. As expected, according to the results of the implementation of forecasting methods, the highest improvement percentage across structures belongs to Hyb (IS) and the lowest to Hyb (WC). The evaluation of SS at the 0.95 level for the base models is shown in the last chart (Figure 6). By comparing it with others, the importance of combining modeling with various weighting in SS determination is understood.

In combining the models' accuracy and bias criteria, hybrids with three and two sub-models presented similar results. If the last criterion results are the same, the priority is choosing two-component combinations to save time and calculations. In that case, it is recommended to select the superior model according to the normality percentage criteria, and the results are presented to managers for decision making.

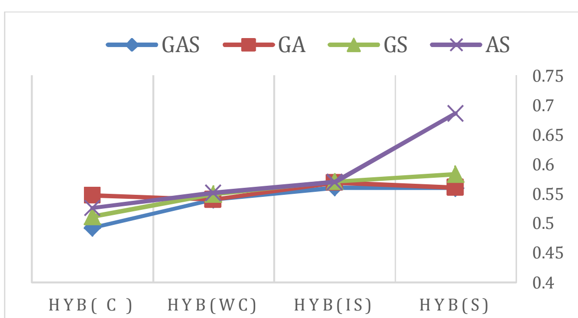

Fig. 1. MAE performance of various hybrid for Ra Modeling

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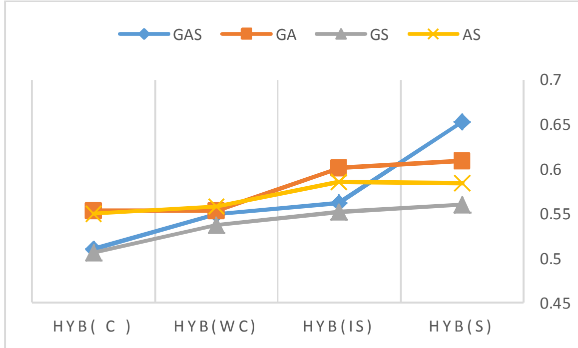

Fig. 2. MAE performance of various hybrid for Cr Modeling

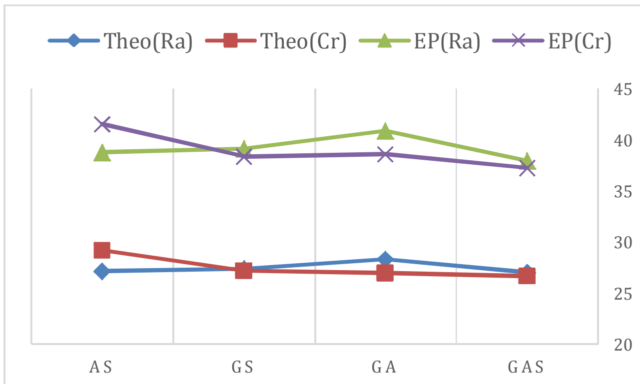

Fig. 3. Theoretical (EP) SS values, using the Hyb(C)

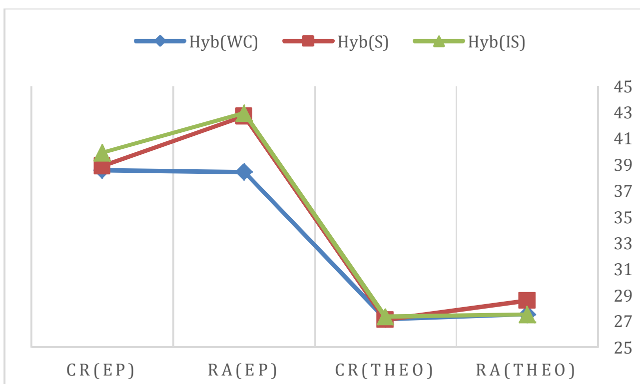

Fig. 4. SS values in Ra(Cr) modeling in its superior structure

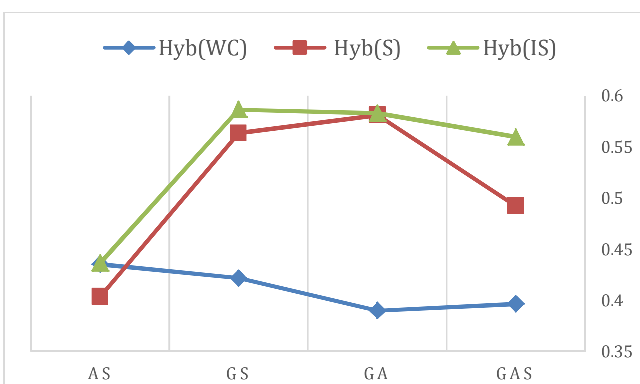

Fig. 5. Calculates the improvement percentage for Hyb(WC-IS-S)

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Fig. 6. SS value of basic models for Ra(Cr) modeling in two perspectives

Here, the independence assumption of Croston's variables is also questioned. It can be argued that considering the two-way relationships and interactions for the variables in the mentioned weighting models has led to a complete understanding of the patterns and structures in intermittent demand. Although hybrid forecasts in intermittent demand are new, there is a great deal of research for researchers in this field. Therefore, it is suggested that all the modeling mentioned in this study be performed by following the variables of Lolly et al. (2017) [41] and compare the results with the present study. The results show better performance of Ra modeling versus Cr in general.

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