

RESEARCH PAPER

# A Multi-objective Optimization Model for Dynamic Virtual Cellular Manufacturing Systems

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## ABSTRACT

*Companies and firms, nowadays, due to mounting competition and product diversity, seek to apply virtual cellular manufacturing systems to reduce production costs and improve quality of the products. In addition, as a result of rapid advancement of technology and the reduction of product life cycle, production systems have turned towards dynamic production environments. Dynamic cellular manufacturing environments examine multi-period planning horizon, with changing demands for the periods. A dynamic virtual cellular manufacturing system is a new production approach to help manufacturers for decision making. Here, due to variability of demand rates in different periods, which turns to flow variability, a mathematical model is presented for dynamic production planning. In this model, we consider virtual cell production conditions and worker flexibility, so that a proper relationship between capital and production parameters (part-machine-worker) is determined by the minimum lost sales of products to customers, a minimal inventory cost, along with a minimal material handling cost. The problems based on the proposed model are solved using LINGO, as well as an epsilon constraint algorithm.*

**KEYWORDS:** *Dynamic virtual cellular manufacturing system; Production planning; Worker flexibility; Epsilon constraint.*

## 1. Introduction

Mounting competition, reduced product life, and elevated product diversity have led companies and firms to make use of new and efficient approaches in the supply, production, and distribution sectors to reduce the costs, as well as increase the use of space and industrial machinery. A great portion of the costs in an industrial unit corresponds to the cost of its production sector. Nowadays, most industrial units try to pursue goals such as efficient use of facility capacity, elevated worker productivity, improvement of quality and production rates due to the use of automation and robotic systems,

reduction of inventory, reduction of material handling, and enhancement of flexibility. To achieve these goals, various production systems have been introduced such as job shop production system, flow shop, mass production, batch production, cellular manufacturing system, virtual cellular manufacturing system, and flexible manufacturing system, each with its advantages and disadvantages. Cellular production system (CMS) is a new production method, being used today as a proper production approach in most large manufacturing centers with relatively high product diversity and operation process routing.

Cell manufacturing is one of the early applications of collective technology rules for manufacturing, in which each cell, consisting of a number of production types of machinery and equipment, is able to do collective processing of the parts. The parts family being based on products and machinery classification and their physical, operational and processing similarities, is formed by several smaller production units called cells. Cells can be both physical and

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virtual in nature. Here, we assume that cells do not have separate physical spaces, and are formed virtually in the manufacturing system. An ideal goal in a virtual cellular manufacturing system is to process all the operations required by each product in just a virtual cell. This goal requires an increase in the use of resource capacity (machines, manpower, etc.) within the manufacturing center. In most cellular manufacturing systems, machines are multipurpose, capable of processing several different operations at a single setup cost/time [1,2,3].

Many companies deal with challenges in production planning. In our study here, by presenting a new model, we aim to determine an appropriate structure for production during planning, so that the product is delivered to customers without delay, with the lowest inventory cost and material handling cost, while determining a proper relationship between the parameters of the capital and production of "part-machine-worker"[4,5,6]. Although cellular manufacturing has been studied and developed by many researchers, an investigation of the available models in various production environments reveals that there are many areas yet to be addressed comprehensively. Some features needed to be addressed are: determining the organization of machinery in a dynamic virtual cell manufacturing environment, distributing production batches between different machineries, determining the flow of materials for timely production of products in different periods and improving the performance of machinery, reducing organizational costs, and increasing market responsiveness [7,8].

The remainder of our work is organized as follows. Section 2 deals with the research thematic literature. The problem statement and mathematical model are discussed in Section 3. Section 4 provides a solution approach to the proposed model. In Section 5 we validate the proposed model and provide our computational results. Finally, our concluding remarks and recommendations for further studies are presented in Section 6.

## 2. Literature Review

The concept of virtual cellular manufacturing system (VCMS) was first introduced in 1978. [9] proposed a four-step approach for virtual cell formation. The first step is supplying the production needs to produce the combination of parts. Then, in the second step, the virtual cells are arranged, and in the third step, the tools are assigned to the machines, and finally, in the last

step, the system performance is evaluated. [10] proposed a mathematical model for virtual cell production, aiming to minimize the total transportation distances between materials. They used a genetic algorithm-based approach to solve their model. In a later study, [11] extended their model by adding another goal, minimizing total product delays. [12] studied a dynamic virtual cellular manufacturing system (DVCMS) considering dynamic demand arrivals through agent-based modeling. Several scenarios were tested to evaluate the model applicability. [13] developed a hybrid genetic algorithm for a DVCMS with supplier selection to solve actual problems. [14] studied a virtual cellular manufacturing system scheduling to minimize the weighted sum of the make span and total travelling distance. A genetic algorithm and a biogeography-based optimization algorithm were developed to solve the model. [15] proposed a new multi-objective model for DCMS considering machine failure and workload balance. An NSGA-II algorithm was developed to solve large-scale problems. [16] proposed a DCMS considering preventive maintenance, production planning and group scheduling. They developed a Benders' decomposition approach to solve the proposed model using the GAMS software package.

Some researchers studied the design of production cells, taking into account the needs of workers. [17] added workers to calculate the number of voids in a cubic network and the number of exceptional elements. [18] using goal programming and tabu search, addressed virtual cell production at the presence of machine and worker constraints. [19] presented a two-step approach to design a comprehensive DVCMS considering the needs of workers. In the first step, due to variability of demand rate in different periods, in turn leading to variability of machine flow, accordingly a dynamic and a static facility layout were developed using the concept of random layout. In the second step, according to the layouts based on the first step, a comprehensive modeling of the virtual cellular manufacturing system and the production planning were made simultaneously. [20] presented a model considering the cost of installing and operating machines and intracellular and extracellular costs at the same time. [21] (2014) developed a genetic algorithm for DVCMS with the ability to buy and sell machines over the periods. [22] presented an optimization model for worker assignment in DCMS. They developed a genetic algorithm to solve the model. [23] studied a cellular

manufacturing system considering working assignment with learning-forgetting effects. A novel memetic algorithm was developed to solve the proposed model.

[24] proposed a VCMS considering group layout planning and scheduling. A metaheuristic algorithm was developed as well as a linear programming approach to solve the proposed problem. [25] developed a VCMS considering group resources allocation to a part family manufacturing. [26] studied an integrated VCMS with supply chain management. They developed a hybrid variable neighborhood search and a genetic algorithm to solve large-scale problems.

### 3. Problem Statement

Workers having prominent roles in working out machines, worker allocation to cells is an important problem in CMS. In our study here, a mixed-integer programming model is developed for DVCMS. Multi-period production planning, dynamic system reconfiguration, machine capacity, worker access time, and worker allocation are considered in the proposed model. The objectives of this problem are, on the one hand, to minimize the total inventory cost of parts, intercellular material handling cost, overhead cost and inventory cost of machines, worker costs, including salaries and the costs of hiring and firing, and on the other hand, to minimize the total lost sale in the system. The main constraints of the problem involve the production capacities of machines, the availability times of workers, and the volume of production.

#### 3.1. Assumptions

- Activity processing time for each product, on each machine, is predetermined.
- Demand for each product is known in each period.
- Machine capacity is known.
- Worker access time is specified.
- Number of cells is given and is constant in each period.
- Only one worker is assigned to process each product on each type of machine.
- Intercellular material handling cost is constant and does not depend on distance.
- Total demand must be met during the planned horizon. In addition, inventory maintenance between periods is possible, and the inventory cost of each product is known. Therefore, the demand for each

product in each period can be met by previous and subsequent periods.

- Machine repair and overhead costs are specified, where the machine is active or idle.
- System reconstruction involves removing or adding machines to each cell, or moving from cell to cell over time.
- Salary for each type of worker is known, whether he/she is active or idle.
- Reconstruction involves the addition and removal of workers in each cell.

#### 3.2. Sets and indices

$Q$ : Quantity of different parts  
 $W$ : Number of different workers  
 $M$ : Number of different machines  
 $C$ : Number of cells  
 $H$ : Number of courses  
 $i$ : Index of different parts ( $i = 1, 2, \dots, Q$ )  
 $w$ : Index of different workers ( $w = 1, 2, \dots, W$ )  
 $m$ : Index of different machines ( $m = 1, 2, \dots, M$ )  
 $k$ : Cell index ( $k = 1, 2, \dots, C$ )  
 $h$ : Period index ( $h = 1, 2, \dots, H$ ).

#### 3.3. Parameters

$r_{imw}$ : 1, if machine type  $m$  can process part  $i$  by worker  $w$ ; 0 otherwise  
 $a_{im}$ : 1, if part  $i$  needs machine type  $m$ ; 0 otherwise  
 $LM_k$ : Minimum size of cell  $k$ , based on the number of machines types  
 $UM_k$ : Maximum size of cell  $k$ , based on the number of machines types  
 $LW_k$ : Minimum size of cell  $k$ , based on the number of workers  
 $RW_{wh}$ : Available time for worker  $w$  in period  $h$   
 $RM_{mh}$ : Available time for machine  $m$  in period  $h$   
 $t_{imw}$ : Processing time of part  $i$  on machine  $m$  by worker  $w$   
 $D_{ih}$ : Demand for part  $i$  in period  $h$   
 $\theta_i^{inter}$ : Handling cost of a unit of an intercellular material for part  $i$   
 $\gamma_{ih}$ : Inventory cost of a unit of part  $i$  in period  $h$   
 $\alpha_m$ : Overhead and inventory costs of machine  $m$   
 $S_{wh}$ : Salary cost of worker  $w$  in period  $h$   
 $HI_{wh}$ : Cost for hiring worker  $w$  in period  $h$   
 $F_{wh}$ : Cost for firing worker  $w$  in period  $h$   
 $A$ : A large number.

#### 3.4. Decision variables

$Y_{ikh}$ : 1, if part  $i$  is processed in cell  $k$  in period  $h$ ; 0 otherwise  
 $X_{imwh}$ : 1, if part  $i$  is processed on machine type  $m$  by worker  $w$  in cell  $k$  in period  $h$ ; 0 otherwise  
 $Z_{ih}$ : 1, if part  $i$  is manufactured in period  $h$ ; 0 otherwise

$NM_{mkh}$ : Number of machines  $m$  assigned to cell  $k$  in period  $h$   
 $NW_{wkh}$ : Number of workers  $w$  assigned to cell  $k$  in period  $h$   
 $P_{ih}$ : Number of part  $i$  for manufacturing in period  $h$

$I_{ih}$ : Inventory of part  $i$  at the end of period  $h$  ( $I_{i0} = 0$ )  
 $B_{ih}$ : Lost sales of part  $i$  in period  $h$  ( $B_{i0} = 0$ )  
 $L_{wkh}^+$ : Number of workers  $w$  applied for cell  $k$  in period  $h$   
 $L_{wkh}^-$ : Number of workers  $w$  fired from cell  $k$  in period  $h$ .

**3.5. Proposed model**

$$Min F_1 = \sum_{h=1}^H \sum_{i=1}^Q \gamma_{ih} \cdot I_{ih} + \sum_{h=1}^H \sum_{i=1}^Q \left[ \left( \sum_{k=1}^C Y_{ikh} \right) - 1 \right] \cdot \theta_i^{inter} \cdot P_{ih} + \sum_{h=1}^H \sum_{k=1}^C \sum_{m=1}^M \alpha_m \cdot NM_{mkh} \\ + \sum_{h=1}^H \sum_{k=1}^C \sum_{w=1}^W S_{wh} \cdot NW_{wkh} + \sum_{h=1}^H \sum_{k=1}^C \sum_{w=1}^W HI_{wh} \cdot L_{wkh}^+ + \sum_{h=1}^H \sum_{k=1}^C \sum_{w=1}^W F_{wh} \cdot L_{wkh}^-$$

$$Min F_2 = \sum_{i=1}^Q \sum_{h=1}^H B_{ih}$$

s. t.

$$\sum_{m=1}^M \sum_{i=1}^Q X_{imwkh} \cdot t_{imw} \cdot P_{ih} \leq NW_{wkh} \cdot RW_{wh} \quad \forall w, h, k \quad (1)$$

$$\sum_{w=1}^W \sum_{i=1}^Q X_{imwkh} \cdot t_{imw} \cdot P_{ih} \leq NM_{mkh} \cdot RM_{mh} \quad \forall m, h, k \quad (2)$$

$$Y_{ikh} = \min(Z_{ih} \cdot \sum_{m=1}^M \sum_{w=1}^W X_{imwkh}) \quad \forall i, h, k \quad (3)$$

$$D_{ih} = P_{ih} + I_{ih-1} - I_{ih} + B_{ih} \quad \forall i, h \quad (4)$$

$$P_{ih} \leq Z_{ih} A \quad \forall i, h \quad (5)$$

$$\sum_{k=1}^C X_{imwkh} \leq r_{imw} \quad \forall i, m, w, h \quad (6)$$

$$\sum_{k=1}^C \sum_{w=1}^W X_{imwkh} = a_{im} Z_{ih} \quad \forall i, m, h \quad (7)$$

$$\sum_{k=1}^C \sum_{m=1}^M \sum_{w=1}^W X_{imwkh} \leq A \times P_{ih} \quad \forall i, h \quad (8)$$

$$\sum_{m=1}^M NM_{mkh} \geq LM_k \quad \forall k, h \quad (9)$$

$$\sum_{m=1}^M NM_{mkh} \leq UM_k \quad \forall k, h \quad (10)$$

$$NW_{wk,h-1} + L_{wkh}^+ - L_{wkh}^- = NW_{wkh} \quad \forall w, k, h \quad (11)$$

$$\sum_{w=1}^W NW_{wkh} \geq LW_k \quad \forall k, h \quad (12)$$

$$Y_{ikh} \in \{0,1\} \quad \forall i, k, h \quad (13)$$

$$X_{imwkh} \in \{0,1\} \quad \forall i, m, w, k, h \quad (14)$$

$$Z_{ih} \in \{0,1\} \quad \forall i, h \quad (15)$$

$$NM_{mkh} \geq 0 \text{ and are integer} \quad \forall m, k, h \quad (16)$$

$$NW_{wkh}, L_{wkh}^+, L_{wkh}^- \geq 0 \text{ and are integer} \quad \forall w, k, h \quad (17)$$

$$P_{ih}, B_{ih}, I_{ih}, Z_{ih} \geq 0 \text{ and are integer} \quad \forall i, h \quad (18)$$

The first objective function includes several types of costs as follows. The first expression shows the inventory cost, comprised of inventory cost of all the products, in all periods, on the production planning horizon. The second expression is the intercellular handling cost of materials. Intercellular material handling cost is incurred when products cannot be manufactured completely in a particular cell. Intercellular handling cost arises when the products can be handled between the cells. In other words, manufacturing a product requires processing in multiple cells. This is due to the fact that not all cells have the potential to manufacture all the products, or the production capacity of a cell is limited. The third expression is the machinery cost. This cost corresponds to the overhead and repair costs of machinery and is calculated according to the number of machinery types used in each period in each cell. The fourth expression is the paid salary cost of workers. The fifth expression is the cost of hiring workers allocated to each cell, in order to reduce the shortage of manpower in a cell. Finally, the sixth expression is the firing cost of workers, who may not be needed any more. When the supply of a firm's product is less than the market demand, the market tries to compensate the shortage from another source. For the unmet market demand, lost sales are considered. The second objective function seeks to minimize the number of lost sales.

Constraint (1) ensures that the activities processing times allocated to workers during each period do not exceed the available times during the period. It should be noted that this constraint also ensures the allocation of operations to be processed by workers in the cells, because according to the right-hand side of the inequality, if  $NW_{wkh} = 0$ , then no operation in that cell is performed by the worker  $w$ , and the left-hand side of the inequality must also be zero. Constraint (2) ensures that activities processing time allocated to the number of machines  $m$  in cell  $k$  during each period does not exceed the available time. This constraint also ensures the allocation of operations in the cells with machines being present, because according to the right-hand side of the inequality, if  $NW_{wkh} = 0$ , then no operation in that cell is performed on the machine type  $m$ , and the left-hand side of the

inequality will also be zero. Constraint (3) determines whether the product  $i$  is produced in cell  $k$  in period  $h$ . Constraint (4) corresponds to the equilibrium of material flow, according to which the amount of demand for each part type in period  $h$  is equal to the sum of the manufactured product in the period, the saved product in the previous period, and the product lost sales in the period and the difference of saved product in the same period. This equation is called "balanced" because the values of  $B_{ih}$  and  $I_{ih}$  cannot be greater than zero simultaneously due to the relationship with the objective function, and thus the product cannot deal with both the lost sale and maintenance. Constraint (5) determines the production (or no production) of part  $i$  in period  $h$ . Constraint (6) indicates that any operation can be performed in only one cell. Constraint (7) ensures that any operation on the relevant machine will be performed only by a single worker capable of working on that machine. It should be noted that this constraint is flexible for worker allocation, and one can choose from a variety of worker options with the ability to process the operation.

According to constraint (8), if the  $i$ th part is not manufactured in the  $h$ th period, no processing of the part should be allocated to the machine and worker. Constraints (9) and (10) indicate the minimum and maximum number of machines that must be assigned to each cell, respectively. Constraint (11) indicates that the number of workers in period  $h$  is equal to the sum of workers in period  $h-1$ , and the number of hired workers in period  $h$  minus the number of fired workers in period  $h$ . Constraint (12) specifies the minimum number of workers allocated to each cell. Constraints (13)-(18) specify the allowable intervals as well as the types of the decision variables.

### 3.6. Mathematical model linearization

We have presented a mixed-integer nonlinear programming model, due to the second expression in the first objective function, as well as the constraints (2), (3), and (4) in the model. Therefore, a number of auxiliary variables are needed to be defined to linearize the model's nonlinear expressions. We first define:

$$F_{ikh} = Y_{ikh} \cdot P_{ih},$$

$$J_{imwkh} = X_{imwkh} \cdot P_{ih}.$$

Based on these expressions, the following constraints are considered:

$$F_{ikh} \geq P_{ih} - A \times (1 - Y_{ikh}) \quad \forall i, k, h \quad (19)$$

$$F_{ikh} \leq P_{ih} + A \times (1 - Y_{ikh}) \quad \forall i, k, h \quad (20)$$

$$J_{imwkh} \geq P_{ih} - A \times (1 - X_{imwkh}) \quad \forall i, m, w, k, h \quad (21)$$

$$J_{imwkh} \leq P_{ih} + A \times (1 - X_{imwkh}) \quad \forall i, m, w, k, h \quad (22)$$

$$F_{ikh} \geq 0 \quad \text{and are integer} \quad \forall i, k, h \quad (23)$$

$$J_{imwkh} \geq 0 \quad \text{and are integer} \quad \forall i, m, w, k, h \quad (24)$$

We then replace constraint (4) with the following two constraints:

$$\sum_{m=1}^M \sum_{w=1}^W X_{imwkh} \leq A \times Y_{ikh} \quad \forall i, k, h \quad (25)$$

$$\sum_{m=1}^M \sum_{w=1}^W X_{imwkh} \geq Y_{ikh} - (1 - Z_{ih})A \quad \forall i, k, h \quad (26)$$

Now, the new forms of constraints (2) and (3) are:

$$\sum_{m=1}^M \sum_{i=1}^Q J_{imwkh} \cdot t_{imw} \leq NW_{wkh} \cdot RW_{wh} \quad \forall w, h, k \quad (27)$$

$$\sum_{w=1}^W \sum_{i=1}^Q J_{imwkh} \cdot t_{imw} \cdot P_{ih} \leq NM_{mkh} \cdot RM_{mh} \quad \forall m, h, k \quad (28)$$

Therefore, the first objective function is changed to

$$\begin{aligned} \text{Min } F_1 = & \sum_{h=1}^H \sum_{i=1}^Q \gamma_{ih} \cdot I_{ih} + \sum_{h=1}^H \sum_{i=1}^Q \left[ \left( \sum_{k=1}^C F_{ikh} \right) - P_{ih} \right] \cdot \theta_i^{\text{inter}} + \sum_{h=1}^H \sum_{k=1}^C \sum_{m=1}^M \alpha_m \cdot NM_{mkh} \\ & + \sum_{h=1}^H \sum_{k=1}^C \sum_{w=1}^W S_{wh} \cdot NW_{wkh} + \sum_{h=1}^H \sum_{k=1}^C \sum_{w=1}^W HI_{wh} \cdot L_{wkh}^+ + \sum_{h=1}^H \sum_{k=1}^C \sum_{w=1}^W F_{wh} \cdot L_{wkh}^- \end{aligned}$$

#### 4. Proposed Solution Approach

The generalized  $\epsilon$ -constraint approach is applied to solve the proposed model. [27] introduced a lexicographic method to calculate the yield objective matrix. In this method, we first solve the first objective function (based on maximum priority) with the main constraints of the problem; let the optimal value of this objective function be  $f_1 = z_1^*$ . Then,  $f_1 = z_1^*$  will be added to the main constraints to maintain the optimal solution of the first optimization sub-problem, and the second objective function will be optimized using these constraints. The hypothesis for the second objective function is maintaining the optimal value of  $f_2 = z_2^*$ ; the next objective function is to be optimized while maintaining the optimal solution of the first two sub-problems, that is,  $f_1 = z_1^*$  and  $f_2 = z_2^*$ . The two optimal values are added to the constraints, and the third objective function will be optimized using these constraints, to get the optimal solution. Proceeding this way, the steps will be carried out to reach optimizing the  $p^{\text{th}}$  objective function and the first row of the yield objective matrix is formed. To form the second row, we first optimize the second objective function considering the main constraints, and for optimizing every other objective function, at a corresponding step, the constraint corresponding to the optimal value of the previous objective function is added to the main constraints as

follows. A lexicographic optimization flowchart is shown in Figure 1.

Using the above approach, a numerical example is worked through. Consider the following 2 objective model:

$$\begin{aligned} \text{min } f_1(x) &= x_2 \\ \text{min } f_2(x) &= -3x_1 - x_2 \\ \text{s.t.} \\ x_1 &\geq 20, \\ x_2 &\geq 10, \\ x_2 &\leq 40, \\ x_1 - x_2 &\leq 30, \\ x_1, x_2 &\geq 0 \end{aligned}$$

To form the yield objective matrix from the lexicographic optimization method, initially the first objective function is used with the main constraints, and the optimal value of  $f_1 = 10$  is obtained; to consider the second objective function,  $f_1 = x_2 = 10$  is added to the constraints, and the second optimal objective function value will be  $f_2 = -130$ .

According to Figure 2, the second objective function is optimized at the point D. In the next step, initially the second objective function is optimized using the main constraints and the optimal value  $f_2 = -250$  is obtained. Then,  $f_2 = -3x_1 + x_2 = -250$  is added to the main constraints, and the first objective function is optimized using these constraints, to obtain the

optimal value of  $f_1 = 40$ . Thus, the yield objective matrix is obtained as shown in Table 1.

**Tab. 1. The yield objective matrix obtained from the lexicographic optimization method.**

	$f_1$	$f_2$
Min $f_1$	10	-130
Min $f_2$	40	-250

Now, using Table 1 and Figure 2, the optimal solution is placed on the Pareto optimal set, i.e., the CD line segment.

Thus, first using the yield objective matrix in accordance with the flowchart given as Figure 1, the  $p - 1$  domains of the objective functions to be constrained are found, where  $p$  is the number of objective functions. From the payoff table we obtain the range of each one of the  $p-1$  objective functions to be used as constraints. Then, we divide the range of the  $i$ th objective function to  $q_i$  equal intervals using  $(q_i - 1)$  intermediate equidistant grid points. Thus, we have  $(q_i + 1)$ , in total, grid points to be used to vary parametrically the RHS ( $e_i$ ) of the  $i$ th objective function. Now, for the considered example, after calculating the yield objective matrix, the domain of the objective function in the constraint (the second objective function) should be divided into  $q_2 = 6$  equal parts, with  $q_2 + 1 = 7$  network points according to Figure 2. The generalized  $\epsilon$ -constraint algorithm is summarized as a flowchart in Figure 3.

### 5. Computational Results

Here, an example is considered and the corresponding model is constructed and solved using the generalized  $\epsilon$ -constraint method using Lingo on an Intel® Core ore i7-3.10GHz CPU-RAM 8GB notebook. After solving the model using the  $\epsilon$ -constraint approach, the results are analyzed.

#### 5.1. Generating a sample problem

First, a standard problem is designed to validate the proposed model and evaluate its efficiency. So, to validate and confirm the proposed model, as well as to examine the results obtained by the  $\epsilon$ -constraint method, a numerical example, consisting of 2 cells, 3 machines, 3 types of parts and 3 workers, is solved. In addition, the minimum and maximum number of machines for cell formation are considered to be 0 and 4, respectively, and the minimum size of each cell is assumed to be zero, depending on the number of workers. Each type of parts has a number of operations, which can be performed by any operator. Table 2 gives the data for the numerical example including inventory and overhead costs and time capacity.

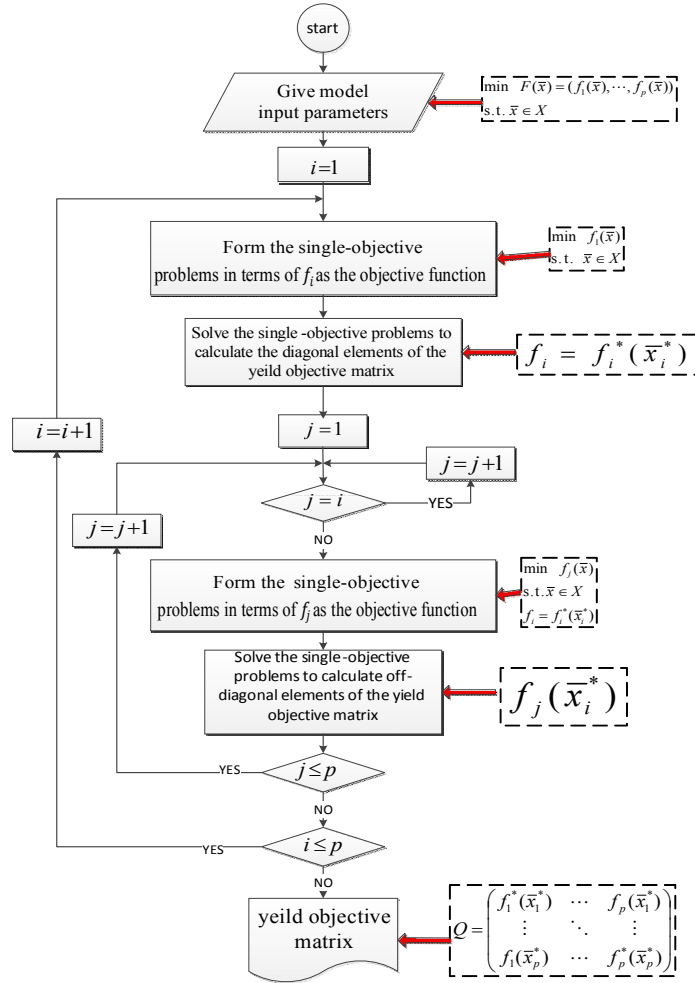


Fig. 1. Flowchart of calculating the yield objective matrix using the lexicographic optimization method (Aghaei et al., 2011)

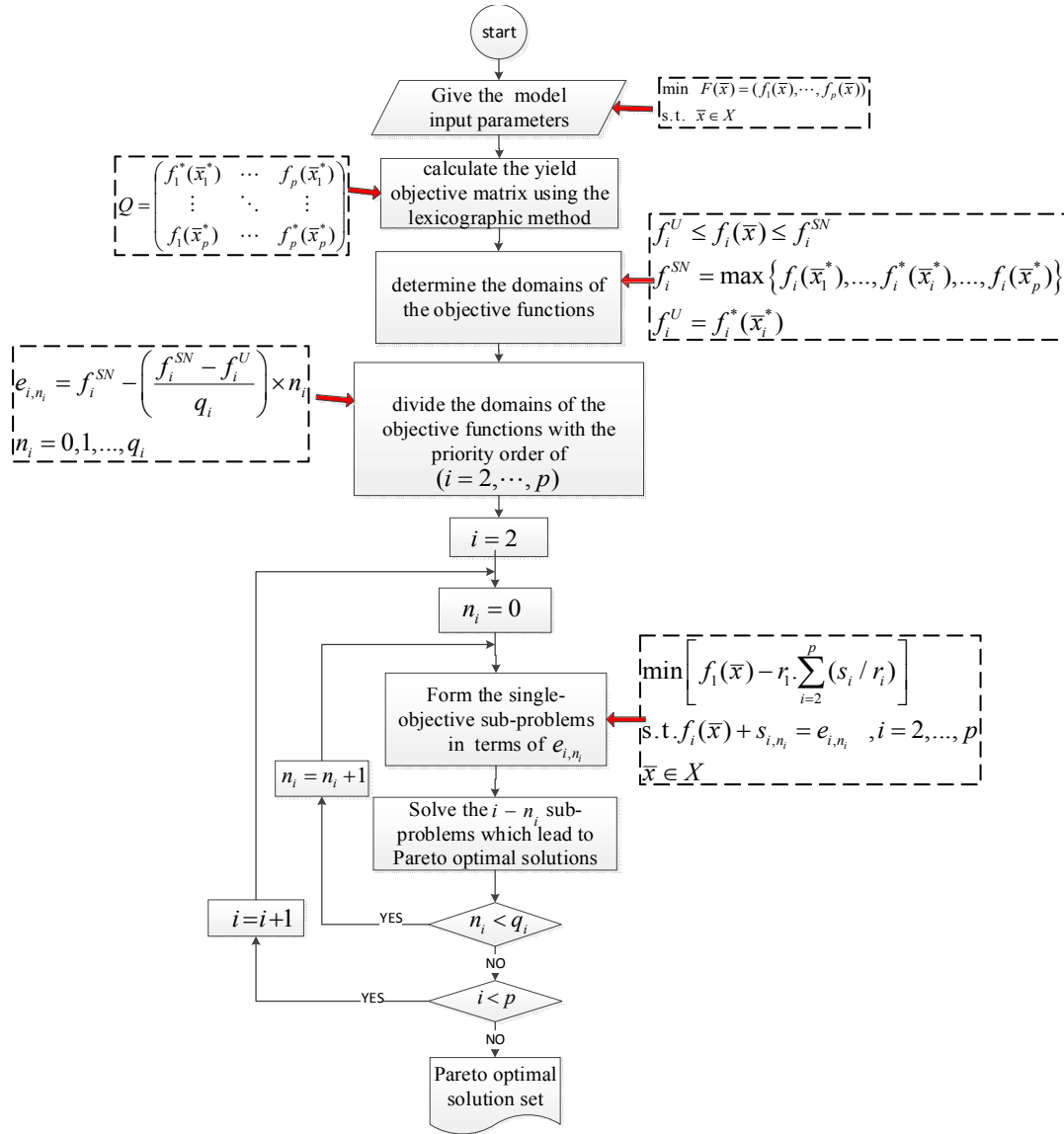
$x_2$

Fig. 2. The result of the epsilon constraint method to calculate the yield objective matrix using the lexicographic method

Tab. 2. Numerical sample data.

Machine type	$\alpha_m$	Information	
		$RM_{m1}$	$RM_{m2}$
1	400	30	30
2	410	30	30
3	430	30	40





**Fig. 3. Flowchart of generalized  $\epsilon$ -constraint method with lexicographic optimization method (Aghaei et al., 2011)**

Tables 3 and 4 show the part-machine and worker-machine data, respectively. For example, according to Table 3, the part type 3 is placed on machine types 1 and 3. Table 4 shows the worker’s ability to work on each machine. For example, worker 1 can work on machines 1 and 3. Therefore,  $\sum_w X_{imwkh}$  is equal to the number

of workers available to process a type  $i$  part on machine type  $m$ . Table 5 shows the processing time matrix for parts on machines. For example, according to the table, part 1 is processed by machine 1, with a processing time of 0.04 hours, using worker 1.

**Tab. 3. "Part-machine" input data for a numerical example.**

		Machine type			$D_{i1}$	$D_{i2}$	$\gamma_{i1}$	$\gamma_{i2}$	$\theta_i^{inter}$
		1	2	3					
Part type	1	1	1	1	0	1550	14	12	11
	2	1	1	0	900	600	12	12	9
	3	1	0	1	1700	500	10	10	8

Tab. 4. "Worker-machine" input data for a numerical example.

	Machine type			Worker information							
	1	2	3	$S_{w1}$	$S_{w2}$	$HI_{w1}$	$HI_{w2}$	$F_{w2}$	$RW_{w1}$	$RW_{w2}$	
Worker	1	1	0	1	470	490	270	285	145	30	30
	2	1	0	0	460	485	260	290	145	30	30
	3	0	1	1	455	475	200	250	155	30	30

Tab. 5. Processing times (hours).

	Part 1			Part 2			Part 3		
	$W_1$	$W_2$	$W_3$	$W_1$	$W_2$	$W_3$	$W_1$	$W_2$	$W_3$
$M_1$	0.04	0.02	0	0.04	0.01	0	0.02	0.03	0
$M_2$	0	0	0.02	0	0	0.04	0	0	0
$M_3$	0.01	0	0.02	0	0	0	0.01	0	0.02

## 5.2. Solving the sample problem

According to the proposed solution approach, each goal is first optimized regardless of the other goals. First, we solve the problem in the LINGO 9 environment, considering the first goal and neutralizing the second goal. Then again, we solve it by considering the second goal and neutralizing the first goal. In this case, there will be the beginning and end solutions of the Pareto edge as follows:

$$\text{Min } Obj_1 = \text{Cost}$$

$$\text{Min } Obj_2 = \text{Lost sale}$$

$$Obj_1^* = 0, \quad Obj_2 = 5250$$

$$Obj_1 = 35059, \quad Obj_2^* = 0.$$

Once the points of the best values of each objective function have been obtained individually, one of the functions must be considered as the motion basis (here the second objective) in order to obtain the other Pareto points, if any. The basic objective function is moved from its optimal value to the size of epsilon each time and is added to the problem as a constraint, and the problem is optimized for other objective function. Here, the value of

epsilon is assumed to be 200, and the following problem is constructed:

$$\text{Min } Obj_2 = \text{Lost sale} \leq 5250 - 200 = 5050.$$

The problem is then optimized for the second objective function, and the solution is as follows:

$$Obj_1^* = 775, \quad Obj_2 = 4750.$$

The same process is then repeated, and each time the result will be noted; this continues until the problem becomes infeasible. Table 6 shows the results obtained from the iterations of the  $\epsilon$ -constraint method.

For example, the outputs of the Pareto point 13, with the objective values  $Obj_1^* = 4080$  and  $Obj_2 = 2550$  are presented in Table 7, showing the appropriate number of productions of each part type in each period to achieve the optimal solution. According to this table, the ending inventory of all parts is zero.

Optimal values of the objective functions corresponding to the point 13 are presented in Table 8. According to this table, the inventory and materials handling costs at this point are zero. Finally, Figure 4 shows the Pareto frontier for the above example.

Tab. 6. Pareto solution of the epsilon constraint method.

Examined points	The objective's upper limit	objective functions values		Efficient (E)	Time
		Obj1	Obj2		
I		0	5250	E	00:01
II		35059	0	E	00:14
1	5050	775	4750	E	00:05
2	4850	775	4750	-	00:01
3	4650	885	4250	E	00:04
4	4450	885	4250	-	00:08
5	4250	885	4250	-	00:04
6	4050	1695	3750	E	00:13
7	3850	1695	3750	-	00:21
8	3650	1770	3550	E	00:21
9	3450	2780	3449	E	00:21
10	3250	3170	3150	E	00:53
11	3050	3600	3000	E	01:01
12	2850	4080	2550	E	02:19
13	2650	4080	2550	-	01:20

14	2450	4350	2249	E	01:01
15	2250	4350	2249	-	01:01
16	2050	4865	2049	E	00:10
17	1850	5150	1751	E	00:12
18	1650	5950	1551	E	00:14
19	1450	6380	1400	E	00:06
20	1250	6835	900	E	00:10
21	1050	6835	900	-	00:11
22	850	7205	800	E	00:07
23	650	8044	649	E	00:07
24	450	11227	449	E	00:09
25	250	13627	249	E	00:21
26	50	33287	49	E	00:09
27	0	infeasible	-	-	-
Total running time					11:24

Tab. 7. The optimal production plan.

Part type Period	Number of parts for production		Ending inventory		Lost sales	
	1	2	1	2	1	2
1	0	0	0	0	0	1550
2	0	500	0	0	900	100
3	1700	500	0	0	0	0

Tab. 8. The optimal values of the objective functions.

Total	Obj <sub>1</sub>						Obj <sub>2</sub>
	Parts Maintenance	Materials Handling Costs	Overhead and Inventory Cost	Salary Cost	Hiring	Firing	Lost Sales
4080	0	0	810	1900	1080	290	2550

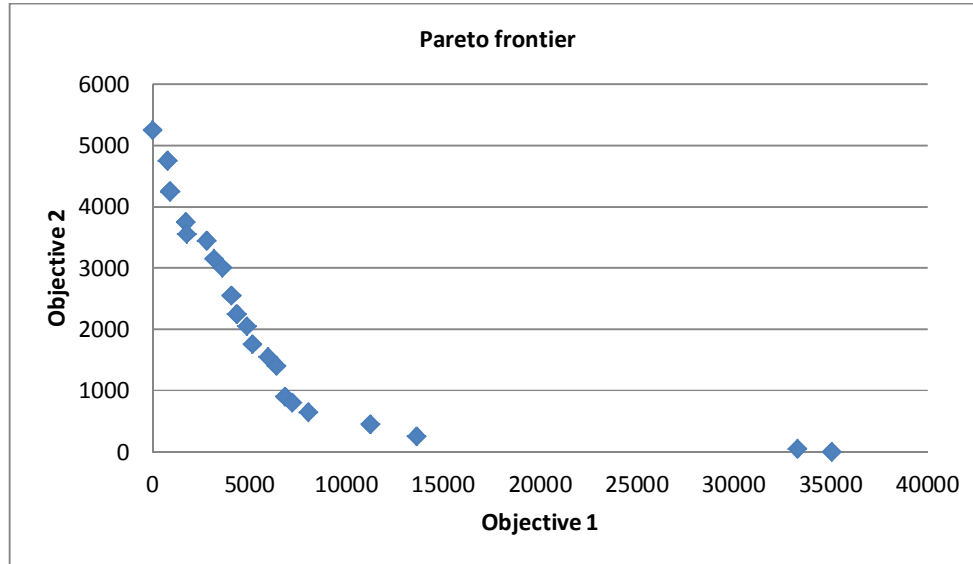


Fig. 4. Obtained Pareto frontier for the numerical example

### 6. Conclusions

A multi-objective optimization model was developed for dynamic virtual cellular manufacturing system, having several operational merits. Significant differences of the present work with available studies are: minimization of the lost sales, incorporation of the worker with

the hiring and firing consideration in each period, consideration of cellular manufacturing issues in part-machine-worker. These considerations appeared to complicate the problem, while appropriates the worker to be a determining and effective factor. The resulting model was first developed as an MINLP problem, and then

linearized appropriately. An  $\varepsilon$ -constraint algorithm was appropriated to solve the model. Finally, several examples were solved to validate the model.

The following extensions of our work may turn to be useful: (1) construction of a model taking into account the machine locations and the distance among the locations, (2) appropriation of a model to determine cellular arrangements in a dynamic virtual environment considering the inventory and repair schedule of the machines and cells, (3) consideration of the sequence of operations in the virtual cell production model and the commissioning times of various parts for machines, and (4) investigation of various meta-heuristic approaches for solving large-scale problems.

### References

- [1] Mahdavi, I., Aalaei, A., Paydar, M. M., & Solimanpur, M. "Designing a mathematical model for dynamic cellular manufacturing systems considering production planning and worker assignment" *Computers & Mathematics with Applications*, Vol. 60, No. 4, (2010), pp. 1014-1025.
- [2] Mahdavi, I., Paydar, M. M., Solimanpur, M., & Heidarzade, A. "Genetic algorithm approach for solving a cell formation problem in cellular manufacturing" *Expert Systems with Applications*, Vol. 36, No. 3, (2009), pp. 6598-6604.
- [3] Mahdavi, I., Firouzian, S., Paydar, M. M., & Saadat, M. "Simulated annealing and artificial immune system algorithms for cell formation with part family clustering" *Journal of Industrial Engineering and Management Studies*, Vol. 7, No. 1, (2020), pp. 191-219.
- [4] Kia, R., Javadian, N., Paydar, M. M., & Saidi-Mehrabad, M. "A simulated annealing for intra-cell layout design of dynamic cellular manufacturing systems with route selection, purchasing machines and cell reconfiguration" *Asia-Pacific Journal of Operational Research*, Vol. 30, No. 04, (2013), p. 1350004.
- [5] Kia, R., Paydar, M. M., Jondabeh, M. A., Javadian, N., & Nejatbakhsh, Y. "A fuzzy linear programming approach to layout design of dynamic cellular manufacturing systems with route selection and cell reconfiguration" *International Journal of Management Science and Engineering Management*, Vol. 6, No. 3, (2011), pp. 219-230.
- [6] Mahdavi, I., Paydar, M. M., Solimanpur, M., Saidi-Mehrabad, M. "A Mathematical Model for Integrating Cell Formation Problem with Machine Layout" *International Journal of Industrial Engineering & Production Research*, Vol. 21, No. 2, (2010), pp. 61-70.
- [7] Behnia, B., Mahdavi, I., Shirazi, B., and Paydar, M. M. "A bi-level mathematical programming for cell formation problem considering workers' interest" *International Journal of Industrial Engineering & Production Research*, Vol. 28, No. 3, (2017), pp. 267-277.
- [8] Mahdavi I., Bootaki B, Bootaki, Paydar M M. "Manufacturing Cell Configuration Considering Worker Interest Concept Applying a Bi-Objective Programming Approach" *International Journal of Industrial Engineering & Production Research*, Vol. 25, No. 1, (2014), pp. 41-53.
- [9] Ratchev, S. M. "Concurrent process and facility prototyping for formation of virtual manufacturing cells" *Integrated Manufacturing Systems*, Vol. 12, No. 4, (2001), pp. 306-315.
- [10] Mak, K. L., and Wang, X. X. "Production scheduling and cell formation for virtual cellular manufacturing systems" *The International Journal of Advanced Manufacturing Technology*, Vol. 20, No. 2, pp. 144-152.
- [11] Mak, K. L., Lau, J. S. K., and Wang, X. X. "A genetic scheduling methodology for virtual cellular manufacturing systems: an industrial application" *International Journal of production research*, Vol. 43, No. 12, (2005), pp. 2423-2450.
- [12] Baykasoglu, A., and Gorkemli, L. "Dynamic virtual cellular manufacturing

- through agent-based modelling” *International Journal of Computer Integrated Manufacturing*, Vol. 30, No. 6, (2017), pp. 564-579.
- [13] Paydar, M. M., and Saidi-Mehrabad, M. “A hybrid genetic algorithm for dynamic virtual cellular manufacturing with supplier selection” *The International Journal of Advanced Manufacturing Technology*, Vol. 92, No. 5, (2017), pp. 3001-3017.
- [14] Zandieh, M. “Scheduling of virtual cellular manufacturing systems: a biogeography-based optimization algorithm” *Applied Artificial Intelligence*, Vol. 33, No. 7, (2019), pp. 594-620.
- [15] Rabbani, M., Farrokhi-Asl, H., and Ravanbakhsh, M. “Dynamic cellular manufacturing system considering machine failure and workload balance” *Journal of Industrial Engineering International*, Vol. 15, No. 1, (2019), pp. 25-40.
- [16] Alimian, M., Ghezavati, V., and Tavakkoli-Moghaddam, R. “New integration of preventive maintenance and production planning with cell formation and group scheduling for dynamic cellular manufacturing systems” *Journal of Manufacturing Systems*, Vol. 56, (2020), pp. 341-358.
- [17] Mahdavi, I., Aalaei, A., Paydar, M. M., and Solimanpur, M. “A new mathematical model for integrating all incidence matrices in multi-dimensional cellular manufacturing system” *Journal of Manufacturing Systems*, Vol. 31, No. 2, (2012), pp. 214-223.
- [18] Hamed, M., Esmailian, G. R., Ismail, N., and Ariffin, M. K. A. “Capability-based virtual cellular manufacturing systems formation in dual-resource constrained settings using Tabu Search” *Computers & Industrial Engineering*, Vol. 62, No. 4, (2012), pp. 953-971.
- [19] Rafiei, H., and Ghodsi, R. “A bi-objective mathematical model toward dynamic cell formation considering labor utilization” *Applied Mathematical Modelling*, Vol. 37, No. 4, (2013), pp. 2308-2316.
- [20] Rezaeideh, H., Mahini, R., and Zarei, M. “Solving a dynamic virtual cell formation problem by linear programming embedded particle swarm optimization algorithm” *Applied Soft Computing*, Vol. 11, No. 3, (2011), pp. 3160-3169.
- [21] Kia, R., Khaksar-Haghani, F., Javadian, N., and Tavakkoli-Moghaddam, R. “Solving a multi-floor layout design model of a dynamic cellular manufacturing system by an efficient genetic algorithm” *Journal of Manufacturing Systems*, Vol. 33, No. 1, (2014), pp. 218-232.
- [22] Karthikeyan, S., Saravanan, M., and Rajkumar, M. “Optimization of worker assignment in dynamic cellular manufacturing system using genetic algorithm” *Journal of Advanced Manufacturing Systems*, Vol. 15, No. 1, (2016), pp. 35-42.
- [23] Chu, X., Gao, D., Cheng, S., Wu, L., Chen, J., Shi, Y., and Qin, Q. “Worker assignment with learning-forgetting effect in cellular manufacturing system using adaptive memetic differential search algorithm” *Computers & Industrial Engineering*, Vol. 136, (2019), pp. 381-396.
- [24] Forghani, K., and Ghomi, S. F. “Joint cell formation, cell scheduling, and group layout problem in virtual and classical cellular manufacturing systems” *Applied Soft Computing*, Vol. 97, (2020), p. 106719.
- [25] Arora, P. K., Haleem, A., Kumar, H., and Khan, S. A. “Recent Development in Virtual Cellular Manufacturing System” In *Recent Advances in Mechanical Engineering*, Springer, Singapore, (2020), pp. 1-7.

- [26] Rostami, A., Paydar, M. M., and Asadi-Gangraj, E. "A hybrid genetic algorithm for integrating virtual cellular manufacturing with supply chain management considering new product development" *Computers & Industrial Engineering*, Vol. 145, (2020), pp. 106565.
- [27] Mavrotas, G. "Effective implementation of the  $\epsilon$ -constraint method in multi-objective mathematical programming problems" *Applied mathematics and computation*, Vol. 213, No. 2, (2009), pp. 455-465.

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