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RESEARCH PAPER

Cost Approximation of a Three-echelon Inventory System with Order Splitting and Information Sharing

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ABSTRACT
This paper develops an approximate cost function for a three-echelon supply chain that has two suppliers, a central warehouse and an arbitrary number of retailers. It takes an integrated approach to multi-echelon inventory control and order-splitting problems. It assumes that all facilities apply continuous review policy for replenishment, demand at the retailers follows a Poisson process, and lead times are stochastic with no predetermined probability distribution. Unsatisfied demand is considered as lost sales at the retailers and backlogged at the warehouse and suppliers. Due to information sharing between the existing echelons, order quantity at each higher level is assumed to be an integer multiple of the lower level. Order placed by the warehouse gets divided between the two suppliers and re-order point is not restricted at the warehouse or suppliers. The main contribution of this paper is its integrated approach and the practical assumption that it uses for the order arrival sequence and the unsatisfied demands. It adds two suppliers as the third echelon to the traditional two-echelon supply chain and considers dynamic sequence of orders arrival to the warehouse at each cycle. The fact that inventory control and sourcing decisions are interdependent and act as the main challenge of supply chain management, considering them in an integrated model can significantly influence operating costs and supply chain's efficiency. Such approach can even have greater impact when blended with practical assumptions that consider lead-time as unpredictable and unsatisfied demand as lost sales. Total cost of the three-echelon inventory system is approximated based on the average unit cost and its accuracy is assessed through simulation. Numerical results with relatively low errors confirms the accuracy of the model. It also shows how to further enhance its accuracy by either increasing the holding cost at all echelons or the penalty cost at the retailers.

KEYWORDS: *Supply chain; Multi-echelon inventory system; Information sharing; Continuous review; Lost sales; Order splitting.*

1. Introduction

Multi echelon Inventory system is one of the most attractive research area in the field of supply chain management. It focuses on placing right order quantity at the right time to reduce the system cost and improve the responsiveness of the supply chain [1]. Due to the competitive environment and high cost of unsatisfied demands, supply chain coordination has become a favorite topic for scholars [2].

This paper integrates the ordering policy and supply decisions in a three-echelon inventory system including two suppliers, a central warehouse and arbitrary number of identical retailers. Orders placed by the warehouse are divided between the two suppliers according to suppliers' lead times and their capacity. Transportation times between all facilities are assumed to be constant while random delay occurs due to the stock out at the warehouse and suppliers which makes the lead time to be uncertain. Adding suppliers as the third echelon to the traditional two-echelon supply chain and stochastic nature of its lead times, result in dynamic sequence of orders arrival to the warehouse at each cycle. Each delivery sequence

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affects the inventory level as well as the operating cost.

The rest of the paper is organized as follows : Section 2 gives a brief literature review. Notations and problem formulation are given in Section 3. The optimal results of the mathematical model are presented and examined by simulation in Section 4. Conclusion and future research opportunities are given in Section 5.

2. Literature Review

Previous researches in the field have focused on numerous areas, some of the important of which are: cost function, optimal ordering policy, decision sourcing, and integrated models. Many recent studies have focused on the cost function and the optimal ordering policy in multi-echelon inventory systems under various conditions.

Akbari Jokar and Seifbarghy [3] considered an inventory system consisting of one warehouse and an arbitrary number of retailers controlled by a continuous review inventory policy. Independent Normal demands were assumed with constant transportation times for all retailers. Unsatisfied demands were assumed to be lost in the retailers and unsatisfied retailer orders were backordered in the warehouse. A cost function was estimated to find the optimal reorder points for given batch sizes in all elements. Hill et al. [4] considered a two-echelon supply chain including a central warehouse and a number of retailers. A single-item with base stock policy was replenished by the warehouse for the retailers. The demand processes on the retailers were assumed to be independent Poisson. Demand not met at a retailer was considered lost. The operational parameter of such a system was estimated and the accuracy was assessed using simulation. Seifbarghy and Esfandiari [5] extended the given research in [3] for the case of non-identical retailers.

Ghahghaei and Seifbarghy [6] studied a two-echelon inventory system consisting of one central storeroom and a number of identical, independent retailers that applied continuous review and modified base stock policy by considering delay between successive orders. Unsatisfied demands were assumed to be backordered at the warehouse while became lost at the retailers. The performance of modified base stock policy was evaluated through simulation and the total cost was compared with that of standard base stock policy. Kok et al. [7] and Ma et al. [8] presented comprehensive literature review on multi-echelon inventory models.

In addition to inventory control, sourcing decisions are the main challenge of supply chain management. Since these challenges are interdependent, considering them in an integrated model can significantly influence operating costs and supply chain's efficiency. Hence, many firms tend to incorporate their supply decisions into ordering policy to reduce their cost and improve their service level. Svoboda et al. [9] carried out a survey analysis on the integrated models of inventory control and multiple sourcing. Although considerable researches have been devoted to the integrated model, more attention has been paid to multi-echelon models with deterministic lead-time [10-14]. Furthermore, these researches investigated the integrated model without allowing shortages. Other researchers extended previous studies to stochastic lead-time while unsatisfied demands are backordered at the retailers [15-18]. Since the lost sales inventory models are less analytically tractable, the researchers have tended to focus on backlogging models, rather than on lost sales models. Hill [19] and Fong et al. [20] studied the integrated model for the case of lost sales. Ghahghaei and Seifbarghy [21] studied the integrated model for the case of lost sales while the reorder point was restricted to be greater than or equal to -1.

Most of the researches, which have investigated the integrated models, assumed that split orders arrive at the same sequence in all cycles [11, 13, 14, 18]. However, stochastic lead-time (constant transportation time plus a delay due to the stock out at the suppliers) can affect the sequence of orders arrival. In other words, even lead times with specific probability distribution, which have been proposed in many previous studies, may not work. Furthermore, most previous studies assume unsatisfied demands to be backordered, while in our competitive environment it may become lost sales especially at the retailers. Table 1 shows a comparative perspective of this study with pertinent previous ones.

Tab. 1. A comparative view of this study with those in the Literature Review

Reference	Number of stages	Stochastic demand	Stochastic lead time	Shortage	Orders Arrival
Chang and Chang [10]	Multi stages	✓		_	Fixed
Duan and Ventura [13]	Multi stages	✓		_	Fixed
Bagul and Muhkerjee [11]	Multi stages	✓		_	Fixed
Knour et al. [14]	Two stages	✓		Backorder	Fixed
Cao and Yao [12]	Two stages	✓		Backorder	Fixed
Abginechi et al. [16]	Two stages	✓	✓	Backorder	Fixed
Song et al. [17]	Two stages	✓	✓	Backorder	Fixed
Saputro et al. [18]	Two stages	✓	✓	Backorder	Fixed
This Study	Three stages	✓	✓	Lost sale	dynamic sequence

As can be noticed, the main contribution of this study is its integrated approach and the practical assumption that it uses for the order arrival sequence and the unsatisfied demands. The fact that random delay with no predetermined probability distribution can occur due to stock out and unsatisfied demands can become lost sales especially at the retailers, this research tries to address these issues in an integrated model for cost estimation. Although such realistic assumptions make the cost estimation more complex, our strategy is to divide the problem into a number of sub-problems; derive an approximate cost function for each case and estimate the total cost based on the weighted average cost of the given cases.

3. Notation and Problem Formulation

A three-echelon supply chain including two suppliers, a central warehouse and an arbitrary number of identical retailers is considered. Transportation times between all facilities are assumed to be constant while random delay on order shipment may occur due to the stock out at the suppliers and the warehouse. The demand at each retailer follows a Poisson process and unsatisfied demand at each retailer is considered as lost sales. Retailers place orders at the central warehouse based on the continuous review inventory policy. The central warehouse and both suppliers also replenish based on continuous review policy.

Warehouse order in each cycle is divided between the two suppliers. Suppliers are assumed to have limited capacity for fulfilling orders that are received from the warehouse. During available stock, orders received from the retailers at the warehouse or from the warehouse at the suppliers are replenished immediately. During backorder, however, a random delay occurs and backordered demands are satisfied based on first-in first-out policy. Each supplier can place order from an outside source that has ample capacity. The independent and dependent decision variables of the model are presented in Table 2; model parameters are given in Table 3.

Tab. 2. Decision variables of the model

Variable	Description
$Q_{s\eta}$	Order quantity at supplier η which is an integer multiple of $Q_{w\eta}$, $\eta = 1,2$
$R_{s\eta}$	Reorder point at supplier η which is an integer multiple of $Q_{w\eta}$, $\eta = 1,2$
$Q_{w\eta}$	Part of the warehouse order quantity placed at supplier η which is an integer multiple of each retailer order, ($\eta = 1,2$)
Q_w	Order quantity of the warehouse which is an integer multiple of each retailer order
R_w	Reorder point of the warehouse which is an integer multiple of each retailer order

Q_r	Order quantity at each retailer *r*
R_r	Reorder point at each retailer *r*
λ_w	Demand rate at the warehouse
$\lambda_{s\eta}$	Demand rate at supplier η, $\lambda_{s\eta} = (Q_{w\eta}/Q_w)\lambda_w$, $(\eta = 1,2)$

Tab. 3. Parameters of the model

Parameters	Description
N	Number of retailers
λ_r	Demand rate at each retailer *r*
$L_{s\eta}$	Lead time of supplier η $(\eta = 1,2)$ assuming that $L_{s1} \leq L_{s2}$
$L_{w\eta}$	Transportation time from supplier η to the warehouse assuming that $L_{w1} \leq L_{w2}$, $(\eta = 1,2)$
$T_{s\eta}$	Delay of received orders at supplier η, $(\eta = 1,2)$
T_w	Delay of received orders at the warehouse
L_r	Transportation time from warehouse to each retailer *r*
TC	Expected total cost of the inventory system
K_η^r	Retailer r's cost per unit when the shipment of supplier η is delivered earlier, $(\eta = 1,2)$
K_η^{ws}	The warehouse and suppliers' cost per unit when the shipment of supplier η is delivered earlier, $(\eta = 1,2)$
h_r	The holding cost per unit and time unit at each retailer *r*
h_w	The holding cost per unit and time unit at the warehouse
$h_{s\eta}$	The holding cost per unit and time unit at the supplier η, $(\eta = 1,2)$
β_r	Penalty cost per unit lost at each retailer *r*
β_w	The shortage cost per unit and time unit at the warehouse
Cap_η	Maximum capacity of supplier η for each shipment to the warehouse, $(\eta = 1,2)$

The presented supply chain is divided into the following two supply chains: SC1 and SC2. SC1 includes supplier *1*, the central warehouse and a number of identical retailers which are replenished by supplier *1*. SC2 consists of supplier *2*, the central warehouse and a number of identical retailers which are replenished by supplier *2*. By replacing N retailer with demand rate λ_r with a retailer with demand rate $N\lambda_r$, we face two serial supply chain with one supplier, a central warehouse and single retailer. The average unit cost in a serial system with base stock policy will be used to derive the operating cost in a similar system with (R, Q) policy. Hence, we initially give the unit cost function in a serial supply chain, which operates under the base-stock policy.

3.1. The unit cost of the serial inventory system operating based on the base stock policy

In this subsection, we consider a basic system including single retailer, a central warehouse and single supplier operating under base stock policy. The inventory holding and shortage costs per unit are obtained for the given basic supply chain. In subsection 3.2, this cost will be used for obtaining the total cost of a three-echelon supply chain including a number of identical retailers, a central warehouse and two suppliers. In the given basic supply chain, the total cost per unit at the retailer, the central warehouse and the supplier is obtained from Eq. (1) in which S_r, S_w and S_s represent the inventory positions at the retailer, the warehouse and the supplier, respectively:

$$unit\_cost(S_r, S_w, S_s) = \Pi_{ws}(S_r, S_w, S_s) + \Pi_r(S_r, S_w, S_s) \quad (1)$$

The given cost function in Eq. (1) involves with the warehouse and supplier's cost per unit (i.e. $\Pi_{ws}(S_r, S_w, S_s)$) and the retailer's cost per unit (i.e. $\Pi_r(S_r, S_w, S_s)$). At the retailer, if the ordered unit arrives before its corresponding demand, it is held in stock and a holding cost is added to the system cost. As mentioned before, unsatisfied demand at the retailer is lost sales. Retailer's cost in Eq. (1) can be written as in Eq. (2).

$$\Pi_r(S_r, S_w, S_s) = TH_r(S_r, S_w, S_s)(1 - P(S_r)) + \beta_r P(S_r) \quad (2)$$

While unsatisfied demand at the retailer is lost sales, the queueing system under Poisson demand follows M/G/S/S queuing model with S servers and generally distributed service time. In the above Equation, $P(S_r)$ is obtained from Erlang`s loss formula while S_r servers are occupied. In this system, the arrival rate is equal to λ_r and the mean service time is $\bar{L}_r$, which is the mean lead time at retailer r (transportation time plus mean delay due to the warehouse stock out). The first statement in Eq. (2) (i.e. $TH_r(S_r, S_w, S_s)$) represents the expected holding cost per unit at retailer r. In the second part, β_r represents the penalty cost per lost demand at retailer r and $P(S_r)$ is the probability of lost demand occurrence.
Note that holding costs at each facility and shortage cost at the warehouse are obtained as mentioned in [22].

3.2. Extension to three-echelon inventory system operating under continuous review policy

In this subsection, the mathematical formulations are given to estimate the cost of an inventory system including a number of identical retailers, a central warehouse and two suppliers operating under the continuous review policy. The system cost is determined using the cost of the basic supply chain given in subsection 3.1. When a facility uses one-for-one ordering policy, the inventory position (on hand inventory plus outstanding order minus backorders) is constant. In general, batch-ordering case, the inventory position is variable so that per each item in a batch, an inventory position is assigned to the facility.
In this system, unsatisfied demands at the warehouse and suppliers are assumed to be backordered and random delays occurs due to shortage of stock at these facilities. If the warehouse is out of stock, the demand at the warehouse is satisfied with a delay of T_w. If the suppliers are out of stock, the demand at supplier *1* is satisfied with a delay of T_{s1} and the demand at supplier *2* is met with a delay of T_{s2}. If the demand is satisfied immediately at the suppliers, the delay in order replenishment is equal to zero; otherwise, the delay will be higher than zero. We consider two suppliers in the proposed model and according to the delay value at each supplier, the following four cases may occur:

1. Case 1 :$T_{s1} = t_1$ and $T_{s2} = t_2$
2. Case 2 :$T_{s1} = t_1$ and $T_{s2} = 0$
3. Case 3 :$T_{s1} = 0$ and $T_{s2} = t_2$
4. Case 4 :$T_{s1} = 0$ and $T_{s2} = 0$

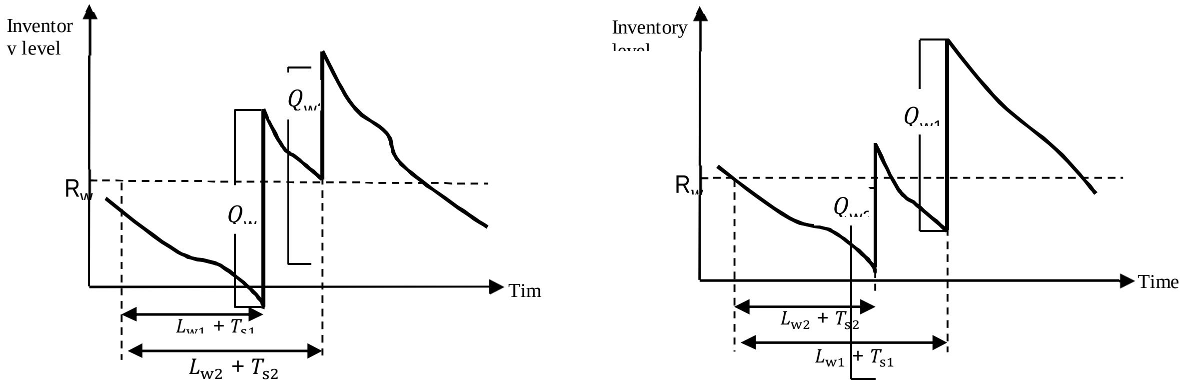
The delay density function ($f(t)$) in each facility applying base-stock policy can be given as in Eq. (3) in which λ represents the rate of Poisson demand and S represents the inventory position [22].

$$f(t) = \frac{\lambda^S (L-t)^{S-1} e^{-\lambda(L-t)}}{(S-1)!} \quad (3)$$

For $t = 0$, the probability distribution function is given by Eq. (4).

$$P(t = 0) = \sum_{k=1}^{S} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad (4)$$

Due to the stochastic nature of the delay, which occurs at the suppliers, orders will be delivered to the warehouse in different sequences in each ordering cycle. Each delivery sequence affects the inventory level and consequently the operating cost at the warehouse. Accordingly, for each of the above-mentioned cases, we face at least one of the two following situations at the warehouse (Fig.1):
Situation *1*: Order from supplier *1* arrives earlier than that of *2* at the warehouse
Situation *2*: Order from supplier *2* arrives earlier than that of *1* at the warehouse



Situation 1: Order from supplier 1 arrives earlier **Situation 2: Order from supplier 2 arrives earlier**

Fig.1. The Warehouse inventory level

Considering the aforementioned explanations, we derive an approximate cost function for each case based on the average unit cost. This way, the total cost function of the inventory system can be obtained from the weighted average cost of the given cases.

3.2.1. Cost evaluation for case 1: $T_{s1} = t_1 > 0$ and $T_{s2} = t_2 > 0$

Considering P_1 as the probability of the first case occurrence, note that $P_1 = P(T_{s1} = t_{s1} > 0) \times P(T_{s2} = t_{s2} > 0)$ and is determined using Eq. (3). In case 1, according to delay value and transportation time from suppliers to the warehouse ($L_{w1} \leq L_{w2}$), we may face both Situation *1* and Situation *2*:

Situation 1: order from supplier 1 arrives earlier at the warehouse

If the first arriving order comes from supplier *1*, the average cost per unit at the retailer r is equal to K_1^r and the average cost per unit at the warehouse and suppliers is equal to K_1^{ws}; this situation occurs with probability of $S1 = P(L_{w1} + t_{s1} < L_{w2} + t_{s2})$. $S1$ can be evaluated using delay density function at the supplier *1* ($f(t_{s1})$) and supplier *2* ($f(t_{s2})$) given in Eq. (3).

In Situation *1*, by averaging over different combinations of indices including i, j and k_η ($\eta = 1,2$) the average cost per unit at the retailer r (K_1^r) can be given as in Eq. (5) :

$$K_1^r = \frac{1}{Q_{s1}Q_wQ_r}\sum_{k_1=R_{s1}+1}^{R_{s1}+Q_{s1}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(1,R_w+1)^+}^{R_w+Q_{w1}} c_{ijk_1} + \sum_{j=R_w+1}^{0} d_{ijk_1}\right) + \frac{1}{Q_{s2}Q_wQ_r}\sum_{k_2=R_{s2}+1}^{R_{s2}+Q_{s2}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+Q_{w1}+1)^+}^{R_w+Q_w} c_{ijk_2} + \sum_{j=R_w+Q_{w1}+1}^{0} d_{ijk_2}\right) \quad (5)$$

In Eq. (5), the unit cost at the retailer c_{ijk_η} ($\eta = 1,2$) is obtained from Eq. (6):

$$c_{ijk_\eta} = \begin{cases} \sum_{l=I_l}^{I_u} \Pi_r\left(i, l, k_\eta Q_w Q_r\right) \times p_{lj}, & k_\eta > 0 \\ \sum_{l=I_l}^{I_u} \Pi_r\left(i, l - \left(k_\eta - R_\eta - 1\right)Q_w Q_r, 0\right) \times p_{lj} & k_\eta \leq 0 \end{cases} \quad (6)$$

Π_r in Eq. (6) represents the retailer cost per unit and can be obtained using the equations shown in subsection 3.1. The inventory positions at the retailer, warehouse and supplier η ($\eta = 1,2$) are i, l and $k_\eta Q_w Q_r$, respectively. As an example, to derive $\Pi_r(S_r, S_w, S_s)$ in Eq. (2), we substitute arguments as follows: $S_r = i$, $S_w = l$ and $S_s = k_\eta Q_w Q_r$ ($k_\eta > 0$). The inventory position at supplier η is $k_\eta Q_w Q_r$ and k_η has been distributed uniformly between the two values of $R_{s\eta} + 1$ and $R_{s\eta} + Q_{s\eta}$. Besides, the inventory position at the retailer varies between the values of $R_r + 1$ and $R_r + Q_r$. In this supply chain, there is more than one retailer and both the retailers and the warehouse (as the higher level) order in batches, each customer demand may trigger a retailer order from the warehouse. The l_{th} customer demand (after the warehouse order) will trigger an order for the j_{th} batch at the warehouse with probability of p_{lj} while j has been distributed uniformly between $R_w + 1$ and $R_w + Q_w$. p_{lj} can be calculated as mentioned in [23].

Note that the inventory position is equal to physical inventory plus outstanding orders minus backorders. This means that for $k_\eta \leq 0$ we can express the inventory position at the warehouse as $l - (k_\eta - R_\eta - 1)\, Q_w Q_r$, while $(k_\eta - R_\eta - 1)\, Q_w Q_r$ denotes the backorders at the warehouse.

When retailer *r* places an order and the inventory position at warehouse is negative, the unit cost at the retailer d_{ijk_η} $(\eta = 1{,}2)$ in Eq. (5) can be calculated by Eq. (7):

$$d_{ijk_\eta} = \begin{cases} \Pi_r\left(i, 0, k_\eta Q_w Q_r\right) & k_\eta > 0 \\ \Pi_r(i, 0{,}0) & k_\eta \leq 0 \end{cases} \tag{7}$$

The average cost per unit at the warehouse and suppliers (K_1^{ws}) can be given as in Eq. (8):

$$K_1^{ws} = \frac{1}{Q_{s1} Q_w Q_r} \sum_{k_1 = R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{i=R_r+1}^{R_r+Q_r} \left(\sum_{j=(0, R_w+1)^+}^{R_w+Q_{w1}} x_{ijk_1} + \sum_{j=R_w+1}^{0} y_{ijk_1} \right) + \frac{1}{Q_{s2} Q_w Q_r} \sum_{k_2 = R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{i=R_r+1}^{R_r+Q_r} \left(\sum_{j=\left(0, R_w+Q_{w1}+1\right)^+}^{R_w+Q_w} x_{ijk_2} + \sum_{j=R_w+Q_{w1}+1}^{0} y_{ijk_2} \right) \tag{8}$$

In Eq. (8), the unit cost at the warehouse and the supplier η, x_{ijk_η} $(\eta = 1{,}2)$ is calculated as in Eq. (9):

$$x_{ijk_\eta} = \begin{cases} \sum_{l=I_l}^{I_u} \Pi_{ws}\left(i, l, k_\eta Q_w Q_r\right) \times p_{lj}, & k_\eta > 0 \\ \sum_{l=I_l}^{I_u} \Pi_{ws}\left(i, l - (k_\eta - R_\eta - 1) Q_w Q_r, 0\right) \times p_{lj} & k_\eta \leq 0 \end{cases} \tag{9}$$

When a retailer places an order and the inventory position at warehouse is negative, the unit cost at the warehouse and the supplier η, y_{ijk_η} $(\eta = 1{,}2)$ is obtained from Eq. (10):

$$y_{ijk_\eta} = \begin{cases} \Pi_{ws}\left(i, 0, k_\eta Q_w Q_r\right) + c_w \left(\frac{j - R_w - 1}{\lambda_w} \right), & k_\eta > 0 \\ \Pi_{ws}(i, 0{,}0) + c_w \left(\frac{j - R_w - 1}{\lambda_w} + \frac{k_\eta - R_{s\eta} - 1}{\lambda_{s\eta}} \right) & k_\eta \leq 0 \end{cases} \tag{10}$$

When the inventory position at both the warehouse and supplier is less than or equal to 0, $(j - R_w - 1/\lambda_w) + (k_\eta - R_{s\eta} - 1/\lambda_{s\eta})$ denotes the average time it takes the warehouse to replenish its inventory position and results in additional shortage cost. For $k_\eta > 0$, $(j - R_w - 1/\lambda_w)$ is the average time it takes the warehouse to fill its inventory position which cause additional shortage cost at the warehouse.

Π_{ws} in Eq. (9) and Eq. (10) represents the warehouse and supplier's cost per unit as mentioned in subsection 3.1. In Eq. (6) and Eq. (9), I_l and I_u are calculated as in Eqs. (11)-(13):

$$I_l = j \qquad j < N \qquad (11)$$
$$I_l = N - 1 + (j - N + 1)Q_r \qquad j \geq N \qquad (12)$$
$$I_u = (N-1)(Q_r - 1) + jQ_r \qquad (13)$$

Situation 2: order from supplier 2 arrives earlier at the warehouse

If the first order arrives from supplier *2*, the average cost per unit at the retailer *r* is equal to K_2^r and the average cost per unit at the warehouse and suppliers is K_2^{ws}; this situation occurs with a probability of $1 - S1$. In this situation, the average cost per unit at the retailer *r* (K_2^r) can be given as in Eq. (14):

$$K_2^r = \frac{1}{Q_{s2}Q_wQ_r}\sum_{k_2=R_{s2}+1}^{R_{s2}+Q_{s2}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+1)^+}^{R_w+Q_{w2}} c_{ijk_2} + \sum_{j=R_w+1}^{0} d_{ijk_2}\right) + \frac{1}{Q_{s1}Q_wQ_r}\sum_{k_1=R_{s1}+1}^{R_{s1}+Q_{s1}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+Q_{w2}+1)^+}^{R_w+Q_w} c_{ijk_1} + \sum_{j=R_w+Q_{w2}+1}^{0} d_{ijk_1}\right) \qquad (14)$$

In Eq. (14), c_{ijk_η}, $\eta = 1{,}2$ is obtained from Eq. (6) and d_{ijk_η}, $\eta = 1{,}2$ is calculated from Eq. (7). The average cost per unit at the warehouse and suppliers (K_2^{ws}) can be given as in Eq. (15):

$$K_2^{ws} = \frac{1}{Q_{s2}Q_wQ_r}\sum_{k_2=R_{s2}+1}^{R_{s2}+Q_{s2}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+1)^+}^{R_w+Q_{w2}} x_{ijk_2} + \sum_{j=R_w+1}^{0} y_{ijk_2}\right) + \frac{1}{Q_{s1}Q_wQ_r}\sum_{k_1=R_{s1}+1}^{R_{s1}+Q_{s1}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+Q_{w2}+1)^+}^{R_w+Q_w} x_{ijk_1} + \sum_{j=R_w+Q_{w2}+1}^{0} y_{ijk_1}\right) \qquad (15)$$

In Eq. (15), x_{ijk_η}, $\eta = 1{,}2$ is calculated as in Eq. (9) and y_{ijk_η}, $\eta = 1{,}2$ is obtained from Eq. (10); thus, the expected cost per unit at the retailer *r* (K^r) in Case *1* is computed as in Eq. (16):

$$K^r = K_1^r \times S1 + K_2^r \times (1 - S1) \qquad (16)$$

The expected cost per unit at the warehouse and suppliers (K^{ws}) is calculated as in Eq. (17):

$$K^{ws} = K_1^{ws} \times S1 + K_2^{ws} \times (1 - S1) \qquad (17)$$

As unsatisfied demand at each retailer is lost sales, the demand rates at the warehouse and both suppliers are less than the demand rate at the retailers. Therefore, the retailers' cost per time unit and the warehouse and suppliers' cost per time unit are calculated separately. Finally, the system cost per time unit in Case *1* is given by Eq. (18):

$$TC_1 = N\lambda_r K^r + \lambda_w Q_r K^{ws} \qquad (18)$$

3.2.2. Cost evaluation for case 2: $T_{s1} = t_1 > 0$ and $T_{s2} = 0$

Considering P_2 as the probability of case *2* occurrence, $P_2 = P(T_{s1} = t_{s1} > 0) \times P(T_{s2} = 0)$ can be calculated based on Eqs. (3)-(4) .In this case, similar to case *1*, both Situation *1* and Situation *2* may occur:

Situation 1: order from supplier 1 arrives earlier at the warehouse

If the supplier *1*'s shipment arrives earlier, the average cost per unit at the retailer *r* (K_1^r) is calculated using Eq. (5) and the average cost per unit at the warehouse and suppliers (K_1^{ws}) is obtained from Eq. (8). This situation occurs with probability of $S2 = P(L_{w1} + t_{s1} < L_{w2})$ can be obtained using Eqs. (3)-(4).

Situation 2: order from supplier 2 arrives earlier at the warehouse

If the supplier *2*'s shipment arrives earlier, the average cost per unit at the retailer *r* (K_2^r) is calculated using Eq. (14) and the average cost per unit at the warehouse and suppliers (K_2^{ws}) can be obtained from Eq. (15); this situation occurs with a probability of $1 - S2$.

Then, similar to Case *1*, the expected cost per unit at the retailer *r* (K^r) is obtained from Eq. (19):

$$K^r = K_1^r \times S2 + K_2^r \times (1 - S2) \quad (19)$$

and the expected cost per unit at the warehouse and suppliers (K^{ws}) can be given by Eq. (20):

$$K^{ws} = K_1^{ws} \times S2 + K_2^{ws} \times (1 - S2) \quad (20)$$

Finally, the system cost per time unit in the case *2* is given by Eq. (21):

$$TC_2 = N\lambda_r K^r + \lambda_w Q_r K^{ws} \quad (21)$$

3.2.3. Cost evaluation for case 3: $T_{s1} = 0$ and $T_{s2} = t_2 > 0$

Denote P_3 as the probability of case *3* occurrence, then, $P_3 = P(T_{s1} = 0) \times P(T_{s2} = t_{s2} > 0)$ and is evaluated using Eqs. (3)-(4) .In this case, the delay at supplier *1* is zero and $L_{w1} \leq L_{w2}$; therefore, the first arriving order arrives from supplier *1* and Situation *1* occurs with probability of $S3 = 1$.

Since the first arriving order comes from supplier *1*, then the expected cost per unit at the retailer *r* (K^r) is clearly given by Eq. (5). Furthermore, the expected cost per unit at the warehouse and suppliers (K^{ws}) is evaluated using Eq. (8). Finally, the system cost per time unit in case *3* is obtained from Eq. (22):

$$TC_3 = N\lambda_r K^r + \lambda_w Q_r K^{ws} \quad (22)$$

3.2.4. Cost evaluation for case 4: $T_{s1} = 0$ and $T_{s2} = 0$

Defining P_4 as the probability of case *4* occurrence, $P_4 = P(T_{s1} = 0) \times P(T_{s2} = 0)$ obtained from Eq. (4). In this case, the delays at both suppliers are equal to zero and $L_{w1} \leq L_{w2}$; therefore, the first arriving order comes from supplier *1* and Situation *1* occurs with probability of $S4 = 1$.

The expected cost per unit at the retailer *r* (K^r) is calculated using Eq. (5) and the expected cost per unit at the warehouse and suppliers (K^{ws}) is obtained from Eq. (8). Therefore, the system cost per time unit for case *4* can be obtained from Eq. (23):

$$TC_4 = N\lambda_r K^r + \lambda_w Q_r K^{ws} \quad (23)$$

3.3.Mathematical model

The expected cost of the given inventory system and the related constraints considering the above-mentioned cases can be given as in Eqs. (24)-(29):

$$TC = TC_1 \times P1 + TC_2 \times P2 + TC_3 \times P3 + TC_4 \times P4 \quad (24)$$

$$s.t:$$

$$Q_w = Q_{w1} + Q_{w2} \quad (25)$$

$$Q_{w1} \leq Cap_1 \quad (26)$$

$$Q_{w2} \leq Cap_2 \quad (27)$$

$$R_r \geq 0 \quad (28)$$

$$R_w \geq -NQ_r \quad (29)$$

In Eq. (24), TC_i represents the system cost per time unit for Case i and P_i represents the probability of Case i occurance ($i = 1,2,3,4$). Eq. (25), ensures that the placed order from warehouse at the two suppliers is exactly divided between the two considered suppliers. Eqs. (26)-(27) ensure that placed orders at the suppliers do not exceed the maximum capacity of the suppliers. Eq. (28) prohibits the occurrence of order crossover at the retailers as there is no more than one outstanding order at any time. For a continuous review inventory system with lost sales demand and N identical retailers, R_w should be greater or equal to $-NQ_r$ to reach the reorder point at the warehouse (Eq. (29)).

3.4. Approximating demand rate at the warehouse and suppliers

It is required to determine λ_w, λ_{s1} and λ_{s2} in order to evaluate the warehouse and suppliers' cost function. We propose an algorithm for determining demand rate at the warehouse and suppliers.

Due to the stock out at each supplier, warehouse orders are delivered with a random delay. Also, as backorders occur at the warehouse, retailers' orders are replenished with a random delay. Delayed orders result in unsatisfied demands to be lost sales at the retailers; thus, the demand rate and backordered demands decrease at the warehouse and suppliers. The backordered demands and demand rate at the warehouse and the suppliers have a proportional relation together, which can be calculated through the following algorithm (Table 4):

Tab. 4. An algorithm to approximate demand rate at the warehouse and suppliers

Step 1:	$n = 0$ Calculate the initial demand rate at the warehouse from $\lambda_w^1 = N\lambda_r$
Step 2:	$n = n + 1$ Obtain the demand rate at supplier *1* and supplier *2* from $\lambda_{s1}^n = \lambda_{s2}^n = \frac{\lambda_w^n}{Q_w}$
Step 3:	Determine the expected backorder at the supplier *1* $B_{s1}^n = \sqrt{(R_{s1} - \lambda_{s1}L_{s1})}[\varphi(z_{s1}) - z_{s1}(1 - \Phi(z_{s1}))$ While, in the above equation, $\varphi(.)$ and $\Phi(.)$ are, respectively, normal density and cumulative distribution functions.
Step 4:	Compute average lead time from supplier *1* to the warehouse by $\bar{L}_{w1}^n = L_{w1} + \frac{B_{s1}^n}{\lambda_{s1}^n}$
Step 5:	Determine the expected backorder at the supplier *2* $B_{s2}^n = \sqrt{(R_{s2} - \lambda_{s2}L_{s2})}[\varphi(z_{s2}) - z_{s2}(1 - \Phi(z_{s2}))]$
Step 6:	Compute average lead time from supplier *2* to the warehouse by $\bar{L}_{w2}^n = L_{w2} + \frac{B_{s2}^n}{\lambda_{s2}^n}$
Step 7:	Calculate the effective lead time at the warehouse $L_w^n = \min(\bar{L}_{w1}^n, \bar{L}_{w2}^n)$
Step 8:	Obtain the expected backorder at the warehouse $B_w^n = \sqrt{(R_w - \lambda_w L_w)}[\varphi(z_w) - z_w(1 - \Phi(z_w))]$
Step 9:	Compute average lead time from the warehouse to the retailer by $\bar{L}_r^n = L_r + \frac{B_w^n}{\lambda_w^n}$
Step 10:	Compute the expected length of time per cycle that the retailer is out of stock by $T_r^n = L_r \times P(R_r, \lambda_r\bar{L}_r^n) - \frac{R_r}{\lambda_r} \times P(R_r + 1, \lambda_r\bar{L}_r^n)$
Step 11:	Update the demand rate at the warehouse by $\lambda_w^{n+1} = \frac{N\lambda_r}{Q_r + \lambda_r T_r^n}$
Step 12:	Convergence evaluation $\lvert\lambda_w^{n+1} - \lambda_w^n\rvert < \varepsilon$, otherwise, return to Step 2

4. Numerical Examples

A number of designed numerical examples, which are shown in Table 5, are intended to assess the accuracy of the proposed model. A three-echelon supply chain including two suppliers, a central warehouse and five identical and independent retailers is considered. The retailers face Poisson demand. A basic numerical example is designed; the values of parameters are considered as: the number of retailers ($N = 5$), transportation time from the supplier *1* to the warehouse ($L_{w1} = 1$); transportation time from supplier *2* to the warehouse ($L_{w2} = 1.5$); transportation time from warehouse to each retailer ($L_r = 0.5$); maximum capacity of the supplier *1* for each shipment to the warehouse ($Cap_1 = 6$) and maximum capacity of the supplier *2* for each shipment to the warehouse ($Cap_2 = 5$). In order to determine the effect of the parameters on the results, the basic example is modified by varying some parameters as demand rate at each retailer ($\lambda_r = 3,5,7$), holding cost per unit and time unit at each retailer ($h_r = 1,2$), penalty cost per unit lost at each retailer ($\beta_r = 5,25$), holding cost per unit and time unit at the warehouse ($h_w = 1,2$), holding cost per unit and time unit at supplier *i* ($h_{si} = 1,2$), supplier *1*'s lead time ($L_{s1} = 1.5,3$) and supplier *2*'s lead time ($L_{s2} = 2,4$).

Tab. 5. Numerical problems related to retailers, warehouse and suppliers

No	λ_r	β_w	β_r	h_{si} ،h_w ،h_r	No	λ_r	β_w	β_r	h_{si} ،h_w ،h_r
1	3	5	5	1	13	5	5	5	2
2	3	25	5	1	14	5	25	5	2
3	3	5	25	1	15	5	5	25	2
4	3	25	25	1	16	5	25	25	2

5	3	5	5	2	17	7	5	5	1
6	3	25	5	2	18	7	25	5	1
7	3	5	25	2	19	7	5	25	1
8	3	25	25	2	20	7	25	25	1
9	5	5	5	1	21	7	5	5	2
10	5	25	5	1	22	7	25	5	2
11	5	5	25	1	23	7	5	25	2
12	5	25	25	1	24	7	25	25	2

Optimal ordering policies including the order quantities and reorder points at the retailers, warehouse and suppliers have been obtained utilizing Optimization toolbox; then, we have approximated the total system cost including holding costs at all echelons (retailers, central warehouse and suppliers), backorder cost at the warehouse and lost sale cost at the retailers. The accuracy of these results is then assessed using simulation [21]. The simulation time length is considered to be 110000 unit times with 10000 unit times as a warm-up period. The performance of the model is assessed by comparing the approximated cost with that of simulation. As illustrated in Table (6)-(7), the average error of the mathematical model compared to the simulation is relatively low and equal to 4.02% and 4.86%.

The approximated total cost has no significant difference (with P-value = 0.455) with the simulation cost and this shows the high accuracy of the given model .Assuming different values for retailer’s demand rate i.e. $\lambda_r = 3,5,7$ the average error turn out to be 4.43%, 4.37% and 4.53%, respectively which means that demand rate of the retailers has no effect on the accuracy of the model. When $L_{s1} = 1.5$ and $L_{s2} = 2$, the average error is 4.02% and when $L_{s1} = 3$ and $L_{s2} = 4$ the average error increases to 4.86%. When the holding cost at the suppliers, warehouse and retailers is equal to 1, the average error turns out to be 4.61% while the holding cost is equal to 2, the average error decreases to 4.27%. When the penalty cost at the retailers is equal to 5, the average error turns out to be 4.82% while the penalty cost is equal to 25, the average error decreases to 4.06%. This shows that increasing the holding cost at all echelons or increasing the penalty cost at the retailers increases the accuracy of the model.

Tab. 6. Comparison of approximated and simulated total cost considering $L_{s1} = 1.5$ and $L_{s2} = 2$

No	Q_{S1}	R_{s1}	Q_{s2}	R_{s2}	Q_w	R_w	Q_{w1}	Q_{w2}	Q_r	R_r	Approximation	Simulation	Error
1	5	2	2	1	2	1	1	1	2	1	75.36	74.19	2%
2	4	1	2	2	2	1	1	1	1	1	115.00	114.14	1%
3	2	1	3	-2	2	-1	1	1	3	2	318.26	310.75	2%
4	3	1	4	2	3	-2	2	1	1	3	373.70	388.29	4%
5	5	1	2	2	4	1	3	1	1	2	80.60	80.09	1%
6	5	3	4	1	3	0	1	2	2	1	192.27	195.76	2%
7	5	-1	3	-2	3	2	1	2	2	1	331.02	306.78	7%
8	4	1	5	2	3	-2	1	2	1	1	458.66	406.92	12%
9	3	2	2	-1	2	1	1	1	2	1	142.86	133.51	7%
10	4	1	2	-2	2	1	1	1	1	1	206.13	198.70	4%
11	4	1	3	2	2	-1	1	1	3	2	498.35	472.04	5%
12	4	2	4	2	4	-3	3	1	2	2	742.13	733.66	1%
13	2	2	3	1	3	-2	2	1	1	1	147.74	138.07	7%
14	4	1	2	2	2	1	1	1	1	1	167.96	172.27	3%
15	2	2	2	1	3	-3	2	1	2	1	592.62	593.11	0%
16	3	1	2	2	4	-3	1	3	3	2	793.26	814.91	3%
17	5	2	2	1	5	-2	2	3	1	1	169.78	187.84	11%
18	4	1	2	2	4	1	3	1	1	1	188.22	202.94	8%

19	2	1	3	2	4	-2	3	1	2	1	773.36	791.31	2%
20	3	1	4	-2	4	2	3	1	1	1	836.70	829.02	1%
21	5	1	2	2	3	-2	2	1	2	1	195.83	207.79	6%
22	3	3	4	1	3	-1	2	1	2	2	448.07	430.14	4%
23	5	1	3	2	3	0	1	2	1	1	767.22	807.29	5%
24	4	-1	3	3	6	-4	3	3	2	1	978.12	981.23	0%
								Mean Error					4.02%

Tab. 7. Comparison of approximated and simulated total cost considering $\mathbf{L_{s1}} = 3$ and $\mathbf{L_{s2}} = 4$

No	Q_{S1}	R_{s1}	Q_{s2}	R_{s2}	Q_w	R_w	Q_{w1}	Q_{w2}	Q_r	R_r	Approximation	Simulation	Error
1	2	2	2	1	5	3	2	3	1	1	66.02	67.20	2%
2	3	1	2	2	2	-1	1	1	2	1	240.31	226.77	6%
3	4	1	4	2	3	-1	2	1	1	1	316.60	326.19	3%
4	5	1	4	2	3	2	2	1	1	1	317.52	346.27	9%
5	3	-1	2	1	5	3	3	2	2	1	77.10	79.05	3%
6	6	1	2	2	3	-2	2	1	1	1	159.12	146.85	8%
7	5	-1	3	2	3	2	2	1	1	1	342.05	320.66	6%
8	3	1	5	3	6	-2	3	3	1	1	382.92	394.45	3%
9	3	2	2	1	3	-1	2	1	2	1	146.47	134.46	8%
10	4	1	2	-2	5	-2	3	2	2	1	357.20	331.41	7%
11	2	2	2	1	5	3	3	2	1	1	546.20	523.21	4%
12	3	-1	2	-2	4	2	3	1	1	1	615.39	653.57	6%
13	2	2	3	4	3	-1	2	1	1	3	124.38	128.22	3%
14	4	1	2	2	2	-1	1	1	1	1	211.97	203.21	4%
15	3	2	2	4	4	-3	3	1	2	2	478.13	487.49	2%
16	4	-2	2	1	3	1	2	1	1	2	631.06	657.65	4%
17	3	3	2	-1	5	-3	3	2	1	1	214.92	197.97	8%
18	4	1	2	2	4	-1	3	1	1	1	224.45	230.64	3%
19	4	1	4	-2	2	-1	1	1	1	1	880.31	860.53	2%
20	3	1	4	2	4	1	3	1	1	1	796.40	840.52	6%
21	6	1	5	2	4	1	3	1	2	1	184.14	189.61	3%
22	5	-1	3	1	3	-1	1	2	2	1	367.45	342.24	7%
23	5	1	3	3	3	1	1	2	2	2	730.60	707.51	3%
24	4	1	5	3	6	-2	3	3	1	1	845.90	889.79	5%
								Mean Error					4.86%

5. Conclusions and Future Research

An approximate cost function have been developed for a three-echelon supply chain including two suppliers, a central warehouse and a number of identical retailers while unsatisfied demand is considered as lost sales at the retailers. Transportation times between all facilities are constant while random delay may occur due to the stock out at the suppliers and warehouse. Due to the stochastic nature of the delay, lead-time at the warehouse and retailers is unpredictable. Thus, adding two suppliers as third echelon as well as stochastic lead time causes split orders from two suppliers do not arrive with the same sequence in different ordering cycles.

Since it is not straightforward to derive the cost function, the initial problem had to be divided into a number of sub-problems. Based on the delay value at each supplier, four different cases (sub-problems) were considered. We derived an approximate cost function for each case based on the average unit cost. Finally, the total cost function of the initial problem was estimated based on the weighted average cost of the given cases.

Using the average unit cost to derive the total cost function is more straightforward than applying the demand distribution during the lead-time. Numerical examples with relatively low errors confirmed the accuracy of the presented model. In this paper, the optimal ordering policies are local optimum and the order quantity at the higher level is assumed to be multiple of the order quantity at the lower level. As future research, we can consider a decentralized system and use different types of game theory approaches in order to find order quantity and reorder points of the system elements.

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