# **Price-Takers' Bidding Strategies in Joint Energy and Spinning Reserve Pay-as-Bid Markets**

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**Abstract:** Strategic bidding in joint energy and spinning reserve markets is a challenging task from the viewpoint of Generation Companies (GenCos). In this paper, the interaction between energy and spinning reserve markets is modeled considering a joint probability density function for the prices of these markets. Considering pay-as-bid pricing mechanism, the bidding problem is formulated and solved as a classic optimization problem. The results show that the contribution of a GenCo in each market strongly depends on its production cost and its level of risk-aversion. Furthermore, if reserve bid acceptance is considered subjected to winning in the energy market, it can affect the strategic bidding behavior.

Keywords: Bidding Strategy, Electricity Market, Energy Market, Spinning Reserve Market, Pay-as-Bid pricing.

# 1 Introduction

An electricity market is a system for effecting purchases, through offers and bids. The market operation is implemented competitively based on auctions. In a single-sided electricity auction, the Independent System Operator (ISO) procures the energy and reserve on behalf of the energy and reserve customers. The ISO aggregates the generation bids and clears the auction based on GenCos' bids and the system requirements, such as load level, requested reserve and etc. In this structure, the competition is established between GenCos.

GenCos participate in the market through bidding generation capacities and corresponding prices. From GenCo's point of view, designing proper bid functions is economically a challenging task to make more profit. In a joint energy and reserve market, the interaction between energy and reserve prices forces the GenCos to compromise between their bids in the submarkets. In the simultaneous markets, GenCos must bid in all submarkets at the same time. Consequently, the bidding problem is important and risky for GenCos while joint bidding in energy and reserve markets.

Many literatures can be found which consider the bidding problem in only-energy markets such as [1, 2].

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However, there are a few literatures which considered the joint bidding problem. In [3] and then in [4, 5] utilizing a bi-level optimization model, a bidding problem is presented in separate energy and spinning reserve markets. It was assumed that the bidding coefficients of rivals obeyed a joint normal distribution.

Bidding in separate uniform-priced energy and spinning reserve markets, is presented in [6]. Bidding parameters of the rivals are forecasted. The bidding parameters of the GenCos are calculated using the evolutionary programming approach.

In [7, 8], optimal allocation of resources to variety of markets is presented, but the strategic bidding problem is not considered.

Reference [9] presented a scenario generation technique for bidding and scheduling in Italian sequential power market using a multi-stage mixedinteger stochastic programming model with linear constraints.

References [10, 11] used game theoretic approaches in bi-level optimization problems, creating optimal bidding strategies at one level by a GenCo, while searching Nash equilibrium of the hybrid markets at the other level.

Simultaneous bidding problem into the separate German energy and reserve markets is introduced in [12]. Some probability distribution functions for market prices, based on the previous finding in [13] are defined for energy market and two independent reserve markets, and a stochastic optimization problem has been solved.

The bidding problem of Virtual power plant (VPP) in a day-ahead joint market of energy and spinning reserve service is investigated in [14] and a model based

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on the deterministic price-based unit commitment is presented for bidding strategy of VPP.

Reference [15] proposed a quadratic mixed-integer stochastic programming model to solve the optimal bidding strategy problem in sequentially cleared Iberian day-ahead market.

In [16], the authors of this paper formulated the joint bidding problem in energy and spinning reserve markets, from the viewpoint of a GenCo considering a joint Probability Density Function (PDF) for the market clearing prices in a PAB pricing mechanism. Participation and acceptance in reserve market is assumed independent of the energy bid acceptance. In other words, it is assumed that the generating unit can be dispatched as a reserve provider even if its energy bidding price is rejected. The mean-variance portfolio theory is utilized for consideration of the risk.

In this paper, a joint PDF for the energy and spinning reserve marginal accepted bidding prices is utilized to formulate the strategic biding problem. In the field of simultaneous bidding in the energy and spinning reserve markets, considering the portfolio theory in order to risk management, the GenCo is a portfolio manager that wants to distribute its production capacity between energy and spinning reserve markets. To find the optimal bidding parameters from a price-taker GenCo's point of view, a mathematical approach is applied with or without considering the risk. The method, which is generalization of a previously presented only-energy market bidding method [2], has been developed similar to the method presented in [16] to model and to solve the joint energy and spinning reserve bidding problem. The reserve provision is considered subjected to energy bid acceptance. Definitely, this technical constraint makes the model more realistic and more complex. It is shown that, considering this condition as a market rule, GenCos have more tendencies to bid in the energy market.

In the rest of the paper, using a model-based approach, the optimal bidding prices for the energy and spinning reserve markets and optimal reserve bidding capacity are extracted numerically. In addition, the effects of GenCos' production cost and risk-aversion degree and the correlation value between energy and spinning reserve prices on the optimal values and on GenCos' bidding strategies are analyzed.

# 2 The Market Structure

To study bidding problem, a one-hour-ahead singlesided and single-node wholesale electricity market is considered. The environment consists of the energy and spinning reserve (from now on, referred to as "reserve") markets. The main agents of the market are GenCos and the independent system operator, as seen in Fig. 1.

In this paper, the step-wise bidding protocol is selected. The method can be applied in linear bidding protocol.



Fig. 1 The joint market structure

As like some of electricity markets, for example New York and California electricity markets, the GenCo submits two stepwise functions, including capacities and corresponding prices, to the joint energy and reserve market for the next hour. The two markets are cleared simultaneously by the ISO under PAB pricing mechanism through a joint optimization program. Then the ISO informs each GenCo of its contribution to energy and reserve markets. According to market rules, the spinning reserve can be provided by energy market winners.

In the following, the strategic bidding problem in a single-sided auction is formulated from the viewpoint of a GenCo. The proposed method can be easily extended to a multi-unit bidding problem. Moreover, double sided auction can be considered and the assumption of a single-sided auction does not affect the generality of the method.

## 3 Price Modeling

Similar to [16], the interaction between energy and reserve market prices is considered assuming a joint PDF for these prices, in order to model the strategic bidding problem, mathematically.

In pay-as-bid auctions, after clearing the market in each trading period, each GenCo is informed by its own accepted bidding prices in the energy and reserve markets. The GenCo can construct a joint PDF of its Marginal Accepted Bidding Prices (MABPs), in order to design its bidding strategy utilizing the method that will be presented in the next section. Therefore, it is reasonable to assume a joint PDF in order to model the energy and reserve prices. This joint PDF is constructed using historical accepted and/or rejected bidding prices, from the viewpoint of the GenCos. The construction of the stated above joint probability density function is beyond the scope of this paper. The joint PDF of MABPs is assumed to be known as  $f_{\rho_e^m,\rho_r^m}(\rho_e^m,\rho_r^m)$ ,

where  $\rho_e^m$  and  $\rho_r^m$  are energy and reserve marginal accepted bidding prices, respectively.

## 4 **Problem Description**

In order to simplify the analysis of GenCos' bidding behavior with different production costs, a linear cost function as  $C_j(p_j)=c_jp_j$  is assumed for the  $j^{\text{th}}$  GenCo, where  $p_j$  (MWh) and  $c_j$  (\$/MWh) are the generated power by the  $j^{\text{th}}$  GenCo and its average cost, respectively. It is clear that the average cost of each GenCo depends on technical and economical characteristics of the generating unit. Similar to [3-7, 9, 10], it is assumed that the GenCo does not bear any cost for the reserve provision.

# 4.1. Interpretation of The Electricity Market Bi-Level Problem

The process of bidding and market clearing in joint energy and reserve PAB markets can be modeled as a bi-level optimization problem [9] which consists of GenCos' level and ISO's level. In order to simplicity, transmission system is ignored and one-step bidding in each market is assumed in the following bi-level formulation.

#### GenCos' level:

In this level, the GenCos try to maximize their profit. The objective function of the  $j^{\text{th}}$  GenCo can be formulated as:

$$\max u_{j}(\rho_{ej} - c_{j})p_{ej} + u_{j}\rho_{rj}p_{rj}$$
  
subject to:  
$$\rho_{ej} \leq \overline{\rho}_{e}$$
(1)

$$\rho_{rj} \leq \overline{\rho}_r$$

where,  $\rho_{ej}$  and  $\rho_{rj}$  are the GenCo's energy and reserve bidding prices, respectively.  $p_{ei}$  and  $p_{ri}$  are dispatched capacities of the GenCo in the energy and reserve markets and  $\overline{\rho}_e$  and  $\overline{\rho}_r$  are energy and reserve markets ceiling prices, respectively. Since the provision of spinning reserve requires the unit to be committed, the binary variable  $u_j$  is used which determines that the GenCo is dispatched in the energy market  $(u_j=1)$  or not  $(u_j=0)$ .

# ISO's level:

ISO minimizes the total procurement cost by solving the following market clearing problem at this level:

$$\min\sum_{j=1}^{n} u_j (\rho_{ej} p_{ej} + \rho_{rj} p_{rj})$$

subject to:

$$\sum_{j=1}^{n} u_{j} p_{ej} = Demand$$

$$\sum_{j=1}^{n} u_{j} p_{rj} = Reserve Requirement$$

$$p_{ej} + p_{rj} \leq G_{j\max} \qquad (j = 1, 2, ..., n)$$

$$u_{j}G_{j\min} \leq p_{ej} \qquad (j = 1, 2, ..., n)$$

$$0 \leq p_{rj} \leq R_{j} \qquad (j = 1, 2, ..., n)$$

$$u_{j} = 0 \text{ or } 1 \qquad (j = 1, 2, ..., n)$$

$$(j = 1, 2, ..., n)$$

In the above formulations, the maximum and minimum generation capacities of the  $j^{\text{th}}$  GenCo are  $G_{j\text{max}}$  and  $G_{j\text{min}}$ , respectively. *n* is the number of GenCos and the reserve production capability of the  $j^{\text{th}}$  GenCo,  $R_j$ , is determined according to its generating unit's ramp-rate.

In practical markets, the information of cost and bidding parameters of the rivals is not publicly available. Therefore, solving the stated-above bi-level problem is not possible for GenCos to make strategic bidding functions. Therefore, it is reasonable to develop a method based on the practically available information. In the following, utilizing the previously discussed joint PDF of the MABPs, a model-based approach is developed. This method uses the price information, which is the only available information of the electricity market.

#### 4.2. The Proposed Bid Functions

In this section, two step-wise functions are proposed for energy and reserve bids. Based on the proposed bid functions, the optimal energy and reserve bidding prices and also the optimal reserve bidding capacity are determined.

It is assumed that a GenCo designs two step-wise functions for the energy and reserve bidding prices. Clearly, *G-R*, i.e. total capacity minus the reserve capability, is a fraction of the total capacity that can be offered only to the energy market. Thus, the GenCo should allocate the rest of its capacity, which is equal to *R*, to energy and reserve markets. Let *x* be the reserve capacity bid, which is a fraction of *R* that the GenCo expects to sell it in the reserve market. As Fig. 2-a shows, the reserve offered price is  $\rho_r$ . Likewise, *R-x* is the remainder of the reserve capability that the GenCo prefers to offer it to the energy market. Consequently, *G-x* will be the part of GenCo's capacity that can be sold in the energy market.  $\rho_{el}$  and  $\rho_{e2}$  are the offered prices for this part,  $\rho_{el}$  for *G-R* and  $\rho_{e2}$  for *R-x*.



Fig. 2 The bid functions (a) reserve (b) energy

 $\rho_{el}$ ,  $\rho_{e2}$ ,  $\rho_r$  and x are unknown parameters that should be determined in order to optimize the bidding strategy.

It should be noted that according to electricity market rules, each GenCo must offer its total generation and reserve capacities to energy and reserve markets, respectively. But, there are remaining capacities which have not been shown in the proposed energy and reserve bid functions, x and R-x, respectively. The GenCo should choose very high prices, e.g. the market ceiling prices, for these parts in the energy and reserve markets to ensure the rejection. Consequently, in order to simplicity, these remaining capacities are omitted from Fig. 2 and the following computations.

#### 4.3. GenCo's Expected Profit

For the proposed bid functions, the total profit of the GenCo,  $\pi$ , can be calculated as the summation of energy and reserve profits,  $\pi_e$  and  $\pi_r$ , respectively.

The energy profit can be formulated as:

$$\pi_e = (\rho_{e1} - c)(G - R) + (\rho_{e2} - c)(R - x)$$
(3)

Because the GenCo's energy profit depends on acceptance of the energy bidding prices, the profit is a function of energy price which is a random variable. Since the joint PDF of MABPs is assumed to be known, the energy profit can be rewritten in terms of indicator random variables  $I_{ei}(\rho_{ei}, \rho_e^m)$  as follows [16]:

$$\pi_{e} = (\rho_{e1} - c)(G - R)I_{e1}(\rho_{e1}, \rho_{e}^{m}) + (\rho_{e2} - c)(R - x)I_{e2}(\rho_{e2}, \rho_{e}^{m})$$

$$I_{ei}\left(\rho_{ei}, \rho_{e}^{m}\right) = \begin{cases} 0 & \rho_{ei} > \rho_{e}^{m} \\ 1 & \rho_{ei} \le \rho_{e}^{m} \end{cases}, \quad i = 1, 2$$

$$(4)$$

where  $I_{ei}(\rho_{ei}, \rho_e^m)$  *i*=1, 2 is defined to model the acceptance of the energy bidding prices.

Considering the proposed reserve bid function, the reserve profit can be formulated as

$$\pi_r = \rho_r x \tag{5}$$

Similar to above explanations, reserve profit depends on acceptance of the reserve bidding price. Moreover, the acceptance of the energy bid is considered as a necessary condition for the reserve bid to be accepted. Therefore, the reserve profit is a function of energy and reserve prices and can be rewritten in terms of indicator random variables  $I_{e1}(\rho_{e1}, \rho_e^m)$  and  $I_r(\rho_r, \rho_r^m)$  as follows:

$$\pi_r = \rho_r x \cdot I_r(\rho_r, \rho_r^m) \cdot I_{e1}(\rho_{e1}, \rho_e^m)$$

$$I_r(\rho_r, \rho_r^m) = \begin{cases} 0 & \rho_r > \rho_r^m \\ 1 & \rho_r \le \rho_r^m \end{cases}$$
(6)

where  $I_r(\rho_r, \rho_r^m)$  is defined to model the acceptance of the reserve bid price.

Consequently, the total profit of the GenCo can be written as:

$$\pi = (\rho_{e1} - c)(G - R)I_{e1}(\rho_{e1}, \rho_e^m) + (\rho_{e2} - c)(R - x)I_{e2}(\rho_{e2}, \rho_e^m) + \rho_r x I_r(\rho_r, \rho_r^m) I_{e1}(\rho_{e1}, \rho_e^m)$$
(7)

Considering the joint PDF for the energy and reserve MABPs at a specific time or a load level and using the definition of the indicator random variables, the expectation of the profit can be computed as follows:

$$E\{\pi\} = (\rho_{e1} - c)(G - R)[1 - F_{\rho_e^m}(\rho_{e1})] + (\rho_{e2} - c)(R - x)[1 - F_{\rho_e^m}(\rho_{e2})] + \rho_r x [1 - F_{\rho_r^m}(\rho_r) - F_{\rho_e^m}(\rho_{e1}) + F_{\rho_e^m}, \rho_r^m(\rho_{e1}, \rho_r)]$$
(8)

 $F_{\rho_e^m}(), i = 1, 2$ , and  $F_{\rho_r^m}()$  are the marginal cumulative distribution functions (CDFs)of energy and reserve MABPs, respectively.  $F_{\rho_r^m, \rho_e^m}()$  is the joint CDF of MABPs.

#### 4.4. GenCo's Objective Function

In order to make the optimal decision, the GenCo faces with the following problem without considering the risk:

max  $E\{\pi\}$ 

*s.t*.

 $0 \le x \le R \le G$ 

(9)

 $\rho_{e1} \leq \rho_{e2} \leq$  energy market ceiling price

 $\rho_r \leq$  reserve market ceiling price

The optimal value of the above objective function and its corresponding optimal bidding parameters can be calculated numerically. The numerical results will be presented and analyzed in the next section. However, a discussion about reserve allocated capacity is presented here.

Rearranging Eq. (8), it can be shown that the expected profit,  $E\{\pi\}$ , is a linear function of variable *x*. Therefore, without considering the risk, the GenCos with different production costs can be classified into two groups. The expected profit of selling the total reserve capability in the energy market is more than selling it in the reserve market, in the first group. This group comprises low cost GenCos. The other group is composed of the high cost GenCos which prefer to sell their total reserve capabilities in the reserve market. It should be noted that by taking the risk into consideration, a third group can be observed; in which the GenCos tend to sell their reserve capability in both the energy and reserve markets.

# 4.4.1. Consideration of the Risk

In this case, the effect of risk on capacity allocation to energy and reserve markets is considered. To make trade-off between profit and risk, the mean-variance approach is applied using a utility function in the form of  $U{\pi}=E{\pi}-\omega Var{\pi}$ . In this form, profit variance is used as a measure of risk [17-20].  $\omega$  is a weighting factor, specified based on risk-aversion degree of the investor. The objective of a GenCo is:

$$\max \ U\{\pi\} = E\{\pi\} - \omega Var\{\pi\}$$

s.t.

 $0 \le x \le R \le G \tag{10}$ 

 $\rho_{e1} \leq \rho_{e2} \leq$  energy market ceiling price

 $\rho_r \leq$  reserve market ceiling price

## 5 Numerical Results

In this section, considering a joint normal probability density function for energy and reserve MABPs, the effect of GenCo's production cost and risk-aversion degree, and correlation between the energy and reserve MABPs on bidding behavior of the GenCos is analyzed utilizing the proposed bid function.

In real markets, GenCos bid in a joint energy and reserve market, not only based on their expected profit, but also they usually consider the risk, and thus they bid based on their utility function in order to compromise between their expected profit and the risk. In the following subsections the GenCos' bidding behavior is analyzed based on the mentioned objective function shown in Eq. (10).

# 5.1. The Effect of Production Cost

In order to analyze the bidding behavior of different cost GenCos in the simultaneous energy and reserve market, the total generation capacity, G, and reserve capability, R, are selected, among all the GenCos, to be 200 MW and 100 MW, respectively.

Fig. 3 shows the reserve optimal bidding capacity of GenCos with different production costs. In this figure the statistical parameters of MABPs, according to [8, 12, 15], are selected as  $\mu_e$ =40,  $\mu_r$ =5,  $\sigma_e$ =5,  $\sigma_r$ =1 and  $\rho$ =0.2, where  $\rho$  is the correlation coefficient of energy and reserve prices. Also, the risk aversion degree among GenCos is assumed to linearly decrease from 0.0098 at marginal cost of 20 \$/MWh to 0.001 at 40 \$/MWh marginal cost.

The three previously stated groups of the GenCos can be observed in Fig. 3. It can be concluded that the more marginal costs, the more the producer preference to bid in the reserve market. The lower cost GenCos can success in the energy market, by bidding lower prices. However, increasing the generation cost results in decreasing the winning chance in the energy market. Therefore, the GenCos with higher production costs prefer to bid most of their reserve capability in the reserve market.

The dashed curve in Fig. 3 presents the reserve optimal bidding capacity of GenCos when the reserve bid acceptance is assumed independent of GenCos' energy market participation, based on the formulations developed in [16]. It is can be seen that considering the reserve market participation independent of energy bid acceptance, deceptively increases the GenCos' tendency to bid in the reserve market. This is an important result of this paper. It says that it is essential to consider the energy bid acceptance as a necessary condition for reserve market participation, in modeling the electricity market bidding problems.

Comparing the optimal energy and reserve bidding prices, it can be seen that if the production cost grows, the energy bidding prices also grows. It is clear that the reserve bid price decreases slowly, while the generation cost increases. That is because the acceptance probability in the energy market is decreased while increasing the production cost and the GenCo must decrease the reserve bid price to increase the probability of reserve acceptance. This concept is compatible with the model presented in [12].



Fig. 3 Optimal bidding capacities in reserve market

# **5.2.** The Effect of Correlation Coefficient

The correlation between energy and reserve MABPs depends on time, load level, reserve requirement or some other parameters. The discussion about the correlation is beyond the scope of this paper, but the bidding behavior of GenCos in different correlation values between energy and reserve prices can be studied based on the objective function introduced in Eq. (10).

A high and a low cost GenCo are selected to study the effect of correlation between energy and reserve MABPs on their bidding behavior. The average cost of the high cost GenCo is 35 (\$/MWh) and the other GenCo's marginal cost is 29 (\$/MWh). Both GenCos have generation and reserve capacities equal to 200 MW and 100 MW respectively. The statistical parameters of prices are selected as  $\mu_e$ =40,  $\mu_r$ =5,  $\sigma_e$ =5,  $\sigma_r$ =1. The risk weighting factors,  $\omega$ , are selected according the previous subsection as 0.0058 and 0.0032, respectively for the high and low cost GenCos.

It can be concluded from Fig. 4 that high cost GenCos increase their reserve capacity bid while the correlation increases. High cost GenCos are more sensitive to the correlation than low cost GenCos. Since high cost GenCos have less chance in the energy market, they bid more capacity in the reserve market when the two markets are positively correlated.



Fig.4. Sensitivity of reserve capacity bid of (a) low and (b) high cost GenCo to correlation coefficient



**Fig.5.** Sensitivity of reserve capacity bid of (a) low and (b) high cost GenCo to risk-aversion degree

#### 5.3. The Effect of Risk-Aversion Degree

The last two GenCos are considered in order to analyze and compare the bidding behavior of high and low cost price-taker GenCos under different riskaversion degrees. In this case the correlation value between energy and reserve market prices is selected to be 0.2. The optimal values of reserve bid capacity versus risk aversion degree are shown in Fig. 5.

A low cost GenCo, generally have chance to sell its total capacity to the energy market. However, when the GenCo's risk-aversion degree is high, the portfolio theory proposes to distribute the capacity between various markets. This can be seen in Fig. 5-a.

Fig. 5-b also shows that for a high cost GenCo, the reserve capacity bid is increased by increasing the level of risk-aversion.

### 6 Conclusion

In this paper a joint probability density function for a GenCo's historical energy and reserve marginal accepted bidding prices is considered and two step-wise bid functions are proposed for the GenCo while participating in a joint energy and spinning reserve market. The bidding problem is formulated considering the energy bid acceptance as a necessary condition for reserve market participation.

Moreover, analysis of GenCos' bidding behavior with different production costs is addressed. The results show that the contribution of GenCos in the energy and reserve markets depends on their production costs, their risk-aversion degree and the correlation coefficient between energy and reserve prices. In addition, the GenCos with different production costs are classified into three groups: only-energy, only-reserve, and energy-and-reserve markets participants. The results can be summarized as follows:

- The optimal reserve bidding capacity and energy bidding price are increased and the reserve bidding price is decreased while the GenCos' production cost increases.

- The GenCos increase their reserve capacity bid while the level of risk-aversion increases.

- For a high cost GenCo, the optimal reserve capacity bid is increased by increasing the correlation coefficient. But a low cost GenCo behaves inversely.

- The acceptance of the energy bid as a necessary condition for the reserve market participation is very important to be considered in modeling the strategic joint bidding problems in simultaneous energy and reserve markets.

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