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A semi analytical solution for rising limb of hydrograph in 2D overland flow

A. Shokoohi[1,*], **B. Saghafian**[2]

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Abstract

In almost all of the present mathematical models, the upstream subbasins, with overland flow as the dominant type of flow, are simulated as a rectangular plane. However, the converging plane is the closest shape to an actual upstream subbasin. The intricate nature of the governing equations of the overland flow on a converging plane is the cause of prolonged absence of an analytical or semi analytical solution to define the rising limb of the resulted hydrograph. In the present research, a new geomorphologic semi analytical method was developed that tries to establish a relationship between the parallel and converging flows to reduce the complexity of the equations. The proposed method uses the principals of the Time Area method modified to apply the kinematic wave theory and then by applying a correction factor finds the actual discharge. The correction factor, which is based on the proportion of the effective drained area to the analytically calculated one, introduces the convergence effect of the flow in reducing the potentially available discharge in a parallel flow. The proposed method was applied to a case study and the result was compared with that of Woolhiser's numerical method that showed the reliability of the new method.

Keywords: Kinematic wave, Converging plane, Parallel flow, Time to equilibrium, Rising limb, Geomorphologic correction factor

1. Introduction and Background

Rainfall-Runoff mathematical models, based on the physical mechanism of the overland flow, must be able to consider all types of watershed geometries. Because of the inherent complexity of natural watersheds, researchers are interested in simplifying their geometry with using a rectangular, V-shape, converging, or diverging plane. Wooding [1, 2] was the first who solved the kinematic equations of overland flow on a V-shape or open book plane analytically and numerically. According to many researchers, the kinematic wave, as a simplification of Saint Venant's equations, is enough for the simulation of the overland flow [3]. Wooding [4] applied the kinematic wave theory to simulate the overland flow on an open book shaped plane and compared the results with observed data. Although obtaining good results, Wooding insisted on using a better and more detailed geometry for the channels. However, the Wooding model is not able to simulate the sharpness of the hydrograph of a converging plane [5]. Woolhiser [6] proposed a sloped V-shape plane as a part of a truncated cone for simulation of the converging flow, which is basically regarded as a two dimensional (2D) flow (Figure 1).

Because of the geometrical similarity of the upstream subbasins in large watersheds to the converging planes [3], finding a robust and simple solution for kinematic wave has been a concern of the 2D overland flow modeling since the 1960s. Veal [7] obtained the unsteady continuity and

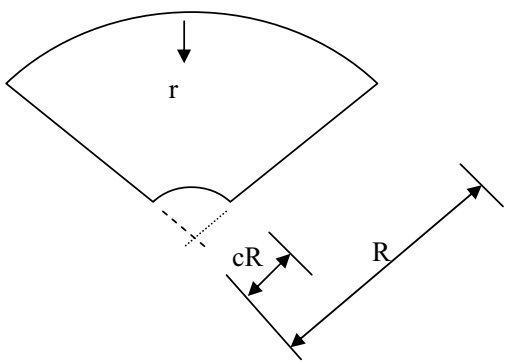


Fig. 1. Plan of a converging plane

* *Corresponding Author: shokoohi@ikiu.ac.ir*
1 Associate Professor, water engineering department, faculty of technical and engineering, International University of Imam Khomeini, Qazvin, Iran
2 Professor in Hydrology, Soil Conservation and Watershed Management Center, Tehran, Iran

momentum equations with the lateral flow for a converging plane. He presented a numerical solution for some special cases. Woolhiser [6] was the first who utilized Veal's proposed equations for hydrological modeling. He imposed a constant rate rainfall on a converging plane and by using the characteristic method in dimensionless form could achieve a numerical and an analytical solution for the rising and the falling limb of the resulted hydrograph respectively. Singh [8], Singh, and Woolhiser [9] both presented a model based on the numerical solution of the converging flow and evaluated the results with observed data. Singh [10, 11] performed many tests to calculate the lag time on the converging planes and introduced some simple methods for deriving the kinematic wave parameters for such 2D flows. Sherman and Singh [12, 13] studied and described the mathematical basis of the kinematic wave modeling for the converging planes. Ellis et al. [14] reported successful application of a kinematic wave based model in an urban watershed. Agiralioglu [15, 16, and 17] studied the overland flow on the converging/ diverging planes. He used the kinematic wave's time to equilibrium on a rectangular plane and derived an equation to relate the obtained results with that of a converging plane with the same length. Campbell and Parlange [18] evaluated some numerical methods for converging flow solution. Saghafian and Julien [19] reported the development and application of a 2D finite difference model named CASC2D for converging flow simulation. When there is a small river and regardless of the shape of the upstream watershed, Mohammad T. Dastorani and Nigel G. Wright [20] used a hydrodynamic model for river flow prediction and then tried to optimize the results by applying artificial neural networks (ANN). Singh [3] reported results of using an explicit single step Lax-Wendrov algorithm for solving the converging flows. As he mentioned, the method uses an analytical method in its boundaries that improves the precision and convergence speed of the algorithm. He reported using this algorithm on more than 50 watersheds and for more than 500 events.

Literature review shows that a simple, robust, and analytical solution for the rising limb of a hydrograph has not been presented since the introduction of the kinematic wave equations for the converging flow. The available numerical methods, in spite of their efficiency, have some limits. All of the numerical explicit methods have conditional stability. They may not converge when the stability conditions are not considered. Taking small time or distance steps (Δt or Δx) regarding the aforementioned conditions may make those methods unfeasible. Implicit numerical methods, although unconditionally stable, have their own problems. For example, by choosing a specific time step, the user may get an illogical answer. However, the inherent errors of numerical approximation and the existence of numerical diffusion, besides bringing no solution state for some situations, may introduce some limitations for solving some problems. Analytical methods, as a tool for checking correctness and precision of numerical methods, always give precise answers. However, many situations are not easy or in some cases impossible. Solving the kinematic wave equation for the rising limb of a hydrograph over a converging plane has been such a case up to now. The method presented in this paper is a semi analytical solution for the 2D converging flow and is based on two major concepts: the Agiralioglu method for simplifying the governing equations, and a modified Time Area method for finding the potential discharge of a converging plane. In addition, this new method uses a geomorphologic property of the plane and introduces a reduction factor to consider the convergence effect of the flow lines in such a domain. The proposed approach is able to solve the kinematic wave equation on a converging plane and derives the rising limb, peak discharge, and time to peak of a hydrograph due to an effective rainfall with constant rate. This research is an attempt to fill a gap that was supposed to be unsolvable for more than 40 years.

2. Materials and methods

Governing Equations

One of the most attractive features of the kinematic wave equation is its simplicity and sufficiency for the overland flow routing [3], but for the converging plane, this attraction could be offset by the complexity of the governing equations. As a common procedure in flow analysis, the equations are defined in the Cartesian coordinate system that for the converging flow, by introducing additional terms, makes them so complicated. The converging flow equations in such a coordinate system have two dimensions, but defining the geometric location of the points in the polar coordinate system can reduce it to one. According to Woolhiser [6], using the 1D equations of the kinematic wave in the radial direction is enough for the simulation of the overland flow over the converging planes.

The continuity and momentum (resistance) equations in the radial form, regarding the primary Veal equations, are as follows [3]:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} = i(r,t) + \frac{uh}{R-r} \tag{1}$$

$$q = uh = \alpha(x,t)h^{\beta} \tag{2}$$

Where h = flow depth; u = average velocity; i = input lateral discharge, which can vary in time and space (in simulation of rainfall-runoff process i is rainfall intensity); q = discharge of unit width; R = Radius of converging plane; β= power and α= friction parameter are used in the resistance equation and can vary in time and space. For the Manning equation, β and α are given as follows:

$$\alpha = \frac{1}{n}S_f^{1/2}, \quad \beta = \frac{5}{3} \tag{3}$$

In which n is the roughness coefficient and S_f is the friction slope for the Manning equation.

By the substitution of Equation 1 into Equation 2 and regarding u as the average velocity, one can write:

$$\frac{\partial h}{\partial t} + \beta\alpha(r,t)h^{\beta-1}\frac{\partial h}{\partial r} = i(r,t) + \frac{\alpha(r,t)h^{\beta}}{R-r} - h^{\beta}\frac{\partial \alpha(r,t)}{\partial r} \tag{4}$$

If α is considered constant in space and time, Equation 4 is simplified as follows:

$$\frac{\partial h}{\partial t}+\beta\alpha\, h^{\beta-1}\frac{\partial h}{\partial r}+h\frac{\partial u}{\partial r}=i(r,t)+\frac{\alpha\, h^{\beta}}{R-r} \tag{5}$$

The Boundary conditions for Equation 5 are as follows:

$$\begin{aligned} &h(r,0)=0 \qquad 0\le r\le R\,(1-c)\\ &h(0,t)=0 \qquad 0\le t\le T \end{aligned} \tag{6}$$

Where c=convergence coefficient (Fig. 1), and T=rainfall duration. It is obvious that for $t <= T$; $i\,(x,\,t) > 0$ and for $t > T$; i(x,t) = 0.

3. Relating converging flow to parallel flow

As it can be seen in the subsequent section, the new method uses a modified version of the Time Area method, which needs to compute the kinematic wave front in each time step. To overcome the complexity of the governing equations in the converging flow and to find an analytical solution for which, it was decided to use the parallel flow as the solution domain and then transfer the results to the converging flow. In this respect, the Agiralioglu method was used. Agiralioglu [15] developed a method for determining the time to equilibrium of the converging plane. The basis of this method is finding the ratio of the time to equilibrium (t_e) of a converging plane to that of a rectangular one. Therefore, upon calculating the time of equilibrium of the rectangular plane, one can find the required t_e of the converging plane. The basic condition for applying this method is the length equality of the two planes. Figure 2 demonstrates the geometry used by Agiralioglu. In this Figure, L is the length of planes, R is the radius of the flow field, and c is the convergence coefficient. It is seen in Figure 2 when c comes to unity, a converging plane changes to a rectangular plane.

The kinematic wave equations, which were used by Agiralioglu, are as follows:

$$\frac{\partial h}{\partial t}+\alpha\beta\, h^{\beta-1}\frac{\partial h}{\partial r}=i \tag{7}$$

$$\frac{\partial h}{\partial t}+\alpha\,\beta h^{\beta-1}\frac{\partial h}{\partial r}=i+\frac{\alpha\, h^{\beta}}{R-r} \tag{8}$$

Equations 7 and 8 are for the rectangular and converging plane respectively. For comparison of the two planes, Equation 8 must be dimensionless. Using Equation 7, one can obtain the depth of flow in the equilibrium state (H_0) and the time to equilibrium (T_0) at the end of the rectangular plane as follows:

$$H_0=\left(\frac{L.i}{\alpha}\right)^{\frac{1}{\beta}} \tag{9}$$

$$T_0=\left(\frac{L}{\alpha i^{\beta-1}}\right)^{\frac{1}{\beta}} \tag{10}$$

Now by introducing the following parameters, one can make Equation 8 dimensionless.

$$h_*=\frac{h}{H_0}, t_*=\frac{t}{T_0}, r_*=\frac{r}{L}, R_*=\frac{L}{R}, i_*=\frac{i}{i_0} \tag{11}$$

$$\frac{\partial h_*}{\partial t_*}+\alpha\,\beta h_*^{\beta-1}\frac{\partial h_*}{\partial r_*}=i_*+\frac{\alpha\, h_*^{\beta}}{R_*-r_*} \tag{12}$$

In Equation 11, L is the length of flow (Fig. 2) and i_0 is the maximum intensity of rainfall. The boundary conditions of Equation 12 are as follows:

$$\begin{cases} h_*=0, r_*=0\\ h_*=0, t_*=0 \end{cases} \tag{13}$$

$$\begin{cases} i_*>0, 0<t_*<T_*\\ i_*=0, t_*>T_* \end{cases} \tag{14}$$

Where $T_*=T/T_0$ and is the dimensionless time of rainfall duration. Equation 12 can be changed to a set of ordinary differential equations as follows:

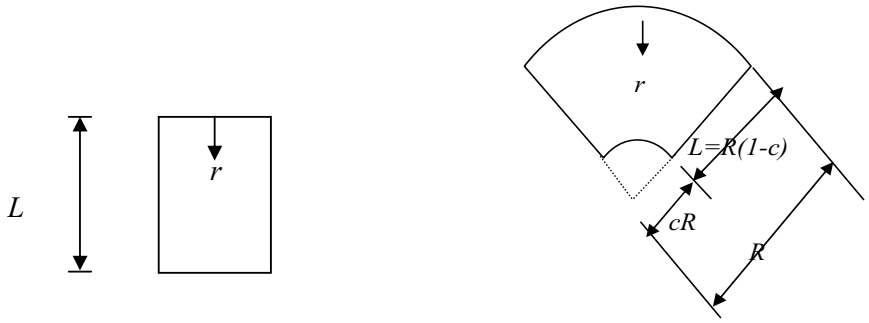
$$\frac{dt_*}{dr_*}=\frac{1}{\beta h_*^{\beta-1}} \tag{15}$$

$$\frac{dh_*}{dr_*}=\frac{i_*}{\beta h_*^{\beta-1}}+\frac{h_*}{\beta\left(\frac{1}{1-c}-r_*\right)} \tag{16}$$

Integration of Equation 16 yields:

$$h_*=\left[\frac{1}{\frac{1}{1-c}-r_*}\int_0^{r_*} i_*\left(\frac{1}{1-c}-x\right)dx\right]^{\frac{1}{\beta}} \tag{17}$$

Equation 17, which is derived by Agiralioglu [15], can be calculated easily. If i is constant, one can write i_* = 1, hence



(a) (b)

Fig. 2. (a) Rectangular plane (b) Converging plane

after some algebraic operations it gives:

$$h_* = \left(\frac{r_*[2 - r_*(1-c)]}{2[1 - r_*(1-c)]}\right)^{\frac{1}{\beta}} \quad (18)$$

Regarding that in the outlet $r_*=r/L=1$ (Fig. 3), one can find h_* in equilibrium state from Equation 18 as follows:

$$(h_*)_e = \left(\frac{1+c}{2c}\right)^{\frac{1}{\beta}} \quad (19)$$

On the basis of h_* from Equation 18 one can find t_* from Equation 15 as follows:

$$t_* = \frac{1}{\beta}\int_0^{r_*} h_*^{1-\beta} dr_* \quad (20)$$

For $r_*=1$, t_* converges to $t_{e*}=t_e/T_0$, in which t_e and T_0 are the time to equilibrium of the converging and rectangular planes respectively. Substitution of h_* from Equation 18 in Equation 20 yields:

$$t_{e_*} = \left(\frac{1}{\beta}\right) 2^{\frac{\beta-1}{\beta}} \int_0^1 \left\{\frac{1 - r_*(1-c)}{r_*[2 - r_*(1-c)]}\right\}^{\frac{\beta-1}{\beta}} dr_* \quad (21)$$

Agiralioglu used a modified form of equation 21 and drew t_{e*} vs. convergence coefficient c (Fig. 4). By β=3/2, 5/3 and 3 Equation 21 yields t_{e*} for the Chezy, Manning, and Darcy-Weisbach equations (as the resistance equation) respectively (Fig. 4). Given c, one can find t_{e*} from Equation 21 or Figure 4, then by calculating T_0 for the equivalent rectangular plane (Equation 10), t_e for the converging plane can be calculated.

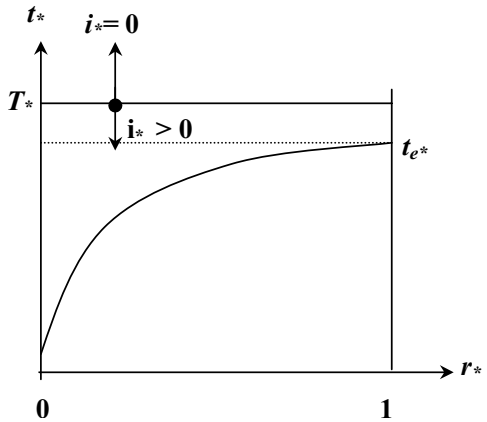


Fig. 3. Non dimensional solution domain for converging flow

4. The Time Area method

The Time-Area model as embedded in the Clark unit hydrograph technique is semi-distributed in nature. The time-area method was originally developed in the recognition of the basic importance of storm temporal pattern effects on runoff [21]. The time-area method was first developed by Clark in 1945 which accounts for spatial pattern of watershed features as well as the temporal pattern of the storm. The most critical step in application of time-area model is the determination of travel time throughout the watershed (delineation of isochrones).

The general equation of Time –Area method which gives a direct runoff hydrograph due to an effective rainfall is as follows:

$$Q_j = \sum_{k=1}^{j} i_k A_{j-k+1} \quad (22)$$

where j = time step, Q = discharge, i = effective rainfall intensity and A = area between two consecutive isochrones (for further study see: [22], [23], and [24]).GIS (Geographic Information system, which its capability has been known in different areas of study [25 and 26], could be a useful tool to derive isochrones effectively.

5. The new method

Some basic assumptions were made to develop the new method for solving the kinematic wave equation on converging planes to find the rising limb of a direct runoff hydrograph. These assumptions are as follows:

a) With q_0 as the unit discharge at the outlet of the rectangular plane ($q_0=\alpha H_0^{\beta}$), and q_c as the unit discharge at the outlet of the converging plane ($q_0=\alpha h^{\beta}$), a relative unit discharge can be defined as follows:

$$q_* = \frac{q_c}{q_0} = \left(\frac{h}{H_0}\right)^{\beta} = h_*^{\beta} \quad (23)$$

$$q_c = q_0 . q_* \quad (24)$$

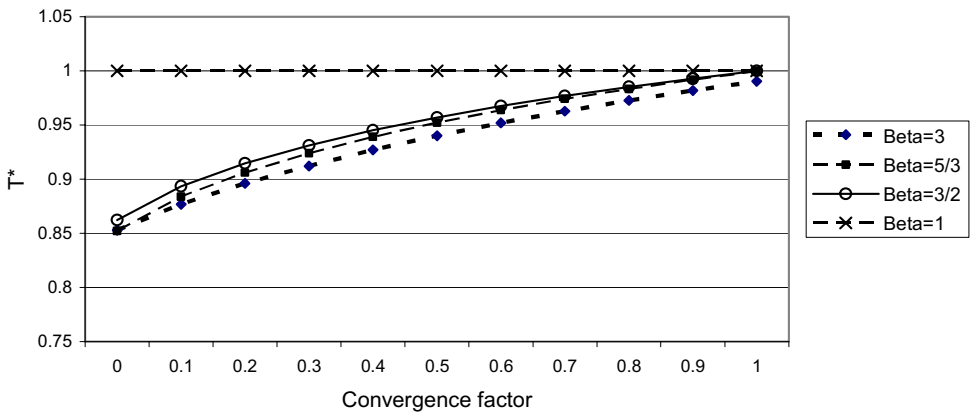


Fig. 4. Time to equilibrium of a converging plane as a proportion of a rectangular plane time to equilibrium (after Agiralioglu, [16])

$$Q = q_c.w \tag{25}$$

$$w = \theta(R - r) \tag{26}$$

In Equation 25, Q = total outflow of converging plane at any desired point, and w= width of flow at the point where Q is calculated.

b) If the rainfall duration is equal to or greater than the time to equilibrium of the plane, discharge at the outlet ($r_* = 1$) reaches equilibrium. In this situation the time to equilibrium can be calculated by using Equation 21 with the specified condition: $r_* = 1$. The result is as follows:

$$t_{e_*} = \frac{1}{\beta}\left(\frac{2c}{1+c}\right)^{\frac{\beta-1}{\beta}} \tag{27}$$

c) The wave moves from the edge of the plane and after a time step traverses the distance r_1. All of the points on a line that lie at the distance r_1 from the edge of the plane with a width equal to w reach equilibrium and are in a steady state (Fig. 5-a and 5-b). In the second step, the wave front reaches to a distance, say r_2 (Fig. 5-a, and 5-c), in a similar manner.

d) Each step in the wave propagation is considered as an independent step. The wave front at the end of each time step is considered as the end of a converging plane. Then one can find the discharge in each step using the above-mentioned equations. The boundary conditions (Equation 13) remain unchanged in each step.

Regarding the above principals, the kinematic wave equation is solved for overland flow routing on a converging plane. Accordingly, obtaining the rising limb of a hydrograph due to an effective rainfall needs to go through the following steps:

1- Set the radius equal to r.

2- The convergence coefficient for radius r (c_r) is calculated by Equation 26 as follows:

$$C_r = 1 - \frac{r}{R} \tag{28}$$

3- The values of H_0, T_0 and Q_0 are calculated for a rectangular plane with a length of r.

4- By Equation 21 or using Figure 4, t_{e*} and then by Equation 11, $t_e(r)$ is calculated.

5- The value of $(h_*)_e$ from Equation 19 and hence h_e from Equation 11 is obtained.

6- The value of Q_r for $t_e(r)$ (obtained in step 4) is calculated by using Equations 23, 24, and 25.

7- Add Δr to r and repeat steps 2 through 7, till wave front reaches the end of the plane.

Apparently, the rising limb of the hydrograph can be computed after step 7 and acquiring the required data (Q_r vs. $t_e(r)$). But, there is an inherent mistake in the above mentioned method that needs the modification of the calculated discharge. Because of setting the basis of the whole method on driving the kinematic wave on a rectangular plane and then modifying the obtained result in favor of the converging plane, the effects of convergence of the flow lines has been ignored. Doing this makes the final results overestimated and thoroughly unacceptable (Figure 7).

The proposed method applies the principals of the Time Area method. Based on the basics of the Time Area method, it can be said that the proposed method tries to simulate the wave traversed length by the application of the kinematic wave principals. However, there are two fundamental differences between the proposed method and the Time Area method. Firstly, the first isochrone in the Time Area method is the closest one to the outlet of the basin, while in the proposed method, it is supposed that the first isochrone is the farthest one to the outlet of the basin. Saghafian and Shokoohi [22] and Shokoohi [23, 24] showed that it is necessary to modify the sequence of the isochrones to achieve the exact solution of the kinematic wave equations for a rectangular plane. Secondly, it is supposed that the wave front works as an outlet in each step. This supposition is important to find a way to consider the flow lines' convergence effect. In respect to this fact that the width of the flow is reducing by advancing the isochrone toward the basin outlet, the idea of introducing a reduction factor to the proposed method was considered. Recalling that the outflow of the kinematic wave is nonlinearly dependent to the upstream drainage area, this factor is seen as the most important geomorphologic property of a basin for this purpose.

Suppose that the wave after its first step of translation travels a distance r from the upstream edge of the plane. After this translation, a new sector with radius r is created. As it can be seen in Figure 6, the upstream area of this location is equal to A_1, while for the actual outlet the drainage area is equal to A'_1 . The reduction factor was set to be the proportion of these two areas. The area of a sector with the central angle of θ and the radius R is obtained by a simple formula: $A = \theta R^2/2$, then one

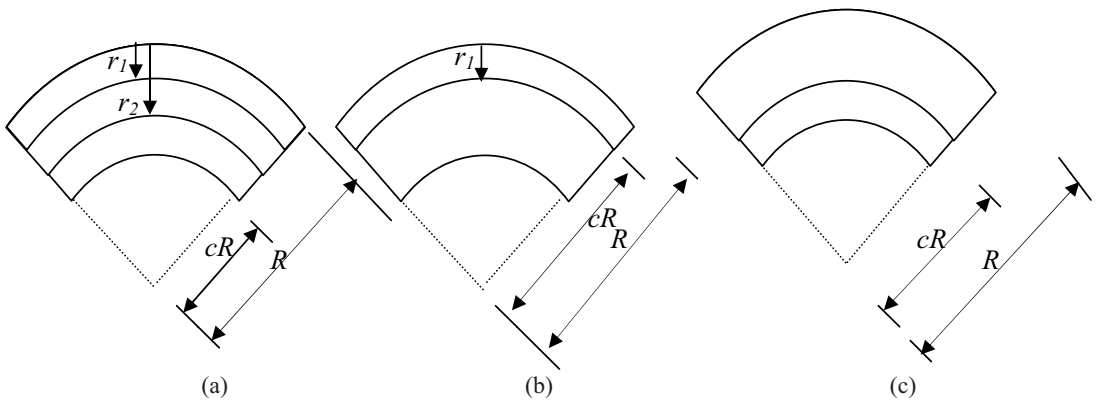


Fig. 5. Parameters of converging plane when wave translates from upstream edge to outlet

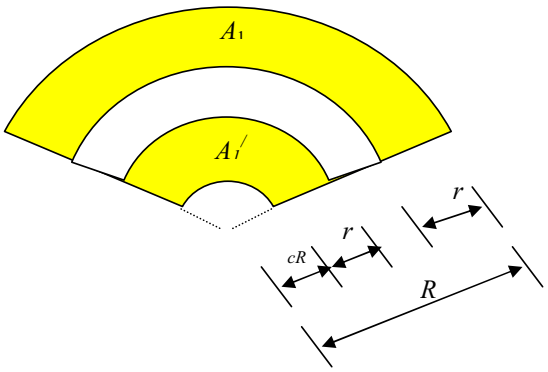


Fig. 6. Illustration of areas used to derive correction factor

can write the following:

$$\begin{cases} A_1 = \dfrac{\theta}{2}[R^2 - (R-r)^2] \\ A_1^{'} = \dfrac{\theta}{2}[(r_0 + r)^2 - r_0^2] = \dfrac{\theta}{2}[(cR + r)^2 - c^2 R^2] \end{cases} \quad (29)$$

Hence, the geomorphologic reduction factor can be introduced as:

$$f = \frac{A_1}{A_1^{'}} = \frac{2R - r}{2cR + r} \quad (30)$$

By this tuning function, the proposed method will have an additional step (between steps 6 and 7) as follows:

- The calculated Q in step 6 is multiplied by f.

6. Results

The most reliable approach to verify the proposed method is to compare the results of its application on a theoretical converging plane with those of a well recognized and acceptable method. For the case study, a basic problem that has been solved by the Woolhiser numerical method for the rising limb of a hydrograph was chosen [3]. The assumed plane is a converging plane with S_0=0.05, c=0.1, R=100 ft, θ=30^o, and a Chezy resistance coefficient of 10 ft$^{1/2}$/sec. A net rainfall with the intensity of i=10 in/hr and the duration of 90 sec was applied throughout the plane. The results of applying the Woolhiser method, according to Singh [3], have been shown in Figure 7. The problem was then solved with and without the geomorphologic correction factor f using the new method (Equation 30). The new method result after the application of the geomorphologic reduction factor completely coincides with the Woolhiser solution.

7. Summary and conclusion

Because of the similarity of the upstream subbasins in large watersheds to converging planes, finding a robust and simple solution for the kinematic wave is a concern of 2D overland flow modeling since the 1960s. For the sake of simplicity, robustness, and speed, almost all of the hydrological models use the rectangular planes for the simulation of the kinematic overland flow, even though it is widely recognized that this assumption for upstream subbasins could increase inaccuracy and the burden of the calibration process of the models. In this regard, the model calibration may change some of the physical parameters that are important in flow routing. In such a condition, the calibrated parameters lose their value to reflect the actual condition of the basin and the probable usage for reliable long-term prediction.

The convergence of the flow lines complicates the continuity equation, so there is no analytical solution for the rising limb of the hydrograph in the overland flow routing. In this paper, a new analytical/semi analytical geomorphologic based method for solving the kinematic wave equations on converging planes was presented. This method is analytical because of using a basic procedure that is founded on finding the kinematic wave propagation velocity in order to compute the discharge at the outlet. This method is semi analytical because of the successful application of a correction factor onto the results of the analytical portion of the proposed method. This method is a geomorphologic one because it relies on the most important feature of watersheds: the drainage area.

The main idea in this research was to modify the intricate governing equations of the converging flow so that they could be solved analytically. In this regard, solving the kinematic equations for parallel flow and then modifying the results in favor of converging flow was considered. In this respect, the Agiralioglu method was used. This method, which is based on

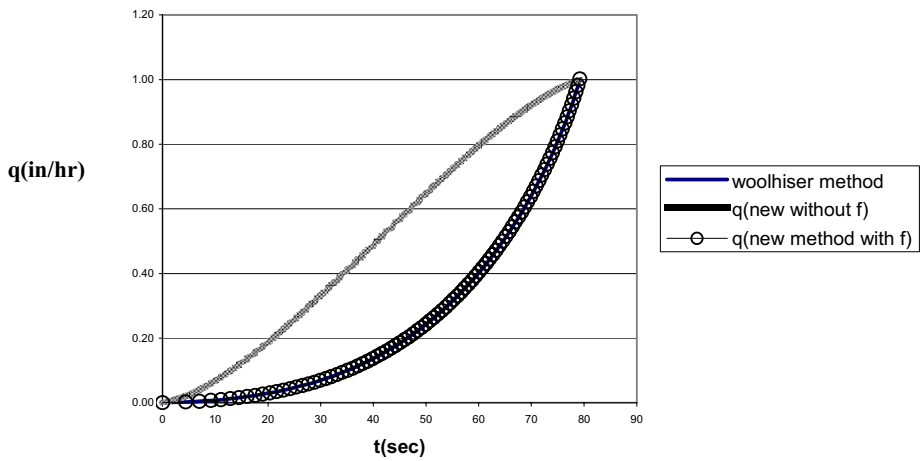


Fig. 7. Comparison of New method and Woolhiser method

the original works of Wollhiser, uses the dimensionless form of Veal's equations and determines the ratio of the converging plane time to equilibrium to that of a rectangular plane. In the new presented method, the location of the wave front, the corresponding equilibrium depth (h_e), and equilibrium discharge (Q_e) at each time step are calculated. In this respect, the performance of the new method is the same as the Time Area method, while there are two distinct differences between them. In the proposed method, the first isochrone is the farthest one to the basin outlet, and the isochrone in each time step is regarded as an outlet as if the location of the outlet of the basin changes with time. For the translation of the calculated discharge in the wave front toward the actual outlet, a reduction (correction) factor, based on the varying degree of convergence in each time step, was proposed. The multiplication of this factor by the calculated Q_e yields the discharge at the basin's outlet. This procedure must be iterated until the wave front reaches the basin's outlet.

The results of applying the new method and the Woolhiser numerical technique to a representative problem were compared. The outcomes indicated that the two methods gave similar results, but the proposed method in this paper has an analytical nature without any kind of limitations.

Some further studies are needed for coding and incorporating the proposed method in a comprehensive hydrological model. It helps to evaluate the new method in the actual basins and appraise its effects on enhancing the calibration processes in order to reduce the burden of calibration in large watersheds. In doing this, the physical meaning of the involved parameters can be appropriately interpreted.

References

[1] Wooding, R.A.: 1965a, A hydraulic Model for the Catchment-stream Problem, I. Kinematic Wave Theory., Journal of Hydrology Vol. 3,pp. 254-267.

[2] Wooding, R.A.: 1965b, A hydraulic Model for the Catchment-stream Problem, II. - Numerical Solution. J. Hydrology, Vol. 3, pp. 268-282.

[3] Singh, V.P.: 1996, Kinematic wave modeling in water resources engineering,
John Wiley & Sons, Inc, New York.

[4] Wooding, R.A.: 1966, A hydraulic Model for the Catchment-stream Problem, III. Comparison with Runoff Observations, J. Hydrology, Vol. 4,pp. 21-37.

[5] Ponce, V. M.: 1989, Engineering Hydrology., Prentice Hall Inc.

[6] Woolhiser, D.A.: 1969, Overland flow on converging surface., Transaction of the ASAE, 12(4),pp. 460-462.

[7] Veal, D.G.: 1966, A computer solution of converging, subcritical overland flow., Unpublished M.S. Thesis, Cornell University .

[8] Singh, V.P.: 1975b, Hybrid formulation of kinematic wave models of watershed runoff., J. Hydrology, 27,pp. 33 –35.

[9] Singh, V.P. and Woolhiser, D.A.: 1976, A nonlinear kinematic wave model for watershed surface runoff., J. Hydrology, 31,pp. 221 – 243.

[10] Singh, V.P. 1975a. Estimation and optimization of Kinematic wave parameters, Water Resources Bulletin, 11 (6), pp. 1091 – 1102.

[11] Singh, V.P.: 1976, A distributed converging overland flow: 3. application to natural watersheds., Water Resources Res., 12(5),pp. 902 – 908.

[12] Sherman, B. and Singh, V. P.: 1976 a, A distributed converging overland flow model: 1. Mathematical solution, Water Resources Res., 12(5), pp. 889 – 896.

[13] Sherman, B. and Singh, V. P.: 1976 b, A distributed converging overland flow model: 2. Effect of infiltration., Water Resources Res., 12 (5), pp. 897 – 901.

[14] Ellis, F.W., Ramsey, F.V and Hornberger, G.M.: 1980, Converging flow model applied to urban Catchment., J. Hydraulics Division, ASCE, 106(9),pp. 1457 – 1470.

[15] Agiralioglu, N.: 1981, Water routing on diverging – converging watersheds, J. Hydraulic Division, ASCE, 107)8(, pp. 1003 – 1017.

[16] Agiralioglu, N.: 1984, Effect of Catchment geometry on time of concentration, Proc. Of Urban Storm Drainage, Gutenberg, Sweden, 1,pp. 177 – 184.

[17] Agiralioglu, N.: 1988, Estimation of the time of concentration for diverging surfaces, J. of Hydrological Sc., 33(2): pp.173 – 179.

[18] Campbell, S.Y. and Parlange, J.Y.: 1984, Overland flow on converging and diverging surfaces: assessment of numerical schemes, J. of Hydrology. 70, pp. 265 – 275.

[19] Saghafian, B. and Julien, P.Y.: 1991, CASC2D users manual. , Civil Engineering Rep. CER 90-91 PYJ-BS-12 Colorado State University , FT Collins , Co

[20] Mohammad T. Dastorani and Nigel G. Wright: 2004, A Hydrodynamic/Neural Network approach for enhanced river, International journal of civil engineering , 2(3), pp. 141-148

[21] Singh, V.P., (1992), Elementary Hydrology, Chapter 16, Prentice Hall, Englewood Cliffs, New Jersey, USA.

[22] Saghafian, B. and shokoohi, A.R.: 2006, A corrected time-area technique for one dimensional flow, International journal of civil engineering , 4(1), pp. 34-41

[23] Shokoohi, A.R.: 2008, A New Approach for Isochrone Mapping in one Dimensional Flow for Using in Time – Area method. Journal of applied science, 8(3), pp. 1117-1129

[24] Shokoohi, A.R. and B. Saghafian, 2007, Comparison of Isochrones delineation Methods to be used in Time-Area Routing Technique. J. of Iran-Water Resources Research, 2(3): pp. 30-42

[25] M. H. Sebt, E. Parvaresh Karan and M. R. Delavar: 2008, Potential Application of GIS to Layout of Construction Temporary Facilities, International journal of civil engineering , 6(4), pp. 235-245

[26] M. H. Vahidnia, A. A. Alesheikh, A. Alimohammadi and F. Hosseinali: 2009, Landslide Hazard Zonation Using Quantitative Methods in GIS, International journal of civil engineering , 7(3), pp. 176-189

Notation

The following symbols are used in this paper:

h = flow depth
u = average velocity
i = input lateral discharge or rainfall intensity
q = discharge of unit width
R = length of converging plane
β = power used in resistance equation
α = friction parameter used in resistance equation
c = convergence coefficient
T = rainfall duration
H_0 = flow depth in steady state on rectangular plane
T_0 = Time to equilibrium on rectangular plane
L = flow length
i_0 = maximum rainfall intensity
$h_*=h/H_0$
$t_*=t/T_0$
$r_*=r/L$

$R_* = R/L$
$i_* = i/i_0$
$T_* = T/T_0$
q_0 = unit discharge at the outlet of rectangular plane
q_c = unit discharge at the outlet of converging plane
Q = converging plane at any desired point
w = width of flow at the point that Q is calculated.
Q_0 = total outflow of rectangular plane.
A_1 = upstream area of any desired location
A'_1 = actual outlet drainage area
θ = central angle of sector
f = Correction (reduction) factor
S_0 = slope of plane