A Corrected Time-Area Technique for One-dimensional Flow

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Abstract: *Time–Area method is one of the most widely applied techniques of watershed routing, and can be potentially used as a distributed model. In this paper, a fundamental flaw in the arrangement of subareas in the original time-area histogram is identified for one-dimensional flow. This is conducted on the basis of comparing time-area hydrograph with that of the kinematic wave theorem. Accordingly, a revised time-area algorithm is developed as a substitute for the original time-area. It is proved that in the revised approach, subareas must be reversely arranged. It is also shown that the revised time-area hydrograph is in perfect agreement with the hydrograph derived by the kinematics wave theory.*

Keywords: *Time–Area, Kinematic Wave, One-dimensional flow, Isochrone.*

Introduction

Time-area (TA) rainfall-runoff transformation is widely known as a hydrologic watershed routing technique to derive the discharge hydrograph excited by an effective rainfall hyetograph. In this technique, by ignoring the storage effects, the watershed is divided into a number of subareas via isochrones; i.e. the contours of equal travel time to the outlet. The histogram of contributing watershed subareas is known as time-area histogram (TAH) and constitutes the basis for the excess rainfall-runoff transformation. To construct the TAH, the watershed time to equilibrium (usually used interchangeably with the time of concentration in the literature) must be divided into a number of equal time intervals. This time interval is the time difference between adjacent isochrones. After plotting the TAH, one can determine the runoff hydrograph by assuming that the increment in the discharge from time t over a certain time interval (Dt) equals the product of effective rainfall intensity over the interval

from t to (t+Dt) and the subarea corresponding to the same time interval.

There is noticeable tendency towards application of TA method in recent years, in the light of advances in the computer sciences and GIS software. Maidment (1993) introduced a new method which uses the time-area method and GIS to derive a unit hydrograph with the capability of involving spatial distribution of watershed parameters. Ajward and Muzik (2000) presented a time-area model on the basis of Maidment's idea to derive spatial unit hydrograph contrasting geomorphoclimatic instantaneous unit hydrograph (GcIUH) (Rodriguez-Iturbe et al., 1982a and 1982b). They named their procedure "deterministic direct hydraulic simulation". Melesse et al. (2003) have reported development of a routing model for generation of direct runoff hydrograph. The main feature of this model was the concept of travel time. According to available reports, between 40 to 60 percent of the U.S. Army Corp of Engineers projects are handled by the time-area method and its variants (Kull

and Feldman, 1998).

The TA method, as a widely-used runoff routing technique, benefits from the following advantages:

1. Temporal distribution of excess rainfall may be accounted for in the runoff discharge calculations.

2. The shape and drainage pattern of the watershed may be reflected in breaking the watershed by the isochrones of equal travel times.

3. Although many consider TA a lumped model, it actually has the potential to perform as a distributed model by incorporating non-uniform effective rainfall and spatially variable watershed characteristics. Accordingly, geographic information systems (GIS) may play a major role in facilitating spatial data preparation and analysis (e.g., Maidment, 1993, Kull and Feldman, 1998).

The main assumptions and limitations of the original time-area (abbreviated by OTA throughout this paper) and most currently-in-practice TA methods may be outlined as follows:

1. The method accounts for pure translation and not for storage, thus lacking peak attenuation and resulting in higher peak discharges particularly for large watersheds. The remedy for this limitation is to incorporate a linear reservoir as originally proposed by Clark (1945).

2. Except for a few studies (e.g., Saghafian and Julien, 1995), the algorithms to determine isochrones are often conceptual and not based on flow hydraulics, thus leading to substantial error in the runoff discharge calculations. This point will be highlighted in the subsequent sections of this paper.

3. Due to existence of a flaw in the manner TAH is arranged, the resultant runoff hydrograph is incorrect and often resembles a convex rising limb in an overall bell-shaped curve, particularly for long duration uniform excess rainfall.

The second and the third limitations are to be addressed by the revised techniques presented in this paper, while inclusion of storage effects is possible through conceptual reservoirs. This would lead to a revised TA method, which may be considered a more accurate yet robust hydraulic-based method for rainfall-runoff transformations.

Methodology

In this section, the rising limb of the hydrograph computed based on the OTA method is compared with that of kinematic wave for an impervious rectangular plane (parallel one-dimensional flow) to highlight the need to revise the OTA methodology. Consider a rectangular plane of constant width W, length L, and slope S_0 subject to a uniform long duration ($t_r > t_e$) excess rainfall intensity i (Figure 1).t_r and t_e represent rainfall duration and time to equilibrium respectively. The relation between unit-width discharge q and flow depth h in the context of a general resistance equation is represented by $q = \alpha h^\beta$. According to the kinematic wave theory (Woolhiser, 1975) the relationship between the time t_0 and the upstream length of the plane (x_0) that has reached equilibrium (steady-state) is given by:

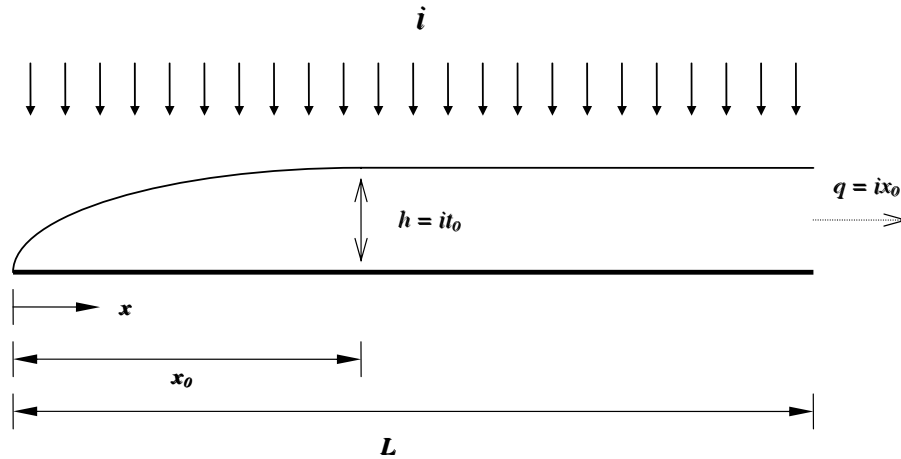$$t_0 = \frac{x_0^{(\frac{1}{\beta})}}{\alpha^{(\frac{1}{\beta})} i^{(1-\frac{1}{\beta})}} \qquad (1)$$

Figure 1- Water surface profile over a rectangular plane under rainfall intensity i at t=t0

Note that for $x_0 \leq x \leq L$, the surface flow depth profile is uniform such that:

$$h(x,t_0) = it_0 \tag{2}$$

and:

$$q(x,t_0) = \alpha(it_0)^\beta \tag{3}$$

However, for the distance upstream of x_0, the flow is non-uniform but steady.

Based on Equation (1), the equilibrium time of the whole plane may be determined by setting $x_0 = L$:

$$t_e = \frac{L^{(\frac{1}{\beta})}}{\alpha^{(\frac{1}{\beta})} i^{(1-\frac{1}{\beta})}} \tag{4}$$

If one wishes to divide the plane into N subareas of equal travel time bounded by isochrones of equal travel times, the x-coordinate for the j-th isochrone (numbered from *upstream*) may be determined by substituting $t_0 = \dfrac{jt_e}{N}$ into Equation (1) and solving for $x_j = x_0$:

$$x_j = L\left(\frac{j}{N}\right)^\beta$$

where $1 \leq j \leq N$

$$\tag{5}$$

The area of the j-th subarea is determined by:

$$A_j = W.l_j = W(x_j - x_{j-1}) = WL\left[\left(\frac{j}{N}\right)^\beta - \left(\frac{j-1}{N}\right)^\beta\right]$$

where $2 \leq j \leq N$

$$\tag{6}$$

and $A_1 = Wl_1 = Wx_1$. The length l_j represents the distance between adjacent isochrones.

According to the time-area methodology, the discharge is expressed by:

$$q_m = \sum_{k=1}^{m} i_k l_{m-k+1} \tag{7}$$

where m is the time step number and q is the outlet discharge.

The nearest subarea to the outlet between *(N-1)-th* and *N-th* isochrones, corresponding to the travel times of t_e/N and $2t_e/N$, contributes to the outlet runoff first and subsequent subareas contribute in the order from downstream to upstream. Therefore the OTA discharge at time $t_k = \dfrac{kt_e}{N}$ is determined simply by $q(t_k) = i(L-x_{N-k})$ or by:

$$q_{OTA}(t_k) = i\sum_{j=N}^{N-k+1} l_j = iL\left[1 - \left(\frac{N-k}{N}\right)^\beta\right]$$

where $1 \leq k \leq N$

$$\tag{8}$$

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If, on the other hand, the aim is to computethe outflow discharge based on kinematic wave theory (Equation 3), one gets:

$$q_{kw}(t_k) = \alpha(it_k)^\beta = iL\left(\frac{k}{N}\right)^\beta \qquad (9)$$

The relative error (e) of q_{OTA} as compared with that of KW discharge may be determined by:

$$e(t_k) = \frac{q_{OTA}(t_k) - q_{kw}(t_k)}{q_{kw}(t_k)} = \left(\frac{N}{k}\right)^\beta - \left(\frac{N-k}{N}\right)^\beta - 1 \qquad (10)$$

As will be seen in the results, the OTA hydrograph shows substantial error. Now, the proposed revised time-area (referred hereafter as RTA) methodology relying on kinematic wave theory is outlined. The revised methodology identifies and corrects the flaw in the manner the time-area histogram is arranged in the original time-area technique. Let's hypothesize that, contrary to the OTA methodology, the RTA discharge hydrograph at time t_k is proportional to the summation of *upstream* subareas that have reached equilibrium at t_k:

$$q_{RTA}(t_k) = i\sum_{j=1}^{k} l_j = iL\left(\frac{k}{N}\right)^\beta \qquad (11)$$

which is also equal to ix_k, where x_k and t_k are related through Equation (1). That is to say, the q_{RTA} is the same as q_{kw} (Equation 9) and equal to the steady state discharge at location x_k. Note again that x_k is indeed the sum of the length of the subareas starting from the upstream (the most distant from the outlet) and not from the downstream as proposed in the OTA methodology. Thus, proper arrangement of subareas in the time-area histogram is by reversing the OTA time-area histogram in such a way that the most upstream subarea is placed first and the most

downstream subarea is positioned last in the *TAH*. This is not to say that the most upstream subarea makes the earliest runoff contributions, rather its steady state discharges is reflected at the outlet first.

Results

The relative location of isochrones are plotted in Figure 2 for $N=10$ and Manning equation ($\beta=5/3$). Figure 3 also shows the time-area histogram arranged based on the OTA methodology. The dimensionless runoff hydrograph from the OTA (Equation 8) and from the kinematic wave (Equation 9) are plotted in Figure 4 in which $q_e=iL$. Note the unreasonable convex (half-bell) shape of the OTA hydrograph.

The relative error (e) of q_{OTA} is also plotted in Figure 5. The relative error is higher for shorter duration rainfall while it drops to zero when rainfall duration is equal to equilibrium time ($t_r=t_e$). For a rainfall duration of say $t_r=t_e/2$, the OTA peak discharge is 2.18 times that of KW for $k=N/2$ and $\beta=5/3$ (Manning equation),i.e. a relative error of 118%.

RTA arrangement of subareas are displayed in Figure 6 which contrasts that in Figure 3. The dimensionless discharge hydrograph based on RTA coincides with that of kinematic wave as shown in Figure 4.

Conclusions

The dimensionless discharge hydrographs of the original time-area was compared to that derived via kinematic wave theory for a rectangular plane with one-dimensioned flow. The OTA method proved erroneous, particularly for shorter duration rainfalls. It was demonstrated that even if the isochrones were determined based on hydraulics of overland flow, the original time-area method
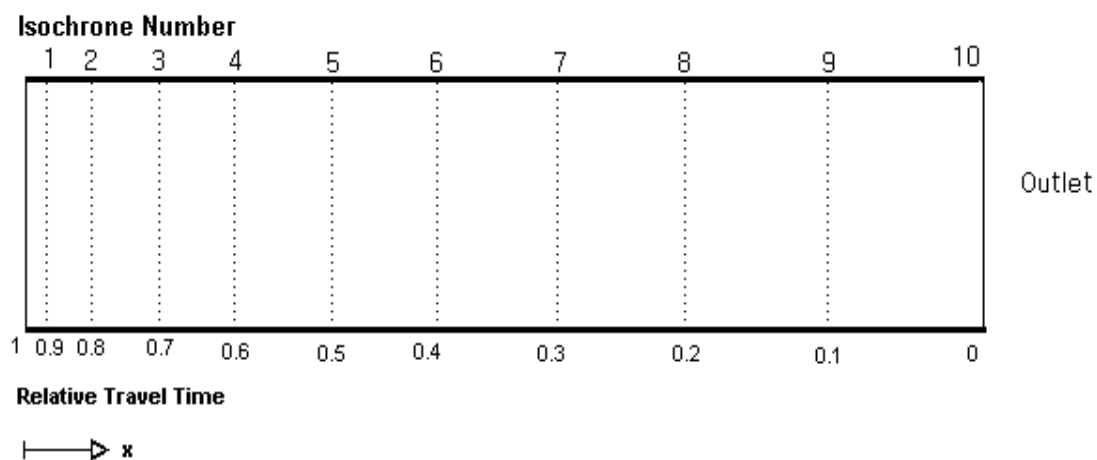
Figure 2- Location of isochrones (dotted lines) and corresponding relative travel time to the outlet for a rectangular plane
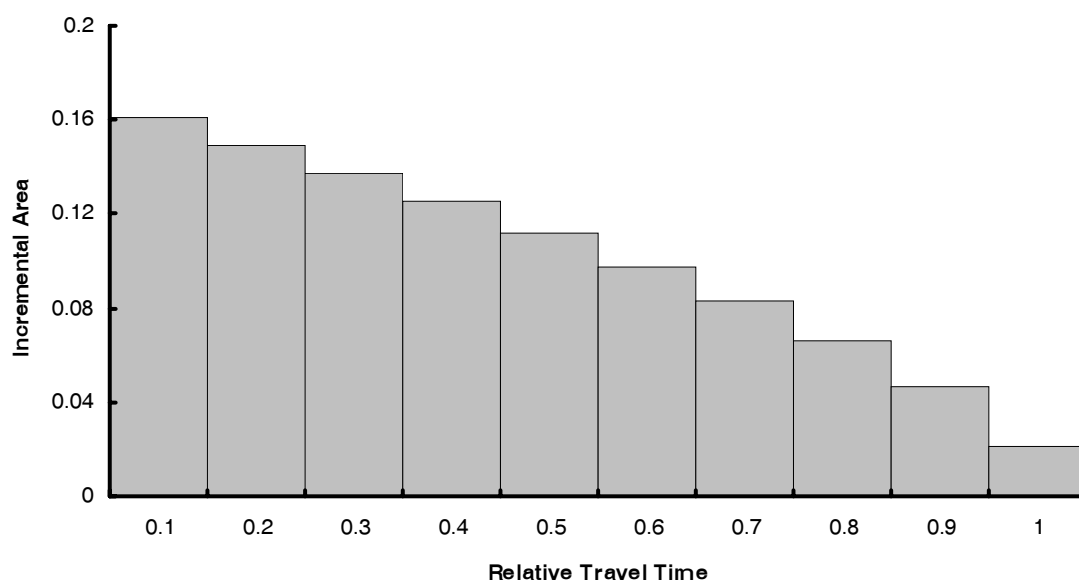

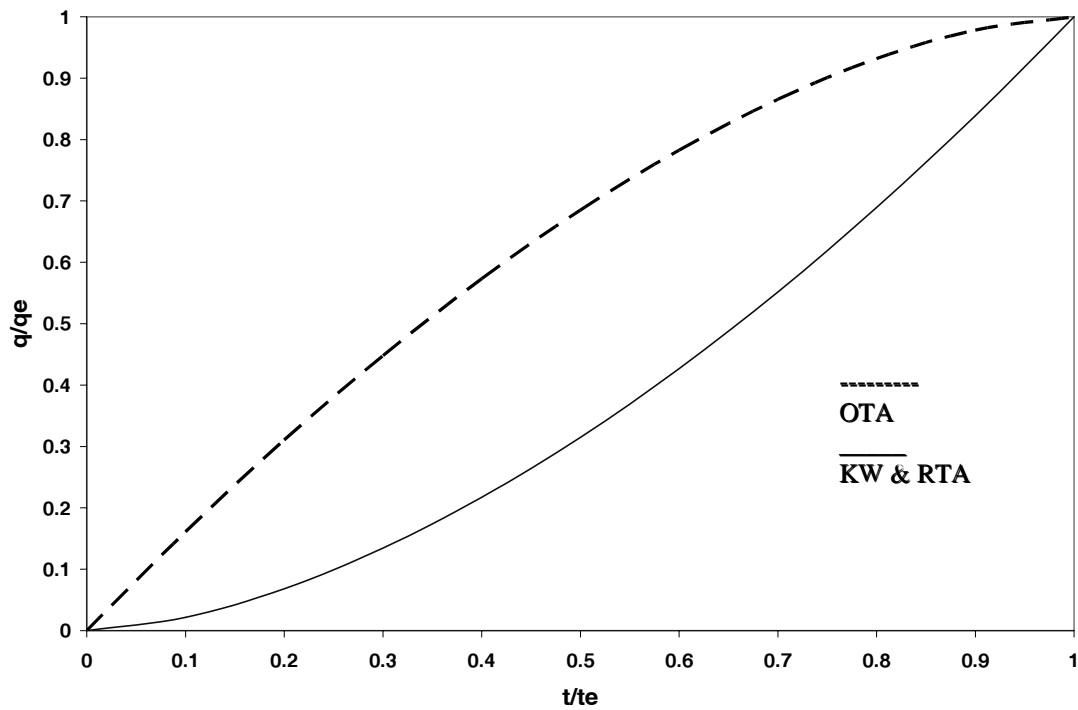**Figure 3.** Time-area histogram arranged based on the OTA method

Figure 4. Dimensionless rising limb discharge hydrograph computed by kinematic wave, RTA, and OTA methods
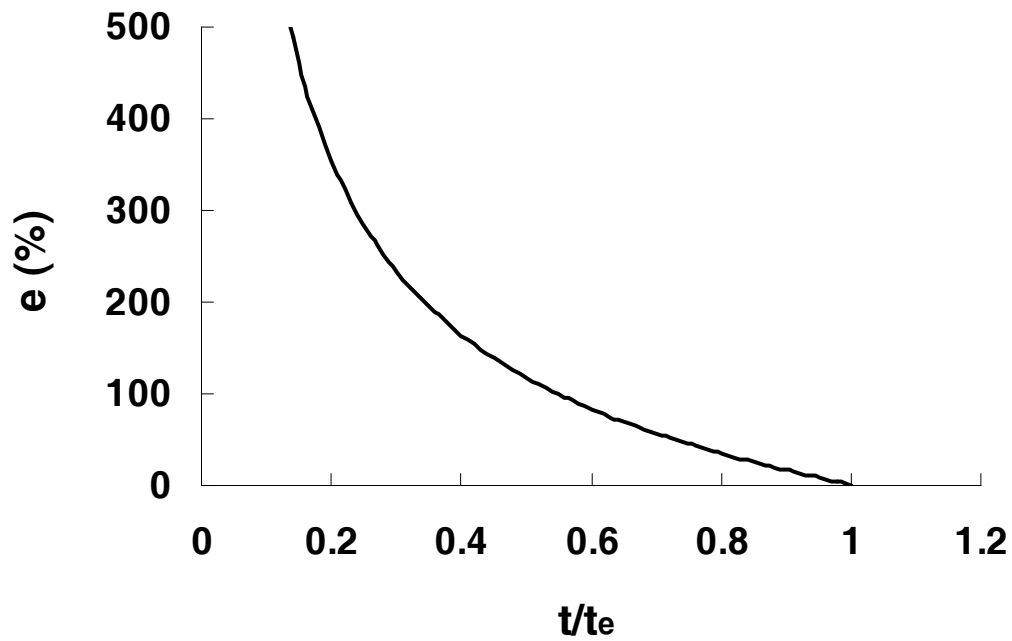
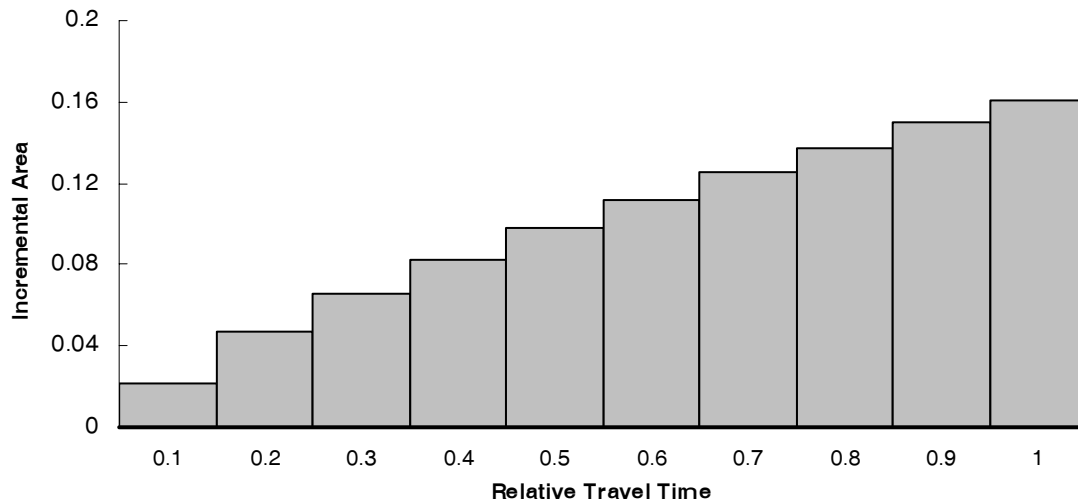

Figure 5. Relative error (e) of q_{OTA}

Figure 6. Revised time-area histogram arrangement for a rectangular plane

suffered from a flaw and required a major revision in the manner the time-area histogram was arranged. This revised time-area approach reverses the way that the sub areas are arranged in the time-area histogram. The RTA hydrograph was in perfect agreement with the hydrograph derived by the kinematics wave theory of one-dimensioned overland flow over a rectangular plane. We therefore propose to use kinematics wave to find the location of isochrones in general watershed geometries (Saghafian and Julien,1995) and apply the RTA arrangement for hydrograph calculations.

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