Comparison of Static and Dynamic Pushover Analysis in Assessment of the Target Displacement

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Abstract: *Pushover analysis is a simplified nonlinear analysis technique that can be used to estimate the dynamic demands imposed on a structure under earthquake excitations. One of the first steps taken in this approximate solution is to assess the maximum roof displacement, known as target displacement, using the base shear versus roof displacement diagram. That could be done by the so-called dynamic pushover analysis, i.e. a dynamic time history analysis of an equivalent single degree of freedom model of the original system, as well as other available approximate static methods. In this paper, a number of load patterns, including a new approach, are considered to construct the related pushover curves. In a so-called dynamic pushover analysis, the bi-linear and tri-linear approximations of these pushover curves were used to assess the target displacements by performing dynamic nonlinear time history analyses. The results obtained for five different special moment resisting steel frames, using five earthquake records were compared with those resulted from the time history analysis of the original system. It is shown that the dynamic pushover analysis approach, specially, with the tri-linear approximation of the pushover curves, proves to have a better accuracy in assessing the target displacements. On the other hand, when nonlinear static procedure seems adequate, no specific preference is observed in using more complicated static procedures (proposed by codes) compared to the simple first mode target displacement assessment.*

Keywords: seismic nonlinear analysis; pushover analysis; FEMA 356; load pattern; target displacement

1. INTRODUCTION

Obviously, some gradual changes are expected in the currently available seismic design methodology implemented in codes based on the assumption of linear elastic structural behavior [1-2]. Complex nonlinear time-history analyses of Multi-Degree-of-Freedom (MDOF) systems are not practical for everyday design use and therefore are not appropriate as a code requirement. The development of a rational methodology that is applicable to the seismic design of new structures as well as to the seismic evaluation and strengthening of existing building has become the main task of many researchers and engineers through the world [3-4]. These methods should take full advantages of presently available ground motion information and engineering knowledge.

In the majority of cases, nonlinear static analysis under monotonically increasing lateral loading (pushover analysis) is an important part of methodology. It represents a relatively simple solution for estimating the nonlinear structural performance. On the other hand, pushover analysis has been proposed, and evaluated in several research studies in somewhat different formats [5-9]. Pushover analysis can be performed using the well-known programs for static and dynamic nonlinear analysis such as DRAIN [10]. However, once the base shear versus roof displacement relationship obtained, the most important problem would be to estimate the target displacement of the structure at which the base-shear demand and corresponding story shears, inter story drifts and plastic hinges rotations are to be calculated. Several static and dynamic procedures are proposed in the literature to assess the target displacement including the first mode roof displacement approximation, the displacement pattern equivalent mode roof displacement and the full dynamic modeling of the structure [3-5]. In the work follows, after a brief review of the basic principles of the above methods, their performance will be compared using five special moment resisting steel frames with 2, 5, 10, 15, 20 stories together with different load patterns and different assumptions in dynamic modeling of the pushover curve.

2. PROBLEM FORMULATION

The equation of motion for a lumped mass shear building system can be written in the following matrix form:

$$M\ddot{U}(t) + C\dot{U}(t) + R(U) = -M\{1\}\ddot{u}_g(t) \qquad (1)$$

where M and C denote the mass and damping matrices respectively. The vector U(t) is the relative displacement of different floors with respect to ground, while R(U) is the vector for resistance force. Also, $\ddot{u}_g(t)$ is the ground acceleration. Assuming a displacement pattern function φ for U(t), the above MDOF system can be approximately converted to an equivalent single degree of freedom (SDOF) system. This shape function corresponds to the deflected shape of the system under the action of a statically applied lateral loads defined by the load pattern function Ψ. The displacement pattern function is normalized with respect to the roof displacement, while the load pattern is normalized with respect to sum of its elements for convenience. Therefore, at any time t, the displacement of the MDOF structural system at each level can be expressed as:

$$U(t) \cong \varphi\, x(t) \qquad (2)$$

where x(t) is the roof displacement of the structure and the resistance vector R is approximated by:

$$R(x) = \psi\, V(x) \qquad (3)$$

in which V(x) is the base shear force of the structure. The load pattern Ψ is usually considered to be a distribution of constant lateral loads and the deflection pattern φ is the average deflection of the structure when the lateral load pattern Ψ is used to push the structure. The average is taken over the domain {x=0, x=x_m}, where x_m is an initial assessment of the anticipated inelastic structural response to design level earthquake and could be approximated by 1% of drift index(Δ_{roof}/H) [5]. Both patterns should be normalized as it was describe before. Having defined the load and the displacement patterns, substituting equations (2) and (3) into equation (1) results in:

$$M\varphi\, \ddot{x}(t) + C\varphi\, \dot{x}(t) + \psi\, V(x) = -M\{1\}\, \ddot{u}_g(t) \qquad (4)$$

Substituting an orthogonal damping matrix for C and pre-multiplying the transpose of the displacement pattern, φ^T, on both sides of equation (4), it transforms to:

$$M^*\ddot{x}(t) + C^*\dot{x}(t) + R^*(x) = -L^*\ddot{u}_g(t) \qquad (5)$$

in which:

$$M^* = \varphi^T \mathbf{M} \varphi \quad , \quad C^* = \varphi^T C \varphi \ ,$$
$$R^*(x) = \varphi^T \psi\, V(x) \quad , \quad L^* = \varphi^T M \{1\}\, \ddot{u}_g \qquad (6)$$

The quantities M^* ,C^*, $R^*(x)$ and L^* can then be defned as the equivalent mass, equivalent damping constant, equivalent resistance function and equivalent modal participation factor respectively. Dividing equation (5) by M^*, approximating $R^*(x)/ M^*$ by a multi-linear equation named r(x), and substituting:

$$C^* / M^* = \alpha + \beta k_0 \tag{7}$$

where k_0 is the initial stiffness of the aforementioned multi-linear system and α, β are the mass and stiffness coefficients in a two terms orthogonal approximation of damping matrix, the equation (5) changes to:

$$\ddot{x}(t) + \left(\alpha + \beta k_0\right) \dot{x}(t) + r(x) = -\ell \; \ddot{u}_g(t) \tag{8}$$

in which $\ell = L^*/M^*$. It can be noticed that the definition of the resistance function of the equivalent SDOF model presented herein does not have the physical meaning of either base shear or base over-turning moment, as compared with other available models. The current definition of the resistance function, $R^*(x)$, can be explained as the inner product of the deflection pattern and the applied load vector. This definition is preferred to either the base shear or base overturning moment, since it takes into account both the distribution of lateral loads and the displacement pattern of the system. In real dynamic response, the ground motion induced inertia forces and the resistance of the structure depends on its displacement pattern. But, this dependence is not reflected by the definition of either base shear or base overturning moment [5].

On the other hand, some simple equivalent static procedures to calculate the target displacement are introduced in the literature. The most comprehensive formula is provided by FEMA356 in the following form [3]:

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^{\,2}}{4\pi^2} g \tag{9}$$

where δ_t is the target displacement, S_a is the spectral acceleration, C_0 is the modification factor of the equivalent mode participation factor. Also, C_2 is equal to 1.0 for a special moment resisting building and C_3 is equal to 1.0 where P-Δ effects are not considered. The parameter C_1 is equal to

$$C_1 = 1.0 \quad \text{for} \quad T_e \geqq T_s \,,$$

$$C_1 = \left[1.0 + (R-1)\frac{T_0}{T_e}\right] / R \quad \text{for} \quad T_e < T_s \tag{10}$$

Also,

$$R = \frac{S_a}{V_y / W} C_m \tag{11}$$

In these equations, T_s is the characteristic period of spectral curve where constant acceleration is changed to constant velocity. The R is the ratio of the elastic capacity to the yield capacity of the structure and C_m is the effective mass factor whose value depends on the number of stories and the fundamental period of the structure. The V_y and W are the yield capacity and the seismic weight of the system respectively. The T_e in equation (9) can be calculated from:

$$T_e = T_i \sqrt{\left(k_i / k_e \right)}$$

3. NUMERICAL EXAMPLES

Five 2, 5, 10, 15, 20 story special moment resisting steel frames are considered. These 2-D structural models are three bays wide, with all floors 3.6(m) high. The side bays are 6.0(m) wide, while the width of the middle bay is 7.5(m). The columns are assumed to be fixed at the base. The gravity loads include a dead load of 400 kg/m^2 and a live load of 100 kg/m^2 for the roof and a dead load of 500 kg/m^2 and live load of 250 kg/m^2 for all floors. Exterior wall panels are assumed to

Table 1- Earthquake records used for the verification study.

No.	Station, Earthquake	Magnitude (ML)	PGA (cm/sec^2)	PGV (cm/sec)	PGD (cm)	Closest Fault Rupture (km)
1	**CASTAIC - O.R.R., NORTHRIDGE, 1994 (N.C)**	6.6	504.22	52.63	2.41	22.6
2	**CORRALITOS, LOMA PRIETA, 1989 (L.C)**	7.0	617.70	55.20	10.88	5.1
3	**GILROY #1, S.Y.S., LOMA PRIETA, 1989 (L.G)**	7.0	426.61	31.91	6.38	10.5
4	**LOS ANGELES- T&H, NORTHRIDGE, 1994(N.L)**	6.6	180.11	20.02	2.74	32.9
5	**PACOIMA-K.C., NORTHRIDGE, 1994 (N.P)**	6.6	424.21	50.88	7.21	8.2

have a weight of 125 kg/m^2. The tributary width of every frame is assumed to be 6.0m.

The frames are designed according to the 1997 Uniform Building Code (UBC97) for a structure located on stiff soil (soil type S_B) in seismic zone 4, with R factor equal to 8.5[11]. The steel members are designed according to Load and Resistance Factor Design (LRFD97) [12]. All members meet the required compactness ratio for local buckling and the joint and member requirements for special moment resisting frames. Therefore beam and column hinging is intentionally allowed in these numerical examples. Since, the lateral stiffness of the bare frames turned out to be very low, the moment of inertia of all members are increased uniformly, so that their fundamental periods be close to those provided by the UBC97 ($T=.085H^{3/4}$). The structural frames are analyzed statically and dynamically using the DRAIN-2DX nonlinear analysis program [10]. Also, P-delta effects are not included in this study. The %5 damping ratios are considered for the first two modes. Moment-rotation relationships for the elements assume 3% strain hardening. Yielding is considered to occur at concentrated plastic hinges at the ends of the elements. Finally, the axial force-bending moment interaction is considered in the columns according to FEMA365.

The dynamic analyses for each frame are performed using five strong ground motion records presented in Table (1). The average acceleration response spectrum of these records is very close to the UBC97 ground motion spectrum for the soil type S_B. All the records are scaled to a peak acceleration value of 0.4g.

The load patterns considered in this study

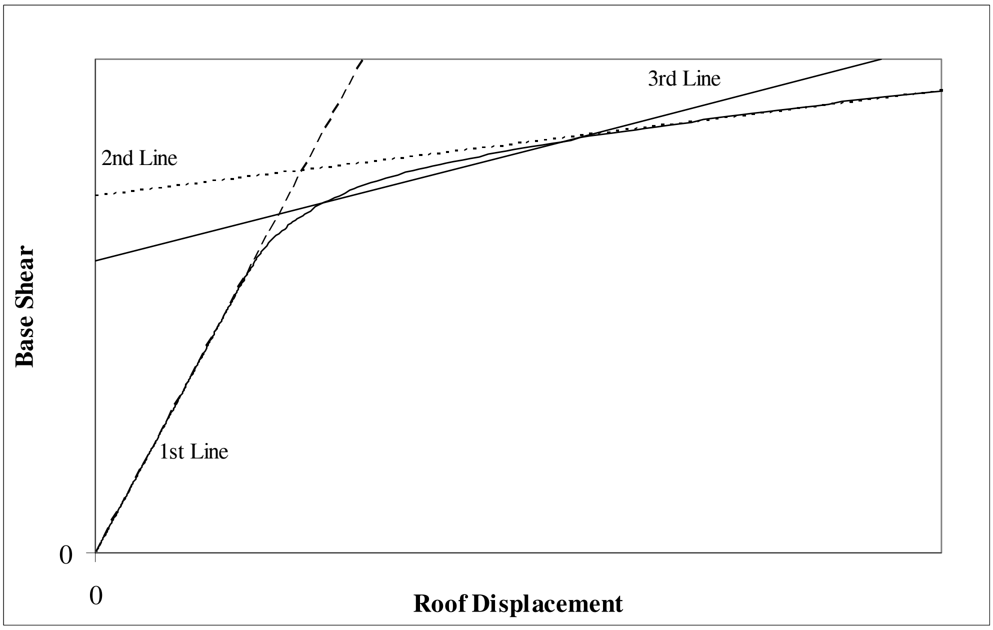

Fig. 1-Tri-linear best fit to the pushover curve for dynamic analysis.

include uniform load pattern (UFM), the IBC2000 [13] equivalent static method load pattern (IBC), the Uniform Building Code equivalent static method load pattern (UBC), FEMA356 modal load pattern (MOD), and the proposed effective modal load pattern (SRM) which is defined as:

$$\Psi_{SRM} = \sqrt{\sum_{i=1}^{n} (EMM_i \, \psi_i)^2} \quad , \qquad (12)$$

$$\psi_i = \omega_i^2 \, M \, \delta_i$$

where EMM_i is the ith effective modal mass and Ψ_i is load pattern of the ith mode that can be calculated using the ith modal shape δ_t. In uniform load pattern (UFM), the amount of load at each story is proportional to its mass. The FEMA356 modal load pattern (MOD) is defined as the SRSS combination of story shears of enough number of modes to include 90% of the total mass of the system.

The bi-linear and tri-linear approximations of the pushover curve are used in the dynamic procedure. The bi-linear curve is constructed according to the NEHRP96 [4] recommendation stating that the 1st line should intersect the main curve at 0.6Fy as it is shown in Figure 1. The third line is placed such that the area under the main curve and the approximate tri-linear curve to be the same. However, it should be noted that unlike FEMA356, the procedure for bi-linear approximation of pushover curves in NEHRP96, does not include the condition of maintaining equal area under the pushover curves.

4. VERIFICATION STUDY

The results of the modal analyses for the 5 the structural models, including their effective modal masses are shown in Table 2. Figures 2 to 6 show the pushover curves for these 5 structural models, using different load patterns. As it is shown in figures 2 & 3, all

Table 2-Building modes and their effective modal mass.

Mode number	2 story building		5 story building		10 story building		15 story building		20 story building	
	period (sec)	E.M.M. (%)	period (sec)	E.M.M. (%)	period (sec)	E.M.M. (%)	period (sec)	E.M.M. (%)	period (sec)	E.M.M. (%)
1st	0.430	88.222	0.858	79.901	1.471	77.581	1.917	74.807	2.339	73.773
2nd	0.188	11.778	0.331	10.572	0.543	10.636	0.702	11.561	0.861	12.021
3rd	-	-	0.215	5.076	0.341	3.877	0.420	4.239	0.512	4.154
4th	-	-	0.150	2.668	0.248	2.339	0.302	2.553	0.362	2.446
5th	-	-	0.115	1.783	0.193	1.617	0.237	1.335	0.276	1.430
6th	-	-	-	-	0.154	1.178	0.194	1.054	0.221	1.091
7th	-	-	-	-	0.126	0.922	0.159	0.975	0.182	0.790
8th	-	-	-	-	0.108	0.628	0.137	0.521	0.156	0.560
9th	-	-	-	-	0.093	0.580	0.119	0.658	0.139	0.462
10th	-	-	-	-	0.077	0.643	0.106	0.491	0.121	0.468

E.M.M: **Effective Modal Mass**

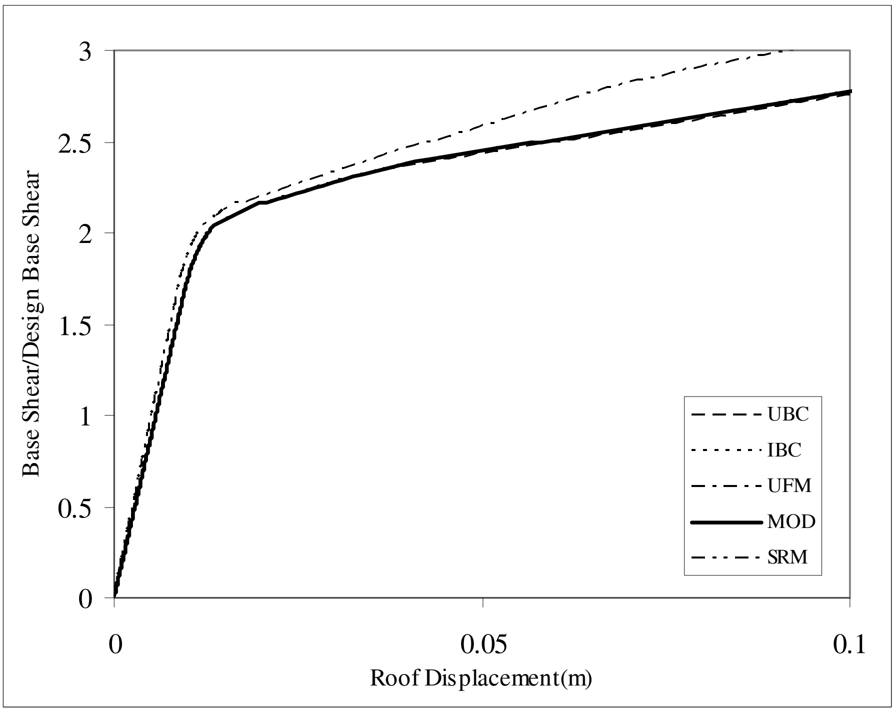

Fig. 2: Pushover diagrams for different load patterns in 2 story structural model.

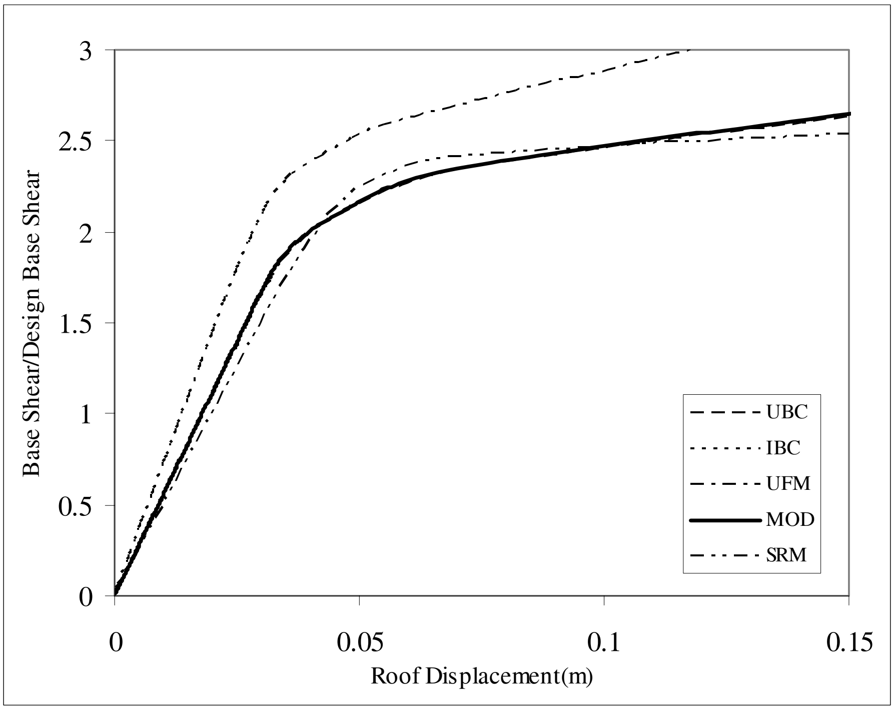

Fig. 3: Pushover diagrams for different load patterns in 5 story structural model.

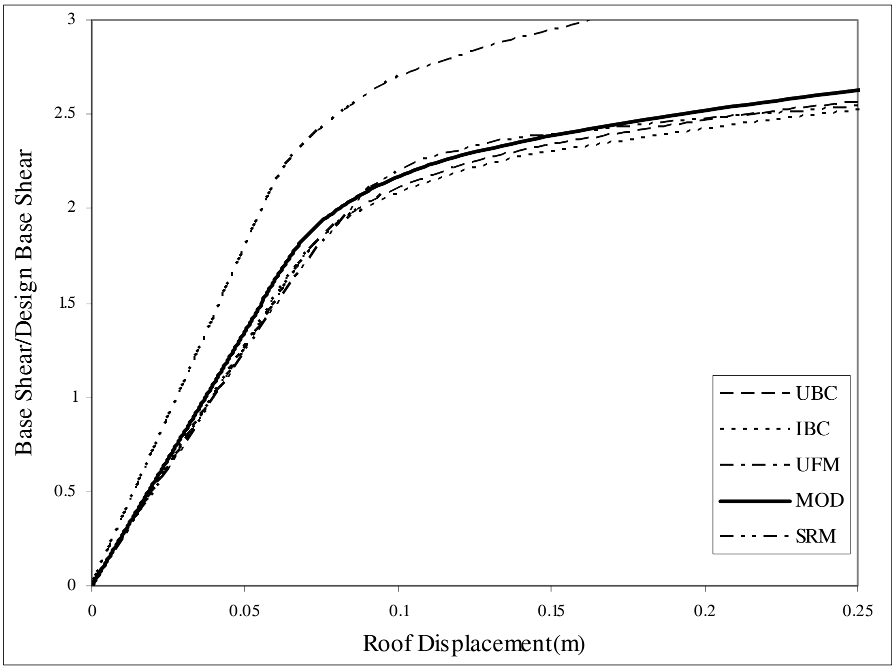

Fig. 4: Pushover diagrams for different load patterns in 10 story structural model.

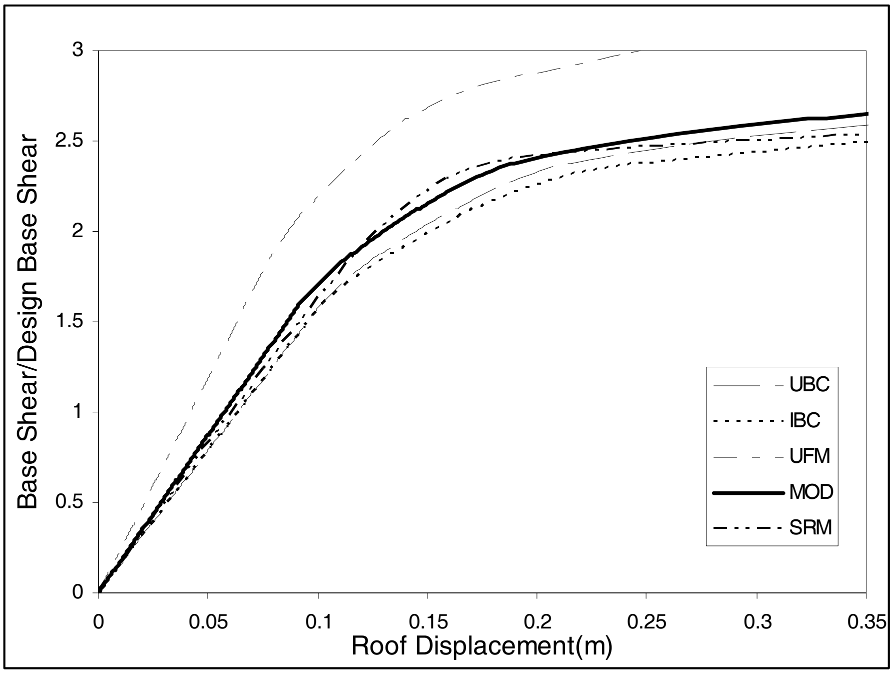

Fig. 5: Pushover diagrams for different load patterns in 15 story structural model.

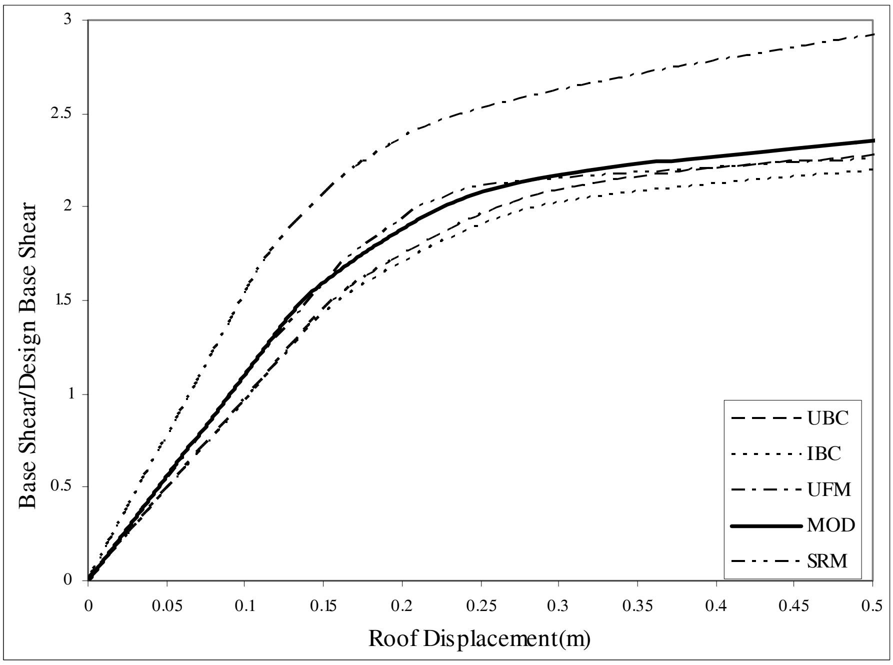

Fig. 6: Pushover diagrams for different load patterns in 20 story structural model.

the load patterns in low rise buildings perform nearly the same except the uniform load pattern (UFM) that causes larger initial slope(higher stiffness) and larger base shear fore in all structural models. The difference between the resulting push-over curves of the considered load patterns in terms of their initial stiffness and maximum base shear force become even more significant when the number of stories is increased. Again, as figures 5 and 6 show, the uniform load pattern (UFM) leads to increasingly higher initial stiffness and larger base shear force in taller buildings.

Tables 3 to 7 compare the target displacements obtained using dynamic time history analysis (THA), dynamic bi-linear and tri-linear pushover analyses, i.e., dynamic time history analysis of an equivalent SDOF system with approximate bi-linear and tri-linear force-deformation behavior, and static FEMA356 procedure for 5 different load patterns for all 5 structural models. By static FEMA356 procedure it is meant to determine the modified target displacement from Eq. 9, using the effective periodobtained from the initial push over diagram.

The first five rows represent the results of the dynamic time history analyses using five earthquake records. The next row is the average and the last row is the existing error of the average value with respect to the result of the time history analysis (THA). For comparison the first mode approximation of the target displacement is also shown in another column. In general, considering the results for all the models, the effective modal load pattern (SRM) and uniform load pattern (UFM) seem to have a better performance in approximating the target displacement. But, dynamic pushover analysis with a tri-linear approximation and SRM load pattern shows the best approximation especially in tall buildings (15 and 20 stories).

Furthermore, as the results indicate, considering equal area in approximating the original push over diagram has a significant effect on the accuracy of the results. One could see that the results for the static FEMA356 procedure that observes the equal area condition in bi-linear approximation of the push over diagram, has less error in estimating the exact target displacement when compared to the dynamic pushover with bi-linear approximation. The tri-linear approximation of the pushover diagram has the best performance among these approaches for the reasons already mentioned. Again, as Tables 3 to 7 depicts, the static FEMA356 procedure's error in assessing the target displacement noticeably increases for taller structural models in comparison with other dynamic methods.

The first mode approximation of the target displacements leads to accurate results in the low rise buildings due to the dominant first mode. However, it seems that none of the static load patterns performs appropriately in high rise buildings for opposite reasons. An important point in using the static procedures is that the first mode approximation of the target displacement has the same degree of approximation as the static FEMA356 procedure. So, for the same level of accuracy, there is no need in going through more complicated calculations.

5. CONCLUSION

Dynamic pushover analysis was introduced as the dynamic time history analysis of an equivalent single degree of freedom model of the original system to determine the target displacement. Also, the code based

Table 3- Comparison of the target displacement obtained using different methods for the 2 story structural model.

EQ	IBC			MOD			UFM			UBC			SRM			1st Mode	THA
	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines		
N.C.		0.039	0.035		0.038	0.035		0.040	0.034		0.039	0.036		0.039	0.035		0.037
L.C.		0.054	0.045		0.053	0.047		0.057	0.046		0.053	0.050		0.054	0.045		0.050
L.G.		0.036	0.041		0.037	0.033		0.037	0.041		0.036	0.038		0.036	0.041		0.033
N.L.		0.064	0.049		0.064	0.067		0.060	0.045		0.064	0.053		0.064	0.049		0.051
N.P.		0.063	0.058		0.064	0.057		0.062	0.057		0.063	0.061		0.063	0.058		0.052
Ave	0.046	0.051	0.045	0.046	0.051	0.048	0.044	0.051	0.045	0.046	0.051	0.047	0.045	0.051	0.045	0.045	0.045
Err(%)	2.74	15.04	1.75	2.20	12.84	6.79	1.73	14.14	0.22	2.97	14.72	6.33	1.57	15.08	1.75	1.55	

Table 4- Comparison of the target displacement obtained using different methods for the 5 story structural model.

EQ	IBC			MOD			UFM			UBC			SRM			1st Mode	THA
	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines		
N.C.		0.113	0.102		0.113	0.100		0.108	0.098		0.114	0.105		0.112	0.100		0.073
L.C.		0.109	0.107		0.114	0.123		0.100	0.109		0.111	0.106		0.108	0.107		0.106
L.G.		0.047	0.047		0.048	0.045		0.047	0.048		0.048	0.048		0.047	0.047		0.056
N.L.		0.119	0.105		0.120	0.108		0.106	0.105		0.120	0.107		0.117	0.104		0.084
N.P.		0.154	0.137		0.155	0.129		0.138	0.135		0.156	0.141		0.152	0.136		0.134
Ave	0.100	0.108	0.099	0.102	0.110	0.101	0.096	0.100	0.099	0.100	0.109	0.101	0.099	0.107	0.099	0.098	0.091
Err(%)	10.22	19.74	9.80	12.22	19.62	11.34	5.57	9.91	9.07	10.82	20.79	11.90	9.53	18.04	8.92	8.60	

Table 5- Comparison of the target displacement obtained using different methods for the 10 story structural model.

EQ	IBC			MOD			UFM			UBC			SRM			1st Mode	THA
	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines		
N.C.		0.130	0.132		0.142	0.146		0.115	0.142		0.129	0.133		0.120	0.126		0.120
L.C.		0.181	0.180		0.187	0.162		0.177	0.171		0.190	0.188		0.183	0.180		0.175
L.G.		0.125	0.125		0.131	0.120		0.128	0.120		0.129	0.129		0.124	0.124		0.116
N.L.		0.207	0.209		0.225	0.201		0.186	0.180		0.211	0.209		0.204	0.205		0.146
N.P.		0.222	0.210		0.229	0.201		0.206	0.231		0.229	0.216		0.225	0.210		0.201
Ave	0.172	0.173	0.171	0.178	0.183	0.166	0.162	0.162	0.169	0.176	0.178	0.175	0.168	0.171	0.169	0.166	0.152
Err(%)	13.73	14.18	12.76	17.48	20.68	9.65	7.10	7.11	11.38	16.35	17.21	15.43	10.80	13.03	11.39	9.58	

Table 6- Comparison of the target displacement obtained using different methods for the 15 story structural model.

EQ	IBC			MOD			UFM			UBC			SRM			1st Mode	THA
	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines		
N.C.		0.264	0.258		0.281	0.265		0.230	0.219		0.271	0.265		0.263	0.258		0.260
L.C.		0.157	0.158		0.173	0.157		0.134	0.165		0.160	0.160		0.157	0.157		0.187
L.G.		0.149	0.149		0.157	0.152		0.149	0.144		0.152	0.152		0.147	0.147		0.139
N.L.		0.333	0.293		0.322	0.292		0.244	0.291		0.301	0.307		0.257	0.280		0.230
N.P.		0.167	0.167		0.169	0.173		0.197	0.170		0.170	0.170		0.156	0.163		0.191
Ave	0.239	0.214	0.205	0.246	0.220	0.208	0.225	0.191	0.198	0.245	0.211	0.211	0.232	0.196	0.201	0.232	0.201
Err(%)	18.64	6.30	1.73	22.08	9.31	3.34	11.51	5.17	1.78	21.46	4.70	4.73	15.43	2.76	0.14	15.02	

Table 7- Comparison of the target displacement obtained using different methods for the 20 story structural model.

EQ	IBC			MOD			UFM			UBC			SRM			1st Mode	THA
	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines	FEMA Static	2 Lines	3 Lines		
N.C.		0.200	0.210		0.249	0.247		0.212	0.215		0.204	0.204		0.197	0.202		0.236
L.C.		0.119	0.125		0.149	0.152		0.117	0.120		0.130	0.130		0.118	0.121		0.203
L.G.		0.128	0.134		0.151	0.144		0.124	0.127		0.132	0.132		0.125	0.129		0.153
N.L.		0.274	0.284		0.259	0.249		0.221	0.242		0.273	0.275		0.252	0.261		0.227
N.P.		0.283	0.297		0.272	0.278		0.265	0.259		0.293	0.293		0.280	0.286		0.261
Ave	0.305	0.201	0.210	0.314	0.216	0.214	0.289	0.188	0.193	0.315	0.207	0.207	0.295	0.194	0.200	0.296	0.216
Err(%)	41.22	7.08	2.84	45.16	0.01	1.00	33.80	13.19	10.89	45.79	4.38	4.28	36.54	10.10	7.67	37.26	

formulations of equivalent static procedures were presented. The accuracy of a number of load patterns, including a new approach in estimating the target displacement was compared and the efficiency of the dynamic procedures in improving the approximation was demonstrated using a numerical example. Furthermore, the newly defined effective modal load pattern (SRM) shown to have acceptable performance in approximating the target displacement. It is shown that the dynamic pushover with a tri-linear approximation and SRM load pattern leads to the least error in approximating the target displacements, especially in 15 and 20 stories structural models. On the other hand, when nonlinear static procedure seems adequate, no specific preference is observed in using more complicated static procedures (proposed by codes) compared to the simple first mode target displacement assessment.

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