Non-invasive Continuous Surface Wave Measurements for In Situ Damping Ratio Profiling of Soils

Abdolhosein Haddad*, Gholamali Shafabakhsh
Civil Engineering Dept. Semnan University, Semnan- IRAN
*E-mail: ahadad@semnan.ac.ir

Abstract: Local site conditions have a strong effect on ground response during earthquakes. Two important soil parameters that control the amplification effects of seismic motions by a soil column are the soil hysteretic damping ratio and shear wave velocity. This paper presents the results of in situ damping ratio measurements performed using continuous surface wave attenuation data at a site in Semnan University campus and analysis used to obtain the near surface soils damping ratio profile. Once the frequency dependent attenuation coefficients are determined, the shear damping ratio profile is calculated using an algorithm based on constrained inversion analysis. A computer code is developed to calculate the shear damping ratio in each soil layer. Comparisons of the in situ shear damping ratio profile determined from continuous surface wave with cross hole independent test measurements are also presented. Values of shear damping ratio, obtained using continuous surface wave measurements, were less than the measured using cross hole tests, possibly because of the higher frequencies used in cross hole tests.

Keywords: Continuous surface waves, Damping ratio, Soil attenuation, Hysteresis loop

1. Introduction

The damping ratio and shear wave velocities of soil and rock deposits are essential parameters for the analysis of subsurface layers under strong earthquake shaking and other dynamic loadings. Geophysical exploration methods have been used for the determination of in situ soil parameters for more than 30 years. These methods are based on extensive theoretical, mathematical and experimental foundations. The majority of this effort has concentrated on soil shear wave velocity and damping ratio measurements which have been used to obtain the shear modulus and site amplification effects [1, 2]. Recent earthquakes such as the 1985 Mexico City, 1989 Loma Prieta in the U.S., 1995 Hyogo-Nabu in Japan [3], and 1999 Izmit earthquake in Turkey [4] demonstrated the importance of site effects in soft clayey and silty soils.

It has become standard practice to determine damping ratios from laboratory tests on soil specimens obtained from test pits and borings. An alternative approach is to determine the in-situ soil damping ratio in the field using geophysical techniques such as continuous surface wave (CSW) testing. Surface wave method provide a non-invasive technique of obtaining soil damping ratio that overcome some of the limitations associated with the more commonly used invasive field methods.

2. Defination of Soil Damping

Damping is the general term given to the dissipation of energy during cyclic loading of an inelastic medium. The soil damping characteristics are usually expressed by the hysteretic or material damping ratio. The hysteresis loop produced from the cyclic loading of a typical soil can be described by the path of the loop itself or by two parameters that describe its general shape.

International Journal of Civil Engineerng. Vol. 5, No. 2, June 2007

93

These parameters are the inclination and the breath of the hysteresis loop, shear modulus and damping, respectively. Figure 1 is a simplified schematic showing one loop of symmetric cyclic loading and its corresponding parameters. The hysteretic damping ratio arising from the nonlinear, inelastic response of soil to cyclic loading is equal to:

$$D = \frac{W_D}{4\pi W_S} = \frac{1}{2\pi}\frac{A_{Loop}}{G_{Sec}\gamma_C^2} \tag{1}$$

where W_D is the energy dissipated during a cycle of loading and W_S is the maximum elastic energy stored in a cycle of constant displacement loading and A_{Loop} is the area of the hysteresis loop, G_{Sec} and γ_c, are secant shear modulus and cyclic shear strain, respectively.

Ishibashi and Zhang [5] developed an empirical expression for the damping ratio of plastic and non- plastic soils as:

$$D = 0.333\frac{1+\exp(-0.0145PI^{1.3})}{2}\times$$
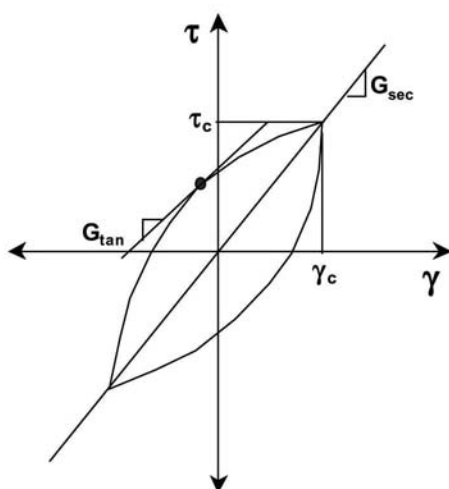$$\left[0.586(\frac{G}{G_{max}})^2 - 1.547\frac{G}{G_{max}} + 1\right] \tag{2}$$

Fig. 1. Hysteresis loop showing secant and tangent shear modulus

where PI is the soil plastic index, G_{max} is small strain shear modulus and G is shear modulus.

Toskoz and Jhonston [6]also showed that the damping ratio is related to the attenuation coefficient α by:

$$D = \alpha V / 2\pi f \tag{3}$$

where V is the wave propagation velocity and f is the frequency of the vibration. Seismic wave attenuation in geo-material is a complex phenomenon resulting from the interaction of several mechanisms that contribute to energy losses during dynamic excitation. Material damping is caused by energy dissipated within the soil skeleton frame. Also the frictional losses between soil particles and fluid flow due to the relative movement between the solid and fluid phases are responsible for material damping.

The attenuation coefficient is a measure of the attenuation of particle motion caused by material damping as a seismic wave propagates through an inelastic media. In this study the attenuation coefficient is determined from measurements of surface wave particle motion.

3. Laboratory Measurements of Soil Damping

Most efforts to measure the damping properties of soil have employed laboratory tests such as resonant-column method. In these tests, solid or hollow cylindrical specimens are subjected to harmonic torsional or axial loading by an electromagnetic loading system. Laboratory testing allows for a large range of soil and environmental parameters to be varied in order to study the sensitivity of the soil to

94

Table 1 Effect of environmental and loading conditions on damping ratio [7]

Increasing Factor	Damping ratio
Increase void ratio, e	Decreases with e
Confining pressure σ'_m	Decreases with σ'_m
Plasticity index, PI	Decreases with PI
Cementation, c	May decrease with c
Over consolidation ratio	Not affected
Cyclic strain γ_c	Increase with γ_c
Number of loading cycles, N	Not significant for moderate γ_c

variations in selected parameters. Damping behavior is also influenced by effective confining pressure, particularly for soils of low plasticity. The influences of various environmental and loading conditions on the damping ratio of soils is described in Table 1. The actual response of a soil specimen in the field under different conditions than those imposed in the laboratory, may be significantly different.

Field tests, enclosing a much larger soil volume which include non-uniform, non-homogeneous conditions, should provide a more representative characterization of the overall soil profile than what is possible using laboratory tests which includes of small volume and relatively homogeneous soil specimens.

4. Field Measuremants of Soil Damping

In-situ measurements of dynamic soil parameters have typically focused on shear wave velocity and soil stiffness (i.e. shear modulus). There have been relatively few efforts on measuring soil damping parameters in field. Studies by Stewart [8] and Mok [9] show the measurement of the soil damping ratio using seismic shear wave amplitude data with the SCPT and cross hole seismic methods. These techniques have several potential disadvantages with respect to obtaining data for damping ratio evaluations. All of these methods required the advancement of a boring or cone penetrometer in to the ground and the installation of a receiver (geophone or accelerometer) at the various depths. Since the coupling between the receiver and surrounding soil can not be physically observed the potential for poor coupling and therefore poor amplitude signal reception, cause some problems in seismic wave amplitude data measurement and interpretation. Additionally, down hole and cross hole testing usually include the installation of grouted casing in the complete boring. This grout and casing (plastic or steel pipe) system made impede or impart disturbance to the incoming signals. The non-
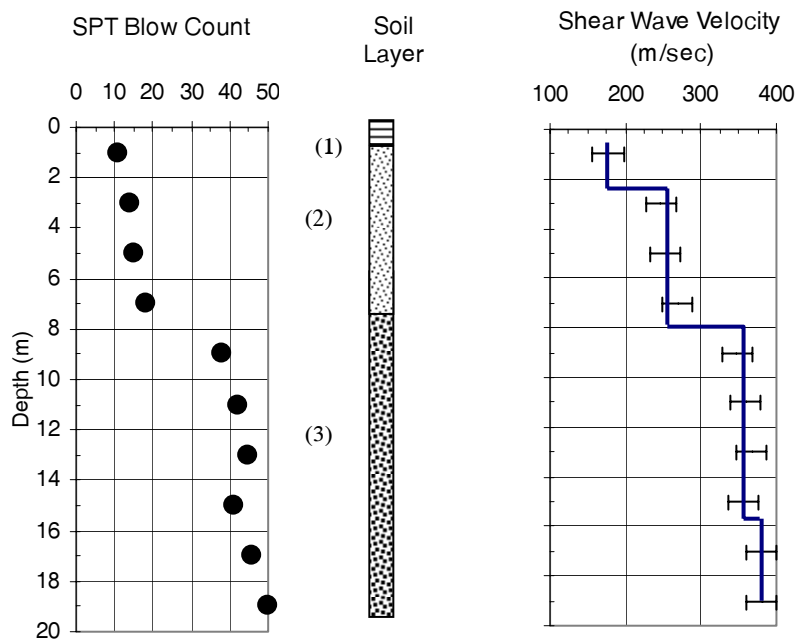
Fig. 2. Interpreted soil profile at the site

invasive techniques like surface wave measurements provide an attractive alternative to more costly intrusive methods such as down-hole and cross-hole tests [10].

5. Site Description

Surface wave attenuation data measurements were performed at a site in Semnan University's campus. The soil conditions at the site consist of approximately 0.6m top soil (layer 1), 7 m of loose to medium silty sand (layer 2), underlain by 11 to 12 m of medium to dense gravelly sand with silt (layer 3). The water table was not encountered to a depth 50 m at this site. The results of a standard penetration test and shear wave velocity made at the site are shown in Figure 2 along with their interpreted soil profile.The shear wave velocity values are obtained from the continuous surface wave tests.

6. Continuous surface wave measurements

The surface wave attenuation coefficient as a function of frequency is required to determine the soil damping ratio profile. In this study continuous surface wave attenuation tests were performed at the site to determine the soil attenuation profile by using an electro-mechanical vibrator as a sine wave source and collecting vertical surface particle motion with geophones connected to the FFT analyzer in the unit control. Rayleigh waves were generated by vertical oscillation of mechanical shaker. The geophones used in this study have a natural frequency of 2 Hz, enable to measure particle velocity in the vertical direction.

The geophones were firmly coupled to the soil by removing the upper 10 to 20 cm of top-soil at the site. The electro-mechanical shaker was placed at the designated location

96

International Journal of Civil Engineerng. Vol. 5, No. 2, June 2007

Fig. 3. Surface wave attenuation measurements at the site

and firmly seated by removing the upper loose soil and preparing a level area for placement of the vibrator base. At each geophone location, ten to fifteen average of the geophones signal were measured at each frequency with mean value recorded by the FFT analyzer. After a measurement has been done, the geophones were moved to new offsets and coupled to the ground surface as previously explained. Measurements of surface wave particle motion were made at 5 to 15 offsets between 0.5 to 10 meters from the vibrator. The equipment configuration used for surface wave attenuation measurements is shown in Figure 3.

7. Attenuation parameter determination

Surface wave observation shows that the geometrical attenuation is a function of $r^{-0.5}$ where r is the distance from the vibration source to the point of measurements. The expression that accounts for both geometrical and material damping is:

$$A(r,f) = \frac{A_0(f)}{r^{-0.5}} \cdot e^{-\alpha(f)r} \qquad (4)$$

Where A is the receiver amplitude value at the distance r, A_0 is the source amplitude and α is the attenuation coefficient. If both sides of Eq. 4 are multiplied by $r^{-0.5}$ and then by taking the natural logarithm, we can linearize the above equation and provide a relationship between receiver amplitude and attenuation coefficient as a function of frequency :

$$Ln\left[A(f).r^{0.5}\right] = Ln(A_0(f)) - \alpha(f) \cdot r \qquad (5)$$

Now by plotting corrected receiver amplitude as a function of distance (r) for a distinct frequency, the slope of the best-fit regression line that will be attenuation coefficient (α). Repeating this analysis for all frequencies, an attenuation coefficient versus frequency curve can be drawn. Figures 4 and 5 show typical corrected geophone amplitude versus offset relationship with the negative slope of the line from regression analysis representing the attenuation coefficient due to soil material damping for frequencies 20 and 60 Hz.

The attenuation of surface waves in a multilayered dissipative medium is only a function of the shear and the compression of
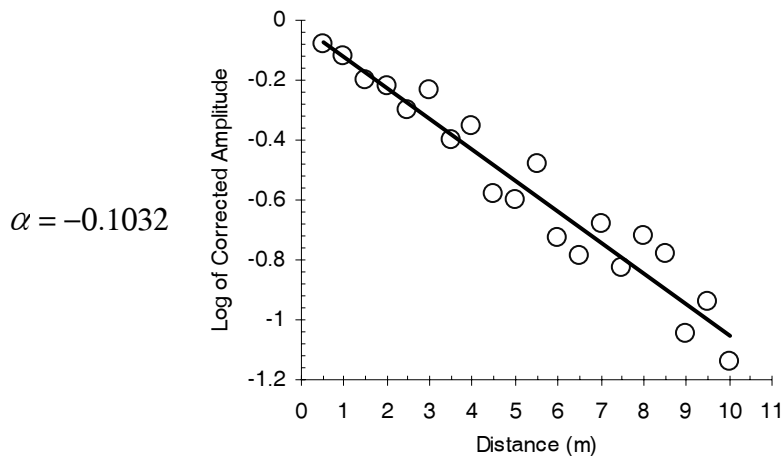
$\alpha = -0.1032$

Fig. 4. Attenuation coefficient for 20 Hz
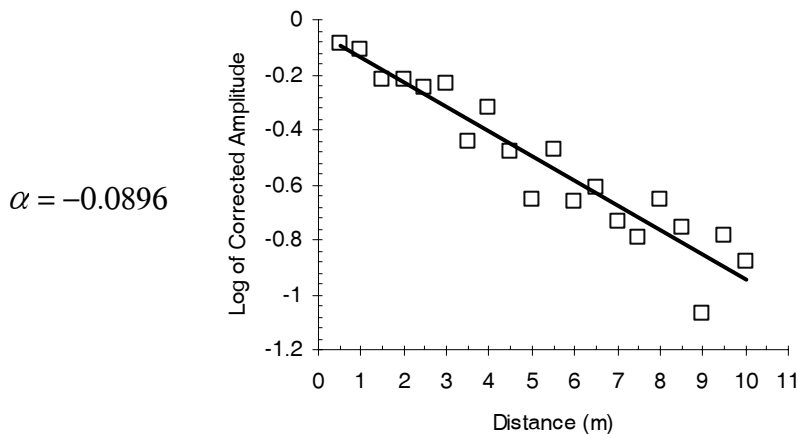

$\alpha = -0.0896$

Fig. 5. Attenuation coefficient for 60 Hz

damping ratios of individual layers as [11]:

$$\alpha_r(f) = \frac{\omega}{V_R^2}\left\{\sum_{i=1}^{n} V_{p,i}(\frac{\partial V_r}{\partial V_p})_i D_{p,i} + \sum_{i=1}^{n} V_{s,i}(\frac{\partial V_r}{\partial V_s})_i D_{s,i}\right\}$$

(6)

where the partial derivatives representing the sensitivity of Rayleigh wave phase velocity to compression and shear wave velocity changes in individual layers are defined by:

$$\left(\frac{\partial V_r}{\partial V_p}\right)_i = \frac{\rho V_p}{2UIK^2}\int_{zi-1}^{zi}\left(kr_1 + \frac{dr_2}{dz}\right)^2 dz$$

(7)

$$\left(\frac{\partial V_r}{\partial V_s}\right)_i = \frac{\rho V_s}{2UIK^2}\int_{zi-1}^{zi}\left[\left(kr_2 - \frac{dr_1}{dz}\right)^2 - 4kr_1\frac{dr_2}{dz}\right]dz$$

(8)

where ρ = mass density, $r_1(k,z,\omega)$ and $r_2(k,z,\omega)$ = eigenfunction associated with the solution of the eigenproblem of Rayleigh waves in elastic layered media, k= wave number, U = group velocity and the term I is the first energy integral defined by the following expression:

$$I = \frac{1}{2}\int_0^\infty \rho(z)\left[r_1^2 + r_2^2\right]dz$$

(9)

The final expression is obtained by replacing D_p in Equation (6) with KD_s, where K is a parameter defined as the ratio of the compression to shear damping ratio.

$$\alpha_r(f) = \frac{\omega}{V_R^2}\left\{\sum_{i=1}^{n}\left[V_p(\frac{\partial V_r}{\partial V_p})_i K + V_s(\frac{\partial V_r}{\partial V_s})_i\right]D_{s,i}\right\} \quad (10)$$

A parametric study has been carried out to evaluate the influence of the value of K on the inverted damping ratio profiles. The value of the K was found to have a negligible influence on the back calculated shear damping profile. So a value of K=1 was used in all analyses based on the previous studies. Geometric dispersion implies the existence of several Rayleigh wave modes for a given frequency, each characterized by its own wave number and mode shape. Because some of the quantities (k, r_1, r_2 ,U and I) appearing on the right hand side of Equations. (7) and (8) are referred to a specific mode, the corresponding partial derivatives on the left hand side should refer to specific mode. In this study, the partial derivatives have been computed for the fundamental mode of vibration. Equation (10) can be expressed in matrix form as:

Gm =d (11)

where G =M×N matrix with M equal to the number of frequencies and N equal to the number of soil layers; m = unknown vector of shear damping ratio values; and d = vector of experimental attenuation coefficients. Equation (11) is solved using a constrained linear inversion algorithm to calculate the unknown vector m of shear damping ratio values. This inversion method is based on the following strategy; given a set of experimental attenuation coefficients and their associated uncertainties, find the optimum values of $D_{s,i}$ that maximize the smoothness of the resulting damping ratio

profile while predicting the experimental attenuation coefficients with reasonable accuracy. This method of solving inverse problem is largely motivated by the observation that inversions performed with unconstrained least square method too often lead to physically unreasonable profiles of model parameters. Thus the problem is that the inversion of a set of measured data relies on an assumed theoretical model. For this study, the model is a soil profile made up of a finite number of layers; the resulting shear damping ratio profile will depend on the number of layers and the layer thicknesses that have been chosen. The method in this study requires a definition solution or a candidate D_s profile. In a layered soil profile, roughness can be defined by the following expression:

$$R = \sum_{i=2}^{N}(D_{s,i} - D_{s,i-1})^2 \quad (12)$$

Now by using a weighted least-square criterion, it is possible to write the misfit between the measured and the predicted attenuation coefficients as follows:

$$\delta^2 = \left\|\mathbf{Wd} - \mathbf{WGm}\right\|^2 \quad (13)$$

where W = diagonal M×M matrix as:

$$\mathbf{W} = \begin{bmatrix} 1/\sigma_1 & & & \\ & 1/\sigma_2 & & \\ & & ... & \\ & & & 1/\sigma_M \end{bmatrix} \quad (14)$$

where σ_i (i =1 to M) is standard deviations that reflect the uncertainties associated with the measured attenuation coefficients. The symbol $\left\|.\right\|$ means the Euclidean norm. The solution of the linear inverse problem that represented by Equation (11) consists of finding a vector like $\hat{m}$ that minimize R of Equation (12) subjected to the constraint that the residual error function is $\delta^2 \leqq \hat{\delta}^2$,
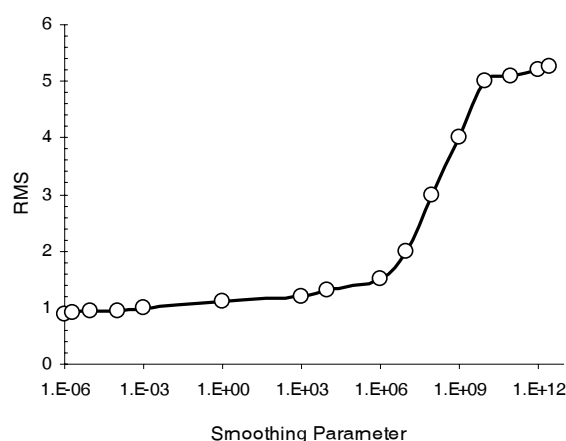
Fig. 6. RMS error versus smoothing parameter

where $\hat{\delta}^2$ is an acceptable error value.

The method of Lagrange multipliers is applied to solve this constrained minimization problem resulting as:

$$\hat{\mathbf{m}} = \left[\lambda\partial^{\mathrm{T}}\partial + (\mathbf{WG})^{\mathrm{T}}\mathbf{WG}\right]^{-1}(\mathbf{WG})^{\mathrm{T}}\mathbf{Wd} \qquad (15)$$

where λ is Lagrange multiplier that may be interpreted as a smoothing parameter, and ∂ is N×N matrix as follows:

$$\partial = \begin{bmatrix} 0 & \cdots & & \\ -1 & 1 & & 0 \\ \cdots & -1 & 1 & \\ & 0 & -1 & 1 \end{bmatrix} \qquad (16)$$

In the solution algorithm the Lagrange multiplier, is chosen subjectively so that the acceptable error is matched with a vector m, composed of non-negative shear damping elements only. This constraint is required as the material damping ratio cannot be negative value. According to the above algorithm, a computer program was developed to solve the Equation 11 in MATLAB workspace. This program used some MATLAB toolboxes facilities to obtain the shear damping ratio in each of the assumed layers by an inverse algorithm presented by Tarantola [12].

8. Results obtained at the site

The stratigraphy used for the inversion analysis at the Semnan University campus site was chosen based on geotechnical data from soil boring, standard penetration tests and in situ shear wave velocity measurements. The variation of the root-mean-square (RMS) error between the measured and estimated attenuation coefficients as a function of the smoothing parameter λ is shown in Figure 6. The value of λ selected is the smallest value that makes a solution vector composed of non-negative damping ratios as described above.

For this site, a value of 1.83×10^4 was selected resulting in an RMS error of 1.31. Although smaller values of λ result in smaller RMS error, the corresponding solutions are not physically reasonable i.e., they contain at least one negative value of damping ratio.

The values of damping ratio resulting from the analysis are shown in Table 2 along with other dynamic soil properties. Comparisons of the in situ shear damping ratio profile determined from continuous surface wave with cross hole test measurements is also plotted in Figure 7. The values range from

**Table 2** Dynamic soil properties at Semnan University campus site from continuous surface wave tests

Layer Number	Thickness (m)	Soil Classification	Shear Wave Velocity (m/s)	Mass Density (Mg/m^3)	Damping Ratio (%)
1	0.6	Top soil	150	1.71	2.8
2	7	SM	240	1.83	2.1
3	12	GM	360	1.94	1.5

approximately 1.5-2.8% for soils in the upper 20 m. As it was anticipated, the inversion algorithm yields damping ratios that vary smoothly with depth consistent with the choice of admissible λ. Rix et.al. [11] have shown the same trend for shear damping ratio profile determination using surface wave measurements in the Treasure Island National Geotechnical Experimentation Site (NGES), in the eastern portion of San Francisco Bay where independent laboratory and in situ values of damping ratio are available for comparison.

Table 3 shows the NGES dynamic soil properties with damping ratio resulting from the surface wave tests. Figure 8 compares the in situ shear damping ratio profile determined from surface wave method with cross hole tests results measured in the Treasure Island (NGES)

The differences between cross hole measurements and the continuous surface wave method can be at-tributed to these possible causes:
- Frequencies used in cross hole tests are usually on the order of several hundred hertz. The shorter wave lengths associated with these higher frequencies are more susceptible to apparent attenuation due to scattering.

- There are substantial differences in the volume of soil sampled by the two methods.
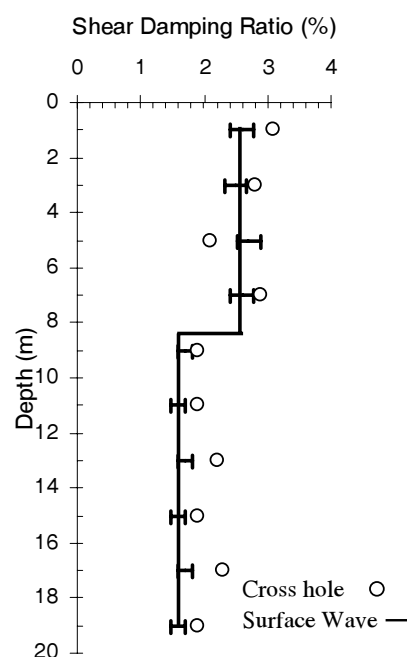

Fig. 7. Comparison of continuous surface wave shear damping ratio profile with cross hole test values at the site

The bore hole spacing in the cross hole measurements was approximately 3 to 4 m and continuous surface wave tests were performed with geophone offsets as large as 10 to 20m. Thus the cross hole measurements yield localized attenuation properties, and the continuous surface wave measurements yields properties that are averaged over a much larger volume of soil.

- At higher frequencies involved in cross hole tests, soil damping may be strongly affected by fluid flow losses in addition to frictional losses. These fluid losses may cause the

Table 3 Dynamic properties at Treasure Island NGES from surface wave tests [11]

Layer Number	Thickness (m)	Soil Classification	Shear Wave Velocity (m/s)	Mass Density (Mg/m^3)	Damping Ratio (%)
1	1.5	SM	145	1.70	0.68
2	1.5	SM	130	1.75	1.99
3	2.0	SP-SM	134	1.75	1.84
4	2.5	CL	145	1.75	1.05
5	3	SM	152	1.75	0.48
6	10	SM	160	1.75	0.30
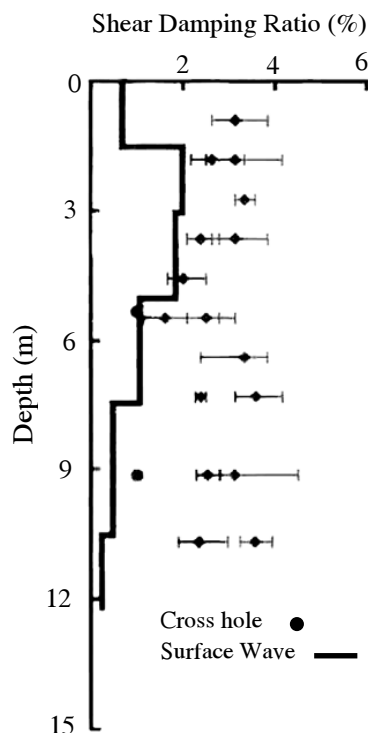
Fig.8. Comparison of surface wave damping values with independent cross hole measured values at Treasure Island NGES [11]

damping is increased and become frequency dependent at higher frequencies.

9. Conclusions

The in situ near surface soil damping ratio profile of a site in Semnan University campus has been determined using non-invasive continuous surface wave attenuation measurements. The test procedure entails determining Rayleigh wave attenuation coefficients as a function of frequency followed by an inversion analysis to determine the shear damping ratio profile. In this study a computer program in Matlab workspace was developed based on the inversion algorithm to obtain shear damping ratio in each layer.

The continuous surface wave method have several advantages over more conventional borehole methods like cross-hole including (1) the adverse effects of the presence of the borehole and poor receiver coupling are avoided; (2) depending on the source of ground vibration, frequencies used in surface wave testing can be much lower than borehole geophysical methods and thus closer to the frequencies encountered during dynamic loading of a site and (3) the non-invasive nature of surface wave measurements makes the test more versatile and economical.

The shear damping ratio values determined from continuous surface wave tests are less than those from cross hole measurements. Difference between the surface wave and cross hole values are attributable to several causes including (1) different amounts of apparent attenuation in the borehole and surface wave in situ measurements; (2)

different mechanisms that control attenuation at higher frequencies and frequency dependent damping ratios; and (3) different volumes of soil was subjected by tests. Finally, it is important to note that continuous surface wave methods should be considered as a complement method and not as a replacement for other methods of geotechnical site investigation

10. References

[1] Sharma, P.V. (1997) Environmental and Engineering Geophysics, Cambridge University Press, Cambridge, U.K., 475 pp.

[2] Ward, S. H. (1990) Geotechnical and Environmental Geophysics, Investigations in Geophysics No. 5, Society of Exploration Geophysics, 3 volumes.

[3] Bardet, J.P., Idriss I. M and D. O'Rourke (1995) The great Hanshin earthquake disaster, Preliminary investigation report, Civil Eng. Dept., USC, Feb. 10, pp.71

[4] EERI (2000) The Izmit (Turkey) Earthquake of A gust 17, 1999: A Reconnaissance Report, Earthquake Engineering Research Institute.

[5] Ishibashi, I and Zhang, X. (1993), Unified dynamic shear moduli and damping ratio of sand and clay, Soils and Foundations, Vol. 33, No:1, pp.182-191

[6] Toksoz, M.N. and Jhonston, D.H. (1981), Seismic wave attenuation, Geophysical Reprint Series No:2 , Society of Exploration Geophysicists, pp.1-5

[7] Kramer, S.L. (1996), Geotechnical Earthquake Engineering, Prentice-Hall Int. Series, 653 pp.

[8] Stewart, W.P. and Campanella, R.G.(1993), "Practical Aspects of In-Situ Measurements of Material Damping with the SCPT," Canadian. Geot. J., Vol. 30, No 2,pp. 211-219

[9] Mok, Y. J., I. Sanchez-Salinero, K. H. Stokoe, and J. Roesset (1988). In situ damping measurements by cross hole seismic methods, Earthquake. Eng. Soil Dyn. II- Recent Advances in Ground Motion Evaluation, Geotechnical Specialty Publication No. 20, pp. 305-320.

[10] Schneider, J.A., Hoyos, L., Jr., Mayne, P.W., Macari, E.J., and Rix, G.J. (1999), Field and laboratory measurements of dynamic shear modulus of Piedmont residual soils, Behavioral Characteristics of Residual Soils, GSP 92, ASCE, Reston, VA, pp. 12-25.

[11] Rix, G. J., C. G. Lai, and A.W. Spang, (2000), "In Situ Measurement of Damping Ratio Using Surface Waves," Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 126, Vol. 5, 472-480,.

[12] Tarantola, A. (1987), Inverse problem theory: Methods for data fitting and model parameter estimation, Elsevier Science Publication, 613p.