# Layout Optimization for X-bracing of Planar Steel Frames Using Ant System

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**Abstract:** *In this paper, the problem of layout optimization for X-bracing of steel frames is studied using the ant system (AS). A new design method is employed to share the gravity and the lateral loads between the main frame and the bracings according to the requirements of the IBC2006 code. An algorithm is developed which is called optimum steel designer (OSD). An optimization method based on an approximate analysis is also developed for layout optimization of braced frames. This method is called the approximate optimum steel designer (AOSD) and uses a simple deterministic optimization algorithm leading to the optimum patterns and it is much faster than the OSD. Several numerical examples are treated by the proposed methods. Efficiency and accuracy of the methods are then discussed. A comparison is also made with Genetic algorithm for one of the frames.*

## 1. Introduction

Bracing systems are well-known solutions for providing sufficient lateral strength and stiffness in steel frameworks; especially for high rise buildings where drifts due to vibration effects become more serious [1]. Due to many practical and architectural considerations, it is usually desirable to optimize not only the pattern (topology) but also the cost of these systems. Such a cost may include structural weight, fabrication, grouping and joint constructions. However, for a pre-determined joint-type, the former comprises of the main structural cost. Despite the role of other frame members, diagonal bracings and their design have major effect on the extra imposed loads to the frame members. These are particularly important for retrofit purposes.

Interaction of the effect of topology of the bracings with sizing of the structural members requires a simultaneous search of optimal topology and sizing, i.e. layout optimization. For practical purposes, the best layout should be chosen from a list of available sections which is in fact a search problem of discrete and combinatorial type. In structural problems, the high cardinality of such a search space together with the behavioral constraints on the resultant forces and deformations, make it a challenge for the designers to find even a suboptimal solution by trial and error approach. For such cases, efficient optimization algorithms such as stochastic search methods are the most suitable tools.

Complexity of such combinatorial discrete search has led the researchers to mimic some heuristic algorithms from natural processes. Ant Colony Optimization is a powerful approach in this class which is employed in the present work.

One of the most popular stochastic methods is the GA. Since 1980's when early works on GA were introduced [2,3], its application has been explored in optimal design of trusses and frames [4-11]. The GA works over a coded space of artificial chromosome genes whose replacement expands this generalized sampling to all regions of the search space via a direct genetic information share. This method is used here for comparison of the results with those of AS for one of the frames.

Another well-known heuristic used in engineering optimization is the AS. This method was first simulated in numerical methods in the pioneering work of Dorigo, Maniezzo and Colorni [12]. Since then, its behavior is studied in

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several optimization problems [13-23]. This method is principally inspired by the rules governing the behavior of the real ants in finding their roots. In this study the AS is chosen for optimization, due to the advantages it has in comparison to the GA.

In all the previous studies carried out on the optimal design of steel braced frames, no distinction had been made between the bracings and the essential frame members, and as a result the entire loads had to be carried by all elements. Though by considering the requirements of the building codes, bracings mostly should be designed for the lateral loads and frame elements have to carry the gravity loads. In order to apply this feature in the optimal design of structures, a method is presented in this paper. This method applies building code requirements in a way that the elements can be designed collectively. Results of this method are compared to those of the previously utilized approaches.

In the present paper, a method is developed for optimizing braced frames based on an approximate analysis approach. Having a relatively low degree of indeterminacy, one can use an approximate method for the analysis, without having all information about the material properties. As a result, if the search space is sufficiently small, then there will be a chance to calculate the fitness for all feasible results and find out the optimum one. Results of such an approximate method, so called AOSD, are compared to those of the AS based method that we call it OSD.

The methods are then applied to layout optimization of some practical X-braced steel frames to study the efficiency and accuracy of the proposed algorithms.

## 2. Formulation of the Optimization Problem

In a frame structure, once a connection type is selected, the fabrication cost has little effect on the optimum design, since it varies proportional to the structural weight [24]. Thus, it is a usual practice to seek the minimum weight design for a given loading state and boundary conditions. Here, the problem is formulated as follows:

Minimize:

$$w = \rho . \sum_{i=1}^{M} A_i . L_i \qquad (1)$$

Subjected to:

$$KU - P = 0 \qquad (2)$$

$$g_1 \geq 0, g_2 \geq 0, ..., g_n \geq 0$$

where $g_1$, $g_2$, ...,$g_n$ are the constraint functions depending on the element type used in each problem, and $K$, $U$ and $P$ are the stiffness matrix, nodal displacement and force vectors, respectively. In this study, the members should satisfy the following constraint on drift, deflection, compaction, strength and stability coefficients according to the Specification for Structural Steel Buildings [25], Minimum Design Loads for Buildings and Other Structures [26], International Building Code 2006 [27] and Seismic Provisions for Structural Steel Buildings [28]:

- Drift

$$Drift \leq 0.02 h_{sx} \qquad (3)$$

- Deflection

$$\begin{cases} \Delta_L < l/360 \\ \Delta_{D+L} < l/240 \end{cases} \qquad (4)$$

Considering the Table 1604.3 of IBC2006 (deflection limit) for steel structural members, the dead load should be taken as zero.

**- Compactness**

For SLRS members these limits are calculated according to the Table I-8-1 (limiting width-thickness ratios for compression elements) of Seismic Provisions for Structural Steel Buildings [28].

**- Strength**

These constrains are based on both AISC 360-05 specification [25] and Seismic Provisions for Structural Steel Buildings [28].

**- Stability**

$$\theta_{max} < \frac{0.5}{\beta C_d} \qquad (5)$$

**- Irregularity**

There is no horizontal irregularity, but vertical irregularity limits are taken into consideration according to the Table 12.3-2 (vertical structural irregularities) of the ASCE/SEI 7-05[26]. It should be noticed that vertical geometric irregularity has not been considered. Since this type of irregularity is supposed to exist when the horizontal dimension of the seismic force-resisting system in any story is more than 130% of that in an adjacent story. As a result, having equal bay sizes in the given structures, this type of irregularity may not let adjacent stories to have different number of bracings, so the feasible bracing placement may reduce considerably, thus it is ignored in this study.

**- Slenderness**

The AISC specification no longer provides a specific maximum slenderness ratio, as it formerly did. The AISC Commentary (E2) does indicate however, that if KL/r is >200 the critical stress will be less than 43.43 MPa. That value was based on engineering judgment, practical economies, and the fact that special care had to be taken to keep from injuring such a slender member during fabrication, shipping and erection. As a result of these important practical considerations, the engineer applying the 2005 AISC Specification will probably select compression members with slenderness values below 200 [29]. Thus this is also considered in the present study.

**- Tensioned column**

Columns placed on the foundation should not have tensile force.

Furthermore, such a constrained formulation is treated in an unconstrained form, using a penalized fitness function as

$$F = F_0 - w * (1 + K_p . V) \qquad (6)$$

$$V = \sum_{nLC} (\max(g^d, 0) + \max(g^s, 0)) \qquad (7)$$

$F_0$ is a constant taken as zero in the class of considered examples. $K_p$ is the penalty coefficient and $V$ denotes the total constraints' violation considering all *NLC* loading combinations.

Calculation of displacements, forces and stresses are based on the second-order elastic behavior of the structure using a finite element structural analysis routine and amplified first-order elastic analysis.

## 3. Representation of the Search Space

Most of the discrete search algorithms work with a coded search space rather than the original design space of the problem. Thus, any set of design variables defines a point in the design space called a *phenotype*, which is mapped to the corresponding point in the coded space known as a *genotype*. Such a mapping is defined by the employed method of encoding between genotypes and phenotypes.

In *Direct Index Coding* (DIC), a genotype is represented as a string of characters chosen from an artificial alphabet depending on the location of each character [30]. For discrete structural sizing problems, such an alphabet will be an index list to be mapped to the corresponding section list locations. Therefore, any such string of integer indices can be decoded to one and only one set of sections in order to define the corresponding model of structure. In this way, the one-to-one correspondence between the phenotypic and genotypic search spaces is preserved.

By DIC, representing a section for any structural member group requires only one index location in the corresponding genotype string. In addition, the alphabet size for this is exactly the same as the number of available sections for the corresponding member group. Hence, this method of encoding will limit the cardinality of genotypic search space to its minimal required value. This is an advantage of DIC over the other representation schemes like the binary coding which imposes larger genotypic spaces for the same phenotype space [31].

### 3.1. Simultaneous topology and size assessment for discrete structural optimization

Topology of a skeletal structure defines the connectivity pattern of its elements between the set of its joint nodes. When such a set of nodes is fixed, the design space of various possible

topologies for that structure can be defined using a prototype pattern called the *proto-morph structure* [32]. In this way, any structural topology will be a subset of possible combinations of patterns out of the member connectivity of the proto-morph. When each node of the structure is connected to all the other nodes, such a proto-morph is called ground *structure*.

Selection of cross-section profiles for any member group in the sizing optimization depends on the chosen topology as connectivity information for that structural model. Therefore, true optimal structural layout is merely found by simultaneous variation of topology and sizing of the structure. Choosing a small section for a member instead of its explicit elimination during the topology optimization is a challenging practice because it not only enforce a degree of numerical error but also may not truly represent the structural stability of the model.

Using DIC, member elimination during genotypic search can be explicitly addressed by a special index, say zero-index, in addition to the indices corresponding to the available section list for any member group. Since there is no extra-imposed preference in the selection of such a zero index with respect to the other section indices, the discrete search will be more efficient and reliable [32].

## 4. Specialization for the Ant Colony Optimization

AS can be considered a class of distributed short-term memory stochastic search over discrete genotypic spaces. Its popular notation is derived and applied to the *Traveling Salesman Problem* (TSP), which is analogous to the simulated natural process of real ants in root finding between their nest and food locations. However, such definitions can be specialized for other (structural) problems preserving the principles of the AS search. The main steps involved in an AS are as follows:

- Representation of any phenotype model to a tour conformed by its states
- Initiation
  o Defining matrix as well as algorithm

control parameters
  o Initiation of pheromone trail matrix
- Solution sorting based on decoded tour and evaluated tour length (or fitness)
- Discovering new tours by *NumAnts* number of artificial agents using *state transition rule*
- Convergence check

Here, the definition and representation of the tours can be generalized. For the structural layout optimization, the *generalized tour* is defined as a string of characters such that the position number of any character corresponds to a state of type 1, while the character value will be filled with a number addressing a state of type 2. The state transition will assign a state of type 2 to any state of type 1. In other word, it will choose an index to fill the corresponding character of that string. As soon as all the string's characters are assigned indices, the generalized tour is completed. The indices address which structural cross-section to be assigned to the corresponding member group.

The tour length for structural weight minimization is defined as the reverse of the fitness value (or penalized weight) for each design phenotype.

In order to enable the search to escape from local optima traps, the pheromone trail is updated using a pheromone deposition and evaporation procedure depending on the chosen variant of AS algorithms.

### 4.1. The utilized Ant Colony Algorithm

Several ant colony algorithms have already been developed for TSP including *Ant System* or AS *Elitist AS, Ranked AS, Min-Max AS* and *Ant Colony System* [12]. For structural sizing problems, Camp et al. [15] have adopted a variant of $AS_{rank}$ which is also employed in the present work.

Let $\tau_{ij}(t)$ be the intensity of the trail on edge (i,j) at iteration t. The intensity of trail at iteration 0, $\tau_{ij}(0)$, is set to a small positive constant $\tau_0$. Each ant at iteration t chooses the next state, where it will be at iteration t+1. The trail evaporation is performed on each edge as soon as it is passed by an ant using the following

relationship:

$$\tau_{ij}(t) = (1-\varphi).\tau_{ij}(t) + \varphi\tau_0 \qquad (8)$$

However, the pheromone deposition is limited to a number of best ranked ants during the local update loop as given by

$$\tau_{ij}(t+n) = (1-\rho).\tau_{ij}(t) + \lambda.\Delta\tau_{ij}^{+}(t) + \Delta\tau_{ij}^{r}(t) \qquad (9)$$

The attractiveness values, $v_{ij}$, are not modified during optimization run. $v_{ij} = 1/d_{ij}$ where $d_{ij}$ is defined as the *i*-th member-group weight using section addressed by the *j*-th index. For the special case of zero index that denotes topological elimination, $v_{ij}$ is taken a large number called $vm$ instead of infinity.

The state transition rule is defined using a roulette wheel lemma with the following probability:

$$p_{ij}^{k}(t) = \frac{[\tau_{ij}(t)]^{\alpha}.[v_{ij}]^{\beta}}{\sum\limits_{l \in allowed}^{n}[\tau_{ij}(t)]^{\alpha}.[v_{ij}]^{\beta}} \qquad (10)$$

Where "*allowed*" is a set of neighboring 2nd type states available from 1st type state i, and $\alpha$ and $\beta$ are parameters to control the relative importance of trail versus attractiveness.

## 5. Design Methodologies

The design methods used in this study are as follows:

1. Simultaneous design of the frame and bracing elements
2. Simultaneous design of the structure for all loads, and the frame for gravity loads

These methods are utilized in design of some building frame systems. Definition, advantages and disadvantages of these methods are explained in this section.

### 5.1. Method 1: Simultaneous design of frame and bracing elements

In optimal design of steel braced frames that has been performed to date, frame elements and bracing elements have been optimized simultaneously. Using this method one can determine the size of all elements and the placement of bracing simultaneously. However, dealing with the problem in such a way may lead to some undesirable results. The most important behavioral problem of this method is exchanging of the responsibility of the members. In such a design all the elements carry the entire loads together. Thus it is possible for some bracing members to carry the gravity load that is devoted to frame members, and it is possible for some columns to carry lateral load which is devoted to bracing members. The main reason of such an unpleasant behavior is that when one optimizes elements collectively, each member of the structure will be able to carry the loads according to its stiffness; as a result, there will be no difference between different types of elements.

### 5.2. Method 2: Simultaneous design of structure for the entire loads, and frame for gravity loads

Most of the building codes such as ASCE have some requirements to share loads between members in a reasonable manner. As an example, ASCE 7-05 defines building frame system as following:

*Building frame system*: A structural system with an essentially complete space frame providing support for gravity loads. Resistance to lateral load is provided by shear walls or braced frames.

Considering these requirements, one is not permitted to design all members simultaneously. The method presented in this study so called "*Simultaneous design of structure for all loads and frame for gravity loads*" helps to a better satisfaction of the building code requirements. Requirements of the essentially complete frame and the entire structure are provided at the same time by the use of this method. In the other words, analysis outputs are achieved in two different steps, one after formation of the essential frame and one for the whole structure. After each step, the requirements of the building code are checked.

## 6. Approximate Optimum Steel Design (AOSD)

Material and sectional properties of the members of an indeterminate structure should be considered to perform an exact structural analysis, but there are some approximate methods that help us to perform analysis without having such information about the members when the degree of indeterminacy is law and also when some kind of symmetry reduces the degree of indeterminacy. In this study, an approximate analysis is used to achieve member forces in different bracing-patterns, and hence an estimate of their weight is achieved. In this method, minimizing the weight of structure is taken as a measure of the fitness.

The following algorithm is used for the AOSD:

1.  Section list of both bracings and columns are sorted according to the cross-section (axial) area in an ascending order.
2.  Compression strength of each member in the section lists of the columns and bracings are calculated.
3.  For all bracing and column members:
    3.1. Axial force is calculated using an approximate method.
    3.2. Starting from the first member in the section list, which is also the lightest one, members are checked to have compression strength more than the calculated axial force of step 3.1. The first section that fulfills the criterion is selected for the member.
4.  For columns located on the base if there is tension in the member, a penalty is chosen for the structure. The penalty used in this study is imposed by adding a considerably big weight to the structure to reduce the fitness. Added load is chosen in a way that all penalized structures are placed after non-penalized ones in the ranking of the optimization.
5.  For each structure all the floors are checked to have at least one bracing, if there is no bracing in one of them, the structure will be penalized by an additional load. Additional load is chosen in a way that all the penalized structures are placed after non-penalized ones in the ranking of optimization.

In this algorithm, member sections are chosen according to their compression strength. This is because the compression is the governing behavior of both columns and bracing members which are optimized here. It should also be noted that when more complicated structures such as dual systems are being treated, it would not be much easy to choose a structural behavior as simple as it is selected here. However, for more complicated structures, it will not be easy to find the governing behavior of the members and the entire structure.

**Approximate analysis:** The approximate analysis used in this study is based on the following two assumptions:

1.  Distribution of the lateral forces in columns and bracings is obtained using the force equilibrium in different joints, considering all the members to act as a simply connected truss member.
2.  Distribution of the gravity loads in the columns is determined according to the load carrying span.

Steps of this algorithm are shown in Figure 1. In this figure we have

$$a = \frac{C_s W}{\sum_{i=1}^{3} i} \tag{11}$$

Where:
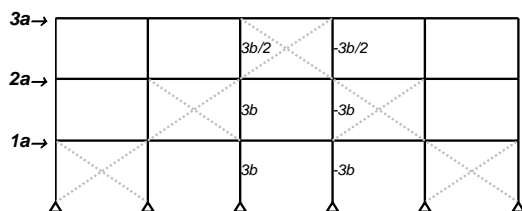$C_S$= The seismic response coeficient
$W$= The effective seismic weight

$$b = a.\tan(\theta) \tag{12}$$

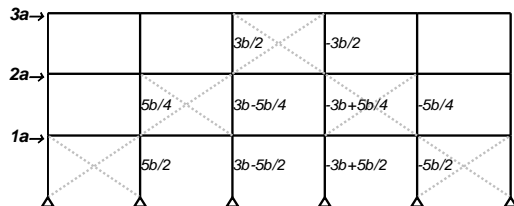$\theta$ = The story height to the span length ratio

$$c = \frac{G_L * L_y * L_x}{2} \tag{13}$$

Where: $G_L$ = Gravity distributed load according to the load combination (N/m²); $L_{y} =$ Frame width (m); $L_x$ = Span length (m).
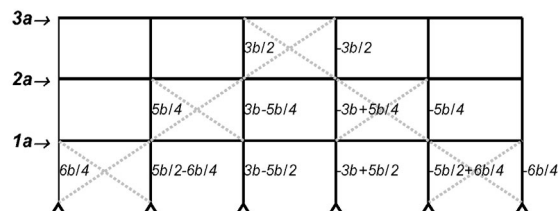
**Step 1: Column forces due to the 3rd floor bracing forces**

3a→
2a→
1a→

3b/2   -3b/2
3b     -3b
3b     -3b

**Step 3: Column forces due to the 3rd & 2nd & 1st floor bracing forces**

3a→
2a→
1a→

3b/2   -3b/2
5b/4   3b-5b/4   -3b+5b/4   -5b/4
6b/4   5b/2-6b/4   3b-5b/2   -3b+5b/2   -5b/2+6b/4   -6b/4

**Step 2: Column forces due to the 3rd & 2nd floor bracing forces**

3a→
2a→
1a→

3b/2   -3b/2
5b/4   3b-5b/4   -3b+5b/4   -5b/4
5b/2   3b-5b/2   -3b+5b/2   -5b/2

**Step 4: Column forces due to gravity loads**

1c   2c   2c   2c   2c   1c
2c   4c   4c   4c   4c   2c
3c   6c   6c   6c   6c   3c
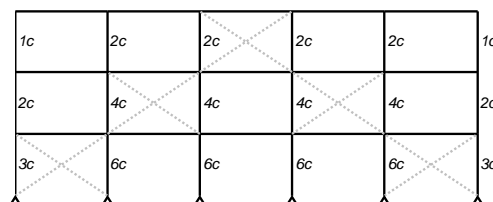
**Fig. 1.** Steps of the approximate analysis method used in this study

## 7. Numerical Examples

### 7.1. Properties of the examples

The following features are common in all the examples:

**Geometry:**

Height of each floor = 3m
Width of the frame = 5.5m
Three degrees of freedom for each joint (x, y translations and z rotation)
All the connections and also the supports are considered as pinned.
Loading condition:

1. Uniformly distributed dead load of 6.3kN/m² in the negative y-direction on all the beam elements
2. Uniformly distributed live load of 1.96kN/m² in the negative y-direction on all the beam elements
3. Earthquake concentrated loads are calculated according to the ASCE 7-05 [26], according to the following parameters:
   R=6, I=1, $S_S$ = 2.29 , $S_I$ = 0.869
   Seismic design category = E
   Earthquake loads acting on the given examples are shown in Table 1.

4. Wind loads are calculated according to the ASCE 7-05 [26], by the following parameters:

1. $\begin{cases} V = 56(m/s) \\ K_d = 0.85 \end{cases}$

2. $\begin{cases} Category = \Pi \\ I = 1 \end{cases}$

3. $\begin{cases} Exposure \ \ category = B \\ Exposure \ \ B \rightarrow \alpha = 7.0 \ \ z_g = 365.76(m) \rightarrow \begin{cases} z < 4.6m \ \ K_z = 2.01(\frac{4.6}{z_g})^{2/\alpha} \\ z < 4.6m \ \ K_z = 2.01(\frac{z}{z_g})^{2/\alpha} \end{cases} \\ K_h = 0.8 \end{cases}$

4. $K_{zt} = 1.0$

5. $G = 0.85$

6. $enclosure \ \ classification \rightarrow Enclosed$

7. $GC_{pi} = \pm 0.18$

8. $\begin{cases} for \ \ walls \begin{cases} Windward \rightarrow C_p = 0.8 \\ Leeward \rightarrow C_p = -0.5^* \end{cases} \\ for \ \ roofs \rightarrow C_p = -0.9, -1.8 \end{cases}$

As the total base shear corresponding to the wind load of the above data is not considerable, in comparison to the earthquake load, thus the wind load does not govern the design.

**Material Properties**

The 50ksi steels are the predominant ones in use today. In fact some of the steel mills charge

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**Table 1.** Earthquake load acting on different frames in the numerical examples

| Floor | Earthquake loads (kN) | | | |
|---|---|---|---|---|
| | Frames 1 & 2 & 3 (6m-span) | | Frame 4 (8m-span) | |
| | 3 story | 5 story | 3 story | 5 story |
| 1th | 173.83 | 115.89 | 231.78 | 154.52 |
| 2th | 347.66 | 231.78 | 463.55 | 309.03 |
| 3th | 521.5 | 347.66 | 695.33 | 463.55 |
| 4th | | 463.55 | | 618.07 |
| 5th | | 579.44 | | 772.59 |
| **Base shear** | **1042.99** | **1738.32** | **1390.66** | **2317.76** |

extra for W-sections if they are to consist of A36 steel. On the other hand, A992 and A500 are preferred material for W-shapes and HSS Rect. Respectively [29]. Data are selected for the members, according to the Table 2, and the following material properties:

$E = 2e8(kN/m^2)$, $\rho = 76.82(kN/m^3)$, and $\nu = 0.3$

### Section List

A designer often selects steel sections of the sizes which are among the rolled sections. Steel beams and bars and plates of unusual sizes will be difficult to obtain during boom periods and will be expensive during any period [29]. Therefore, in this study rolled sections are utilized.

For beams and columns, W-shape sections between W8x10 and W16x89 and for bracing members HSS Rect. sections are specified. Also the following algorithm is used to reduce the size of the section list of bracings:

1. Sections are checked for slenderness, and compaction limits. Members which do not fall in the feasible region are omitted.

2. Considering $A_i$ as the cross-section area of the $i^{th}$ member in the list, and $n$ as the number of sections in the section list, the following steps are taken to reduce the size of the search space in order to increase the efficiency of the optimization process:

2.1. All members that satisfy $\frac{A_n - A_i}{A} < 0.1$, $j=1:n$, are put in one group.

2.2. Section lists are classified by repeating step 2.1, considering the members of the last group to be omitted.

2.3. A new section list is created by substituting all the members of each group by the best of them in carrying compression loads.

### 7.2. Members under optimization

In this study, only the column and bracing members are determined using optimization

**Table 2.** Section types selected for the numerical examples

| Member type | Shape | ASTM designation | $F_y$(MPa) | $F_u$(MPa) |
|---|---|---|---|---|
| Column | W | A992 | 344.70 | 448.20 |
| Beam | W | A992 | 344.70 | 448.20 |
| Bracing | HSS Rect. | A500 | 317.20 | 399.90 |

**Table 3.** Definition of different frames used in the numerical examples

|  | Design method | Width of the bays (m) | Effective length factor of bracing |
|---|---|---|---|
| **Frame 1** | 1 | 6 | 1 |
| **Frame 2** | 2 | 6 | 1 |
| **Frame 3** | 2 | 6 | 0.5 |
| **Frame 4** | 2 | 8 | 1 |

methods. This is because the forces in the beam members do not alter by changes in the bracing patterns, hence can be designed independently. Thus the sizes of these members are kept fixed during the optimization process.

### 7.2.1. Example; Group 1

In this group of examples, four 3-story frames are investigated. All the frames have 5 bays but they differ in the design method, span length, and the effective length factor of bracings. Properties of the frames are depicted in Table 3.

Results are shown in Figures 2-5. In these figures there are 2 lines of description at the top of each frame. The first line contains the rank of the pattern using the OSD, and the second line consists of the ranking of AOSD.

1st line: Rank of the pattern using the OSD
2nd line: Rank of the pattern using the AOSD.

Details of the patterns of Figures 2-5 are provided in the Tables 4-7. In these tables and also in Tables 8-12, the following notations are employed:
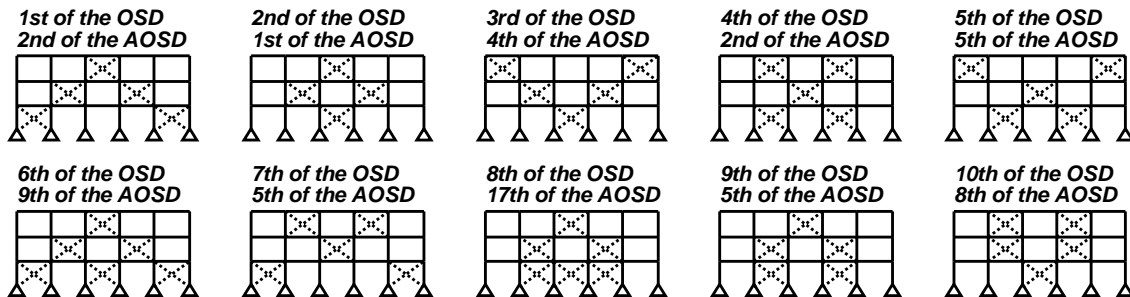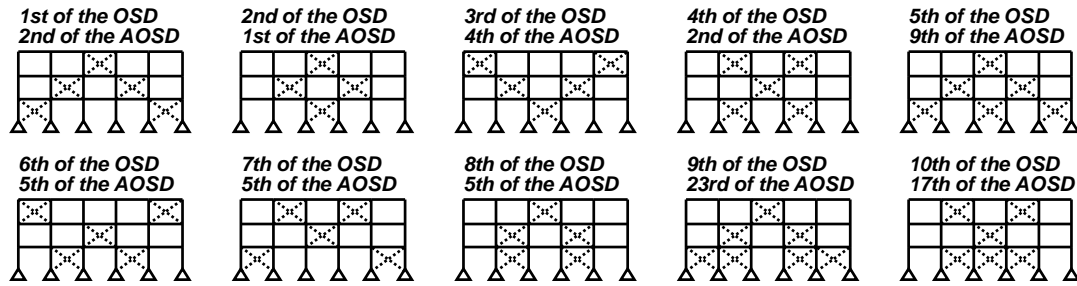


**Fig. 2.** Best results of Frame 1 (3-story structure) using AS

**Table 4.** Best results of the 3-story Frame 1 using AS (Figure 2)

| OSD Rank | AOSD Rank | AOSD Rank 2[1] | Number of Bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | DW[2] | $DW_{Best}$[3] | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 5 | 83.79 | 83.43 | −0.40 | 0.00 | 40.69 | 27.55 | 27.43 | 15.55 | 15.48 |
| 2 | 1 | 1 | 4 | 84.55 | 83.12 | −1.70 | 0.09 | 40.69 | 29.95 | 29.83 | 13.90 | 12.77 |
| 3 | 4 | 3 | 5 | 85.25 | 83.82 | −1.70 | 0.17 | 40.69 | 27.55 | 27.43 | 17.01 | 15.87 |
| 4 | 2 | 1 | 5 | 87.66 | 83.43 | −4.80 | 0.46 | 40.69 | 29.95 | 27.43 | 17.01 | 15.48 |
| 5 | 5 | 4 | 5 | 87.66 | 85.83 | −2.10 | 0.46 | 40.69 | 29.95 | 29.83 | 17.01 | 15.48 |
| 6 | 9 | 1 | 6 | 88.36 | 86.53 | −2.10 | 0.55 | 40.69 | 27.55 | 27.43 | 20.12 | 18.58 |
| 7 | 5 | 4 | 5 | 88.43 | 85.83 | −2.90 | 0.55 | 40.69 | 29.95 | 29.83 | 17.78 | 15.48 |
| 8 | 17 | 9 | 6 | 89.69 | 88.93 | −0.90 | 0.70 | 40.69 | 29.95 | 29.83 | 19.04 | 18.58 |
| 9 | 5 | 4 | 5 | 91.28 | 85.83 | −6.00 | 0.89 | 40.69 | 35.03 | 29.83 | 15.55 | 15.48 |
| 10 | 8 | 7 | 5 | 92.74 | 86.21 | −7.00 | 1.07 | 40.69 | 35.03 | 29.83 | 17.01 | 15.87 |
| **Mean** | **5.8** | **3.5** | **5.1** | **87.94** | **85.30** | **−2.96** | **0.49** | **40.69** | **30.25** | **28.87** | **17.00** | **15.91** |

**Fig.3.** Best results of Frame 2 (3-story structure) using AS

**Table 5.** Best results of the 3-story Frame 2 using AS (Figure 3)

| OSD rank | AOSD rank | AOSD rank 2[1] | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | DW[2] | $DW_{Best}$[3] | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 5 | 83.79 | 83.43 | (0.40 | 0.00 | 40.69 | 27.55 | 27.43 | 15.55 | 15.48 |
| 2 | 1 | 1 | 4 | 84.55 | 83.12 | (1.70 | 0.09 | 40.69 | 29.95 | 29.83 | 13.90 | 12.77 |
| 3 | 4 | 3 | 5 | 85.71 | 83.82 | (2.20 | 0.23 | 40.69 | 27.55 | 27.43 | 17.47 | 15.87 |
| 4 | 2 | 1 | 5 | 87.66 | 83.43 | (4.80 | 0.46 | 40.69 | 29.95 | 27.43 | 17.01 | 15.48 |
| 5 | 9 | 1 | 6 | 87.67 | 86.53 | (1.30 | 0.46 | 40.69 | 27.55 | 27.43 | 19.43 | 18.58 |
| 6 | 5 | 4 | 5 | 88.12 | 85.83 | (2.60 | 0.52 | 40.69 | 29.95 | 29.83 | 17.47 | 15.48 |
| 7 | 5 | 4 | 5 | 88.43 | 85.83 | (2.90 | 0.55 | 40.69 | 29.95 | 29.83 | 17.78 | 15.48 |
| 8 | 5 | 4 | 5 | 91.28 | 85.83 | (6.00 | 0.89 | 40.69 | 35.03 | 29.83 | 15.55 | 15.48 |
| 9 | 23 | 1 | 7 | 91.55 | 89.63 | (2.10 | 0.93 | 40.69 | 27.55 | 27.43 | 23.31 | 21.68 |
| 10 | 17 | 9 | 6 | 91.75 | 88.93 | (3.10 | 0.95 | 40.69 | 27.55 | 29.83 | 23.51 | 18.58 |
| Mean | 7.3 | 2.9 | 5.3 | 88.05 | 85.64 | (2.71 | 0.51 | 40.69 | 29.26 | 28.63 | 18.10 | 16.49 |

- **$W_{OSD}$** defines the weight of the structure using the OSD,
- **$W_{AOSD}$** defines the weight of the structure using the AOSD. The AOSD rank 2 is the rank of the pattern in comparison to the patterns of the same number of bracings,

- **DW** is $\dfrac{W_{AOSD} - W_{OSD}}{W_{OSD}}$ ,

- **$DW_{Best}$** is $\dfrac{W_{OSD} - W_{OSD,Best}}{W_{OSD}}$ , in which $W_{OSD,Best}$ is the $W_{OSD}$ of the first row.

- **$W_{beam}$** is weight of the beams of structure.
- **$W_{column-OSD}$** and **$W_{Bracing-OSD}$** are weights of the columns and bracings of the structure using OSD, respectively.
- **$W_{column-AOSD}$** and **$W_{Bracing-AOSD}$** are weights of the columns and bracings of the structure using AOSD, respectively.

### 7.2.2. Example; Group 2

Frames of this group are the same as those of group 1. The only difference is in the number of stories. Here, 5 story frames are studied. Results are shown in Figures 6-9. Details of the patterns of Figures 6-9 are in the Tables 8-11.

### 7.2.3. Control example; GA based optimization

In this example, the most complicated problem of the example groups 1 and 2, which is 5 story frame designed by method 1 is optimized using genetic algorithm. Results are shown in Figure 10 and table 12. Figure 11 is also added to illustrate the convergence rate of the GA in comparison to the AS.

## 8. Discussion

### 8.1. Comparison of the results of methods 1 and 2

In order to compare the results of the previous design methods with those of the present one, Frames 1 and 2 are compared in different groups of examples. The results of these two frames are illustrated in Figures 2, 3, 6 and 7 and also in
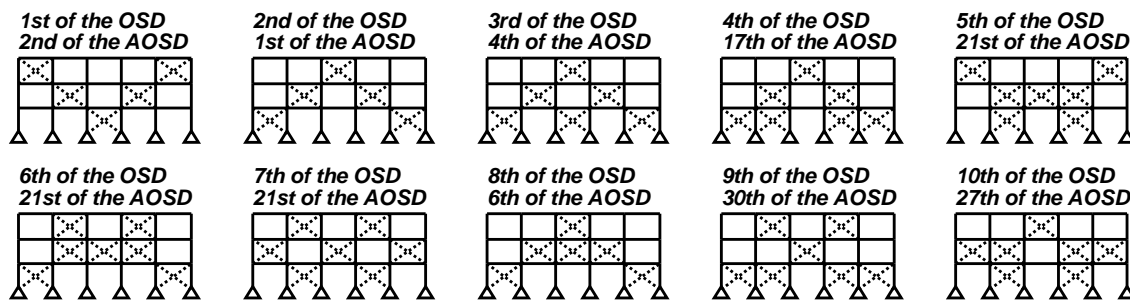
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**Fig. 4.** Best results of Frame 3 (3-story structure) using AS

**Table 6.** Best results of the 3-story Frame 3 using AS (Figure 4)

| OSD rank | AOSD rank | AOSD rank 2[1] | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | $DW^2$ | $DW_{Best}^3$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 5 | 76.86 | 79.13 | 3.00 | 0.00 | 40.69 | 27.55 | 27.43 | 8.62 | 11.18 |
| 2 | 1 | 1 | 5 | 76.97 | 78.49 | 2.00 | 0.01 | 40.69 | 27.55 | 27.43 | 8.73 | 10.54 |
| 3 | 4 | 1 | 6 | 77.24 | 79.81 | 3.30 | 0.05 | 40.69 | 27.55 | 27.43 | 9.00 | 11.86 |
| 4 | 17 | 3 | 7 | 77.65 | 81.73 | 5.30 | 0.11 | 40.69 | 27.55 | 27.43 | 9.41 | 13.78 |
| 5 | 21 | 5 | 7 | 77.79 | 82.04 | 5.50 | 0.12 | 40.69 | 27.55 | 27.43 | 9.55 | 14.09 |
| 6 | 21 | 5 | 7 | 77.91 | 82.04 | 5.30 | 0.14 | 40.69 | 27.55 | 27.43 | 9.67 | 14.09 |
| 7 | 21 | 5 | 7 | 77.91 | 82.04 | 5.30 | 0.14 | 40.69 | 27.55 | 27.43 | 9.67 | 14.09 |
| 8 | 6 | 2 | 6 | 78.23 | 80.42 | 2.80 | 0.18 | 40.69 | 27.55 | 27.43 | 9.99 | 12.47 |
| 9 | 30 | 11 | 7 | 78.26 | 82.60 | 5.50 | 0.18 | 40.69 | 27.55 | 27.43 | 10.02 | 14.65 |
| 10 | 27 | 10 | 7 | 78.27 | 82.34 | 5.20 | 0.18 | 40.69 | 27.55 | 27.43 | 10.03 | 14.39 |
| **Mean** | **15** | **4.5** | **6.4** | **77.71** | **81.06** | **4.32** | **0.11** | **40.69** | **27.55** | **27.43** | **9.47** | **13.11** |

Tables 4, 5, 8 and 9. In these tables the following notations are adopted:

1. **AOSD rank 2** is the rank of the pattern in comparison to the patterns with the same number of bracings.

2. **DW** is $(W_{AOSD} - W_{OSD})/W_{OSD}$

3. **DW$_{Best}$** is $(W_{OSD} - W_{OSD, Best})/W_{OSD}$ , in which $W_{OSD, Best}$ is the $W_{OSD}$ of the first row.

As it can be seen, for these two groups of examples we have:

**Group 1:** Results of a 3-story frame which is designed by method 1 (Frame 1) achieved by OSD varies between 83.79 and 92.74, while for the Frame 2 designed by method 2, the results vary between 83.79 and 91.75.

**Group 2:** Results of a 5-story frame which is designed by method 1 (Frame 1) are achieved by OSD vary between 154.45 and 158.89, while for Frame 2 designed by method 2, these results vary from 154.45 to 158.49.

One may expect that using method 1 for design, the optimum patterns to have closer weights in comparison to method 2, because method 2 is the same as method 1 with some additional limitations. Thus all the results of method 2 can also be considered as an acceptable set of results for method 1. However, this is not true, because in AS optimization problems the size of the search space have considerable effect on the results. As the size of the search space reduces, the accuracy of the results increase. Thus it can be seen that although method 2 has additional limitations, however, the best results of this method are better than those of method 1.

**8.2. Comparison of the Results of the OSD and AOSD**

*8.2.1. Comparison of the Results for Frames with Different Span Length*

As it can be seen from Table 13 which contains the summary of the results presented in Tables 4 to 11, for both groups of the examples, the
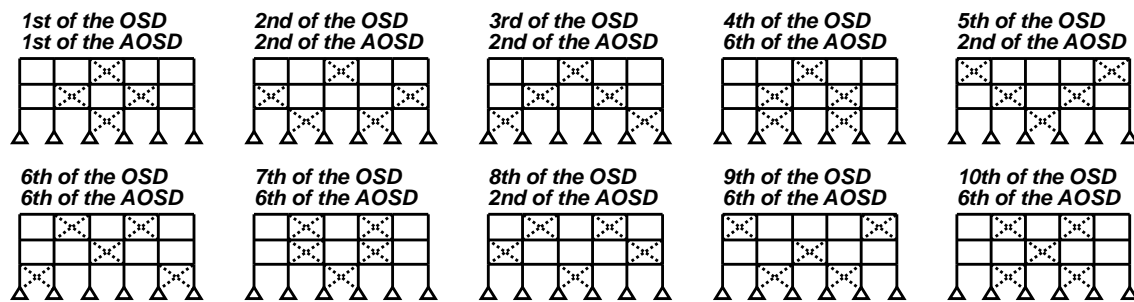
**Fig. 5.** Best results of Frame 4 (3-story structure) using AS

**Table 7.** Best results of the 3-story Frame 4 using AS (Figure 5)

| OSD rank | AOSD rank | AOSD rank 2[1] | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | $DW^2$ | $DW_{Best}{}^3$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 4 | 151.46 | 147.43 | −2.70 | 0.00 | 87.36 | 35.03 | 34.89 | 29.06 | 25.55 |
| 2 | 2 | 1 | 5 | 154.47 | 150.36 | −2.70 | 0.20 | 87.36 | 35.03 | 31.42 | 32.07 | 31.94 |
| 3 | 2 | 1 | 5 | 154.47 | 150.36 | −2.70 | 0.20 | 87.36 | 35.03 | 31.42 | 32.07 | 31.94 |
| 4 | 6 | 5 | 5 | 154.47 | 153.82 | −0.40 | 0.20 | 87.36 | 35.03 | 34.89 | 32.07 | 31.94 |
| 5 | 2 | 1 | 5 | 156.45 | 150.36 | −3.90 | 0.33 | 87.36 | 29.95 | 31.42 | 39.14 | 31.94 |
| 6 | 6 | 5 | 5 | 157.87 | 153.82 | −2.60 | 0.42 | 87.36 | 35.03 | 34.89 | 35.47 | 31.94 |
| 7 | 6 | 5 | 5 | 157.87 | 153.82 | −2.60 | 0.42 | 87.36 | 35.03 | 34.89 | 35.47 | 31.94 |
| 8 | 2 | 1 | 5 | 157.87 | 150.36 | −4.80 | 0.42 | 87.36 | 35.03 | 31.42 | 35.47 | 31.94 |
| 9 | 6 | 5 | 5 | 157.87 | 153.82 | −2.60 | 0.42 | 87.36 | 35.03 | 34.89 | 35.47 | 31.94 |
| 10 | 6 | 5 | 5 | 159.48 | 153.82 | −3.50 | 0.53 | 87.36 | 35.03 | 34.89 | 37.08 | 31.94 |
| **Mean** | **3.9** | **3** | **4.9** | **156.23** | **151.80** | **−2.85** | **0.31** | **87.36** | **34.53** | **33.50** | **34.34** | **31.30** |

number of bracings in the optimum solution reduces by an increase in the bay length. This may look wrong in the first sight, however, it will become reasonable if we take into consideration the exact behavior of the bracings. In other words, when the span length of a frame is small, many available steel profiles may satisfy the requirements of slenderness, thus the number of bracing and their size should be calculated according to the lateral loads acting on them, but when the length of the bays is increased, only some heavy profiles may satisfy the requirements of slenderness. These heavy members often satisfy the allowable strength requirements, and therefore we may not need additional bracings. As a result the number of bracings may be reduced in the frame.

### 8.2.2. Comparison of the Results for Frames with Different Bracing Member Slenderness

If the bracing slenderness of all the frames designed by method 2 are sorted, it can be observed that Frame 4, has the largest slenderness. Frame 2 gains the next rank, and Frame 3 becomes the last. As it can be seen from Table 11, in both groups of examples the mean value of the AOSD Rank is reduced by an increase in the slenderness of bracing. This is because the accuracy of the approximate analysis used in this study reduces when the number of bracings in a floor is more than two and also as it is discussed in subsection 8.2.1, the number of bracings decreases by an increase in the slenderness. In the cases where we have three or more bracings in a story, the forces are not shared equally between them because although roofs are not flexible and horizontal displacements of the points in a floor are the same, the vertical displacement differ in different bays. This can also be observed in the following figures:

Figure 2: The result 8 that contains (a) floor with three bracings achieves higher rank in comparison to the others.
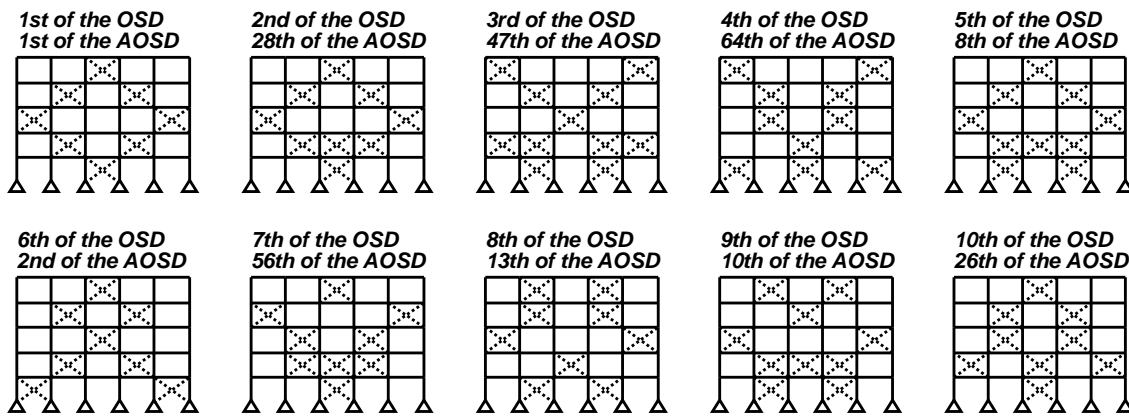
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**Fig. 6.** Best results of Frame 1 (5-story structure) using AS

**Table 8.** Best results of the 5-story Frame 1 using AS (Figure 6)

| OSD rank | AOSD rank | AOSD rank 2[1] | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | $DW^2$ | $DW_{Best}^3$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 8 | 154.45 | 143.16 | −7.30 | 0.00 | 67.82 | 53.47 | 55.68 | 33.16 | 26.69 |
| 2 | 28 | 8 | 9 | 155.79 | 149.98 | −3.70 | 0.09 | 67.82 | 52.61 | 59.41 | 35.35 | 29.79 |
| 3 | 47 | 3 | 11 | 156.02 | 151.52 | −2.90 | 0.10 | 67.82 | 49.14 | 55.68 | 39.06 | 35.05 |
| 4 | 64 | 31 | 10 | 156.44 | 152.35 | −2.60 | 0.13 | 67.82 | 52.61 | 59.15 | 36.00 | 32.42 |
| 5 | 8 | 1 | 10 | 156.88 | 147.43 | −6.00 | 0.17 | 67.82 | 51.28 | 55.68 | 37.78 | 30.96 |
| 6 | 2 | 2 | 8 | 157.20 | 144.36 | −8.20 | 0.18 | 67.82 | 55.29 | 57.82 | 34.09 | 25.76 |
| 7 | 56 | 12 | 9 | 157.82 | 151.90 | −3.70 | 0.22 | 67.82 | 57.41 | 62.42 | 32.59 | 29.79 |
| 8 | 13 | 3 | 9 | 158.56 | 147.98 | -6.70 | 0.27 | 67.82 | 53.40 | 58.96 | 37.34 | 29.32 |
| 9 | 10 | 2 | 10 | 158.79 | 147.82 | -6.90 | 0.28 | 67.82 | 50.79 | 55.68 | 40.17 | 31.35 |
| 10 | 26 | 7 | 9 | 158.89 | 149.72 | -5.80 | 0.29 | 67.82 | 57.48 | 59.15 | 33.59 | 29.79 |
| **Mean** | **25.5** | **7** | **9.3** | **157.08** | **148.62** | **-5.38** | **0.17** | **67.82** | **53.35** | **57.96** | **35.91** | **30.09** |

Figure 3: The result 9 which contains (a) floor with three bracings achieves higher rank in comparison to the others.

Figure 4: The results 4, 5, 6, 7, 9 and 10 all have more than two bracings in each story and none of them have achieved a very good result in the AOSD.

Figure 5: There is no result with more than two bracings, thus all the results also achieved good rank in the AOSD.

Figure 6: The results 2, 3, 4, 7 and 10 all have more than two bracings in each story and none of them have achieved a very good result in the AOSD.

Figure 7: The results 2, 3, 4, 5, 6, 9 and 10 all have more than two bracings in each story and none of them have achieved much good results in the AOSD.

Figure 8: The results 5 and 6 both have more than two bracings in each floor and both achieved not only bad rank but also a penalty for having base columns with tension. And also all the other results achieved have not very good ranks because all of them have a story with more than two bracings.

Figure 9: There is no result with more than two bracings, so all the results have achieved good rank in the AOSD.

Thus it can be concluded that all the results that received bad rank have more than two bracings in at least one story. It should also be mentioned that there is a few other results in Figures 2 to 9 that have more than 2 bracings in a story but they do not receive bad ranks. This is because when a frame has a story with more than two bracings, the approximate analysis method just affects the forces calculated for bracings of that story and related columns. As members are designed in groups,
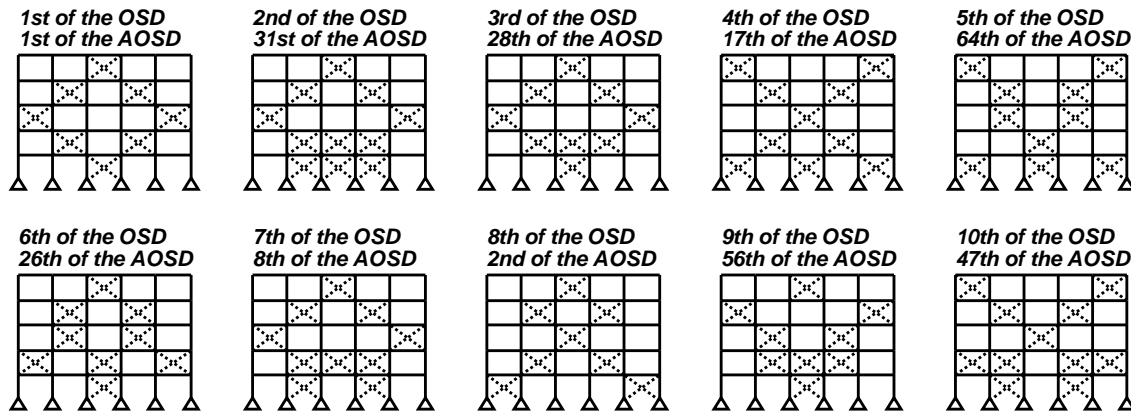
**Fig. 7.** Best results of Frame 2 (5-story structure) using AS

**Table 9.** Best results of the 5-story Frame 2 using AS (Figure 7)

| OSD rank | AOSD rank | AOSD rank 21 | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | $DW^2$ | $DW_{Best}^3$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 8 | 154.45 | 143.16 | −7.30 | 0.00 | 67.82 | 53.47 | 55.68 | 33.16 | 26.69 |
| 2 | 31 | 1 | 11 | 155.31 | 150.53 | −3.10 | 0.06 | 67.82 | 49.14 | 55.68 | 38.35 | 34.06 |
| 3 | 28 | 8 | 9 | 155.79 | 149.98 | −3.70 | 0.09 | 67.82 | 52.61 | 59.41 | 35.35 | 29.79 |
| 4 | 17 | 5 | 10 | 155.84 | 148.43 | −4.80 | 0.09 | 67.82 | 52.61 | 55.68 | 35.40 | 31.96 |
| 5 | 64 | 31 | 10 | 156.32 | 152.35 | −2.50 | 0.12 | 67.82 | 51.28 | 59.15 | 37.22 | 32.42 |
| 6 | 26 | 7 | 9 | 156.76 | 149.72 | −4.50 | 0.15 | 67.82 | 54.81 | 59.15 | 34.13 | 29.79 |
| 7 | 8 | 1 | 10 | 156.88 | 147.43 | −6.00 | 0.17 | 67.82 | 51.28 | 55.68 | 37.78 | 30.96 |
| 8 | 2 | 2 | 8 | 157.20 | 144.36 | −8.20 | 0.18 | 67.82 | 55.29 | 57.82 | 34.09 | 25.76 |
| 9 | 56 | 12 | 9 | 157.82 | 151.90 | −3.70 | 0.22 | 67.82 | 57.41 | 62.42 | 32.59 | 29.79 |
| 10 | 47 | 3 | 11 | 158.49 | 151.52 | −4.40 | 0.26 | 67.82 | 49.14 | 55.68 | 41.53 | 35.05 |
| **Mean** | **28** | **7.1** | **9.5** | **156.49** | **148.94** | **-4.82** | **0.13** | **67.82** | **52.70** | **57.64** | **35.96** | **30.63** |

when the critical members of groups are not the ones which are affected by the approximate analysis, design results may not be affected.

### 8.2.3. Comparison of the Computational Time

The comparison of the computational time for different numerical examples is made in Table 14. It should be mentioned that though the 10-story frames are not treated in this study, however, their computational time are calculated. As it can be seen from this table, the AOSD is more than 35,000 times faster than the OSD of method 2 for the case of 3 story frame. On the other hand the same ratio for 5-story and 10-story are 3,015 and 0.22, respectively. Therefore, it can be seen that the AOSD have advantageous in being faster when a non-complicated structure is treated. Therefore, for complicated structures it cannot be used for

time saving. Although it should be mentioned that this comparison is not quite fair because the AOSD is a deterministic optimization method and the fitness of all the possible patterns are calculated for choosing the best of them, while the OSD is a non-deterministic optimization approach and using this method one cannot be sure that the achieved results are the best of all. In other words, in Table 14 it is assumed that after 100, 300 and 500 loops one can get an acceptable group of best results of 3, 5 and 10 story frames, respectively. However, this does not mean that no better result would have been obtained if the optimization had been continued.

### 8.3. Comparison of the Results of AS and GA

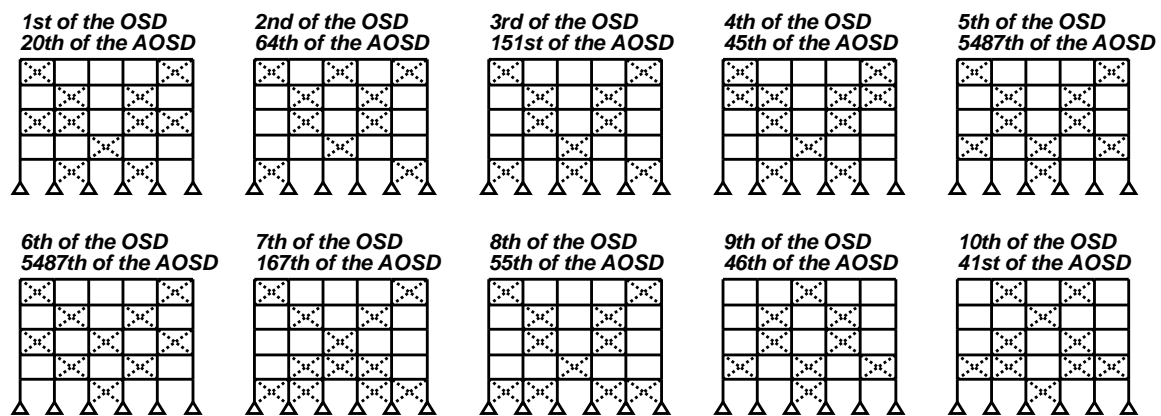Comparison of Figure 10 and table 12 which correspond to GA-based optimization, and Figure

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**Fig. 8.** Best results of Frame 3 (5-story structure) using AS

**Table 10.** Best results of the 5-story Frame 3 using AS (Figure 8)

| OSD rank | AOSD rank | AOSD rank 21 | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | DW2 | $DW_{Best}^{3}$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 2 | 11 | 140.31 | 143.22 | 2.10 | 0.00 | 67.82 | 49.14 | 55.68 | 23.35 | 26.76 |
| 2 | 64 | 19 | 10 | 142.98 | 145.00 | 1.40 | 0.19 | 67.82 | 52.61 | 57.82 | 22.54 | 26.41 |
| 3 | 151 | 42 | 10 | 143.36 | 146.36 | 2.10 | 0.22 | 67.82 | 52.61 | 59.15 | 22.92 | 26.43 |
| 4 | 45 | 12 | 11 | 143.43 | 144.19 | 0.50 | 0.22 | 67.82 | 52.61 | 55.68 | 22.99 | 27.72 |
| 5 | 5487 | 594 | 10 | 143.46 | 967.46 | 574.4 | 0.22 | 67.82 | 49.14 | 881.24 | 26.50 | 25.44 |
| 6 | 5487 | 594 | 10 | 143.69 | 967.46 | 573.3 | 0.24 | 67.82 | 52.61 | 881.24 | 23.25 | 25.44 |
| 7 | 167 | 34 | 12 | 143.95 | 146.72 | 1.90 | 0.26 | 67.82 | 52.61 | 59.15 | 23.51 | 26.79 |
| 8 | 55 | 14 | 11 | 144.01 | 144.59 | 0.40 | 0.26 | 67.82 | 52.61 | 57.82 | 23.57 | 25.99 |
| 9 | 46 | 6 | 9 | 144.05 | 144.23 | 0.10 | 0.27 | 67.82 | 54.81 | 59.15 | 21.42 | 24.30 |
| 10 | 41 | 14 | 10 | 144.28 | 144.07 | -0.10 | 0.28 | 67.82 | 54.81 | 57.82 | 21.65 | 25.47 |
| **Mean** | **1156.3** | **133.10** | **10.40** | **143.35** | **309.33** | **115.61** | **0.22** | **67.82** | **52.36** | **222.47** | **23.17** | **26.07** |

6 and table 8 corresponding AS-based design, reveals the following:

a. Best result of AS is 154.45kN and the best result of GA is 157.12kN. Thus AS leads to better results compared to GA.

b. Mean weight of the top 10 results of AS is 157.08kN, while that of the GA-based optimization is 160.51kN.

Therefore, AS performs better than GA.

Moreover the convergence rate of the AS in comparison to GA, shown in Figure 11, is much higher than the convergence rate of the GA.

Therefore, four important results are achieved:

2. Most of the best results achieved from the OSD receive a good rank using the AOSD, this means that one can guess the best bracing placement of a not complicated structure by having a good knowledge of the governing behavior of the members and the use of the approximate method.

3. Accuracy of the results achieved by the approximate analysis based optimization method increase by an increase in the accuracy of the analysis method being used.

4. There is no general optimum pattern for all structures, and the best pattern differs by changes in the number of stories, the length of bay, effective length factor and all the other parameters that change the governing behavior of the structure.

5. The approximate analysis based optimization method can be used as a time saving tool for estimating the best patterns for simple frames. However, this algorithm may become non-efficient when the size of search space increases.

6. It may be concluded that in general, AS performs better than GA for treating the layout optimization problem.
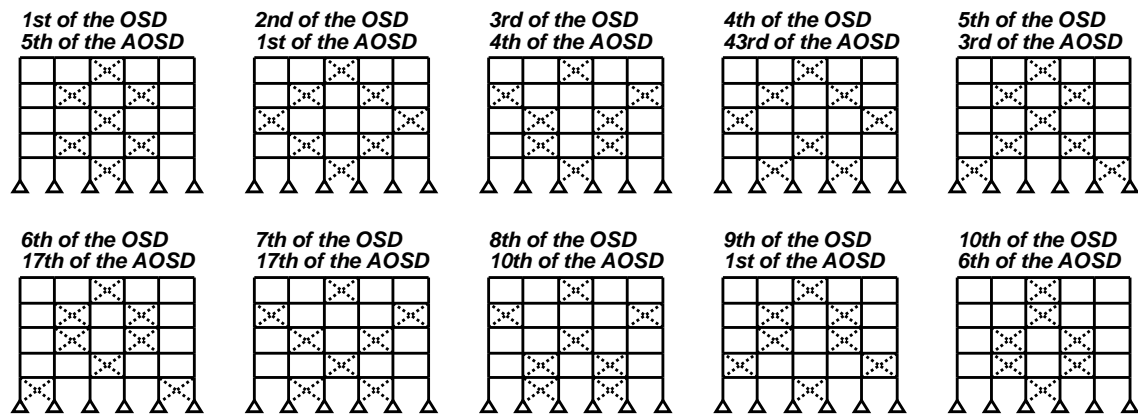
**Fig. 9.** Best results of Frame 4 (5-story structure) using AS

**Table 11.** Best results of the 5-story Frame 4 using AS (Figure 9)

| OSD rank | AOSD rank | AOSD rank 2[1] | Number of bracing | $W_{OSD}$ (kN) | $W_{AOSD}$ (kN) | $DW^2$ | $DW_{Best}{}^3$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 7 | 270.77 | 263.07 | −2.80 | 0.00 | 145.60 | 71.85 | 80.69 | 53.32 | 46.53 |
| 2 | 1 | 1 | 8 | 273.78 | 260.94 | −4.70 | 0.11 | 145.60 | 71.85 | 72.17 | 56.33 | 52.92 |
| 3 | 4 | 4 | 8 | 275.39 | 263.07 | −4.50 | 0.17 | 145.60 | 73.46 | 74.30 | 56.33 | 52.92 |
| 4 | 43 | 21 | 8 | 275.83 | 269.46 | −2.30 | 0.19 | 145.60 | 71.85 | 80.69 | 58.38 | 52.92 |
| 5 | 3 | 3 | 8 | 275.83 | 261.25 | −5.30 | 0.19 | 145.60 | 71.85 | 74.30 | 58.38 | 51.10 |
| 6 | 17 | 11 | 8 | 275.83 | 267.06 | −3.20 | 0.19 | 145.60 | 71.85 | 78.29 | 58.38 | 52.92 |
| 7 | 17 | 11 | 8 | 277.43 | 267.06 | −3.70 | 0.25 | 145.60 | 73.46 | 78.29 | 58.37 | 52.92 |
| 8 | 10 | 8 | 8 | 277.43 | 265.24 | −4.40 | 0.25 | 145.60 | 73.46 | 78.29 | 58.37 | 51.10 |
| 9 | 1 | 1 | 8 | 277.48 | 260.94 | −6.00 | 0.25 | 145.60 | 69.44 | 72.17 | 62.43 | 52.92 |
| 10 | 6 | 2 | 7 | 279.00 | 263.34 | −5.60 | 0.30 | 145.60 | 81.66 | 82.29 | 51.74 | 46.53 |
| **Mean** | **10.7** | **6.3** | **7.8** | **275.88** | **264.14** | **−4.25** | **0.19** | **145.60** | **73.07** | **77.15** | **57.20** | **51.28** |

## 9. Summary and Conclusions

In this article, AS is applied to optimum design of X-braced building frame systems. A new method of design is developed to fulfill the requirements of IBC2006 based on sharing the responsibilities between different types of elements. In this method, which is called a simultaneous design of structure for all loads and frame for gravity loads, the essential frame carry
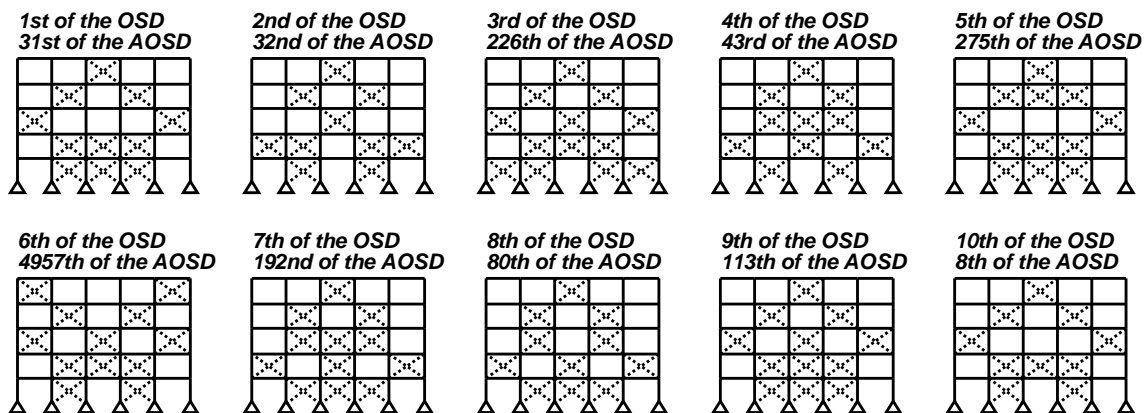


**Fig. 10.** Best results of Frame 1 (5-story structure) using GA

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**Table 12.** Best results of the 5-story Frame 1 using GA (Figure 10)

| OSD Rank | AOSD Rank | AOSD Rank 2 | Number of Bracing | WOSD (kN) | $W_{AOSD}$ (kN) | DW | $DW_{Best}{}^{3}$ | $W_{beam}$ (kN) | $W_{Column-OSD}$ (kN) | $W_{Column-AOSD}$ (kN) | $W_{Bracing-OSD}$ (kN) | $W_{Bracing-AOSD}$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 31 | 1 | 11 | 157.12 | 150.53 | -4.20 | 0.00 | 67.82 | 59.28 | 35.42 | 30.01 | 34.06 |
| 2 | 32 | 13 | 10 | 159.83 | 150.56 | -5.80 | 0.17 | 67.82 | 69.53 | 37.55 | 22.47 | 31.96 |
| 3 | 226 | 1 | 13 | 160.15 | 156.72 | -2.10 | 0.19 | 67.82 | 59.28 | 35.42 | 33.04 | 40.25 |
| 4 | 43 | 2 | 11 | 160.17 | 151.06 | -5.70 | 0.19 | 67.82 | 65.08 | 35.95 | 27.27 | 34.06 |
| 5 | 275 | 67 | 12 | 160.23 | 157.94 | -1.40 | 0.20 | 67.82 | 59.28 | 37.55 | 33.12 | 37.16 |
| 6 | 4957 | 633 | 12 | 160.57 | 979.18 | 509.8 | 0.22 | 67.82 | 59.28 | 860.97 | 33.46 | 37.16 |
| 7 | 192 | 32 | 12 | 160.76 | 155.76 | -3.10 | 0.23 | 67.82 | 62.85 | 37.55 | 30.09 | 37.16 |
| 8 | 80 | 16 | 11 | 161.05 | 152.66 | -5.20 | 0.25 | 67.82 | 69.53 | 37.55 | 23.69 | 34.06 |
| 9 | 113 | 5 | 12 | 162.62 | 154.16 | -5.20 | 0.35 | 67.82 | 69.53 | 35.95 | 25.26 | 37.16 |
| 10 | 8 | 1 | 10 | 162.62 | 147.43 | -9.30 | 0.35 | 67.82 | 69.53 | 35.42 | 25.26 | 30.96 |
| **Mean** | **595.7** | **77.1** | **11.4** | **160.51** | **235.60** | **46.78** | **0.22** | **67.82** | **64.32** | **118.93** | **28.37** | **35.40** |
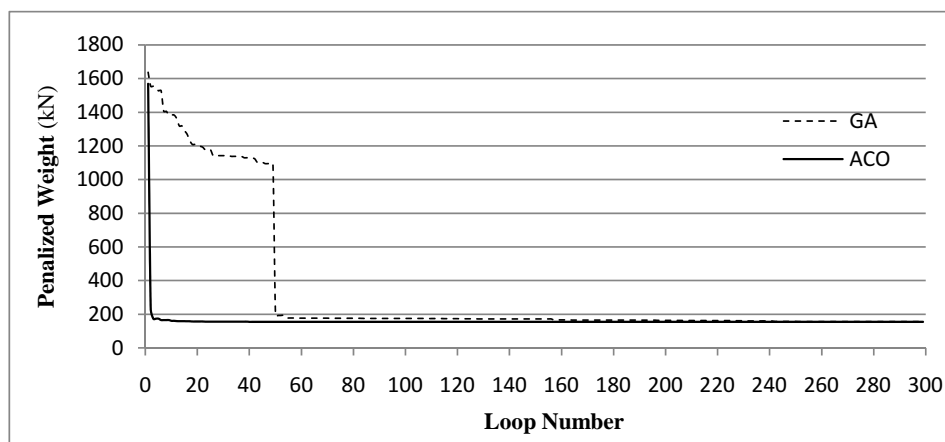


**Fig. 11.** Convergence rate of AS in compare with GA

the gravity loads while the entire building carry all the loads acting on the building. On the other hand, an optimization method based on the approximate analysis method is used to control the results.

Several numerical examples are treated by these two methods on 3, 5 story 5-bay frames and also by the previous optimization methods. The results show that:

**Table 13.** Summary of the results obtained from group examples 1 and 2

| Number of stories | Frame number | Mean of bracing number | Mean of AOSD ranks |
|---|---|---|---|
| 3 | 1 | 5.1 | 5.8 |
| 3 | 2 | 5.3 | 7.3 |
| 3 | 3 | 6.4 | 15 |
| 3 | 4 | 4.9 | 3.9 |
| 5 | 1 | 9.3 | 25.5 |
| 5 | 2 | 9.5 | 28 |
| 5 | 3 | 10.4 | 1156.30 |
| 5 | 4 | 7.8 | 10.7 |

Table 14. Run time comparison of different methods

| | Try Number | Loop number of each try | Time of each loop | Final time | | Final time to the final time of approximate method |
|---|---|---|---|---|---|---|
| | | | | second | hours | |
| 3-story, Method 1 | 100 | 60 | 1.64 | 9,840 | 2.73 | 23,773 |
| 3-story, Method 2 | 100 | 60 | 2.65 | 15,900 | 4.42 | 38,414 |
| 3-story, Approximate method | 511 | 1 | 0.0008 | 0.41 | | 1 |
| 5-story, Method 1 | 300 | 80 | 3.50 | 84,000 | 23.33 | 2,136 |
| 5-story, Method 2 | 300 | 80 | 4.94 | 118,560 | 32.93 | 3,015 |
| 5-story, Approximate method | 32767 | 1 | 0.0012 | 39.32 | | 1 |
| 10-story, Method 1 | 500 | 100 | 11.58 | 579,000 | 160.83 | 0.22 |
| 10-story, Method 2 | 500 | 100 | 14.38 | 719,000 | 199.72 | 0.27 |
| 10-story, Approximate method | 1,073,741,823 | 1 | 0.0025 | 2,684,355 | 745.65 | 1 |

1. Using GA and AS algorithms, one can include building code limitations much easier in the design.
2. AS has some advantages compared to GA in layout optimization.
3. Most of the best results achieved from the OSD received a good rank using the AOSD. This means that one can guess the best bracing placement of a structure (not complicated) by having a good knowledge of governing behavior of members and the use of approximate methods.
4. Accuracy of the results achieved by an approximate method increase by increasing the accuracy of the analysis method being used.
5. No general optimum pattern can be found applicable to arbitrary structure, and also the best pattern differ by changes in the number of stories, the length of bay, effective length factor and the other parameters that change the governing behavior of the structure.
6. Optimization rate of the AOSD over the OSD can be considerable in small structures. Thus for such structures, this simple algorithm can be used instead of the powerful optimization methods for having a fast reasonable estimation of the best results.

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