An Optimization Procedure for Automated Design of Seismic-Resistant Steel Frames

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Abstract: In this paper a general procedure for automated minimum weight design of two-dimensional steel frames under seismic loading is proposed. The proposal comprises two parts: a) Formulation of automated design of frames under seismic loading and b) introduction of an optimization engine and the improvement made on it for the solution of optimal design. Seismic loading, that depends on dynamic characteristics of structure, is determined using "Equivalent static loading" scheme. The design automation is sought via formulation of the design problem in the form of a standard optimization problem in which the design requirements is treated as optimization constraints.

The Optimality Criteria (OC) method has been modified/improved and used for solution of the optimization problem. The improvement in (OC) algorithm relates to simultaneous identification of active set of constraints and calculation of corresponding Lagrange multipliers. The modification has resulted in rapid convergence of the algorithm, which is promising for highly nonlinear optimal design problems. Two examples have been provided to show the procedure of automated design and optimization of seismic-resistant frames and the performance and capability of the proposed algorithm.

Keywords: optimization, weight minimization, earthquake resistant, seismic resistant, seismic analysis, equivalent static loading, Optimality Criteria, steel frames, sensitivity analysis

1. Introduction

The design task in structural engineering field is an iterative process in which the cycle of "Design → Analysis → Control → Design" is repeated until in the Control stage all design requirements are met. Since the seismic loading is not constant and changes due to variation in natural frequencies of the structure, the design of structures under earthquake loading is highly nonlinear and more iterative and complicated than the design under constant loading.

The process of obtaining optimal design may additionally increase the number of design cycles because it tends to reduce the cost besides satisfaction of design requirements especially if it is not associated with a proper optimization algorithm. This process becomes more complicated and more iterative for optimal design of structures under variable seismic loading. However, if a proper optimization algorithm is incorporated within the design cycle, the design process may be guided automatically towards an optimum design with less computational effort. The preference of any optimization algorithm compared to others lies on its efficiency in reducing the number of design and optimization iterations.

Numerous research works concerning optimal design of structures have been reported in the literature; however, the optimal design of structures under seismic

loading constitutes a small part. In fact it can be claimed that the design optimization of structures under dynamic/seismic loading is in its infancy stage.

In a review by Cheng [1], most of research works and reports published in journals and world conferences on earthquake engineering up to 1985, has been cited. The review categorizes the papers based on nature of structures, materials, objective functions, optimization methods and deterministic or non-deterministic in response and resistance. It also includes the research done by the author and his co-workers on optimal design of structures under seismic loading.

Kramer and Grierson in a later research [2], tried to minimize the weight of structure under dynamic loads. They used a Gradient Projection algorithm for the optimization engine and solved problems of combined static and dynamic loadings. McGee and Phan [3,4] proposed Adaptable Optimality Criterion technique for optimal design of structures under frequency constraints. Allowing for the tune up of the step size parameter, and using an extrapolation scheme, they reported good convergence in Optimality Criteria method. Memari and Madhkhan [5] used the Feasible Direction method for optimal design of steel frames subject to combined gravity and seismic loads.

More recently, Papadrakakis et. al. [6] proposed the use of Evolution Strategies in the optimum design of structures under seismic loading. They used response spectrum modal analysis to evaluate the seismic loading. To this end, they created six artificial accelograms. Then they used Evolution Strategy (ES) for obtaining the optimal design of structures. Evolution Strategies are those algorithms that similar to

Genetic Algorithm (GA), do not need gradient information; their major difference with GA is that they use real vectors instead of bit-strings. For the optimal design of a six-storey space frame the authors performed about 160 finite element analysis.

Although most of researchers aim to find an applicable design optimization method, which can be used in practical design of structures that involve in numerous design variables and constraints, there has been limited success. To the knowledge of the authors, there are a few commercial software packages that are concerned with optimal design of structures under static loadings; but there is not any software that does optimization under dynamic/seismic loading. The goal of this research work is to propose a methodology that can be used for optimal design of structures of practical size, under seismic loading. To that end, the common design procedure that is usually done in the consulting design offices is followed. The Kuhn-Tucker based Optimality Criteria method, that has very small dependence to the number of design variables, has been selected as the optimization engine. The drawbacks of the OC method have been remedied and it has been shown that the algorithm is capable of finding optimal design of practical size structures under seismic loading in a few iterations.

2. General Formulation of the Design Problem

The automated design of seismic resistant steel frames may be mathematically formulated in the form of a standard optimization problem that consists of an objective function and a set of constraints. Among various options for the objective function (e.g. weight, cost, natural frequency, etc.), in this paper the objective function is

the weight of structure and is given by equation (1).

$$Z = \sum_{i=1}^{nelem} W_i = \sum_{i=1}^{nelem} \rho L_i A_i \qquad (1)$$

Where ρ is the specific weight of steel, A_i and L_i are cross-section area and length of element i respectively. The constraint set comprises all design limitations that are provided by design codes of practice. Here in this paper, AISC-ASD specifications are used. The constraints include limits on stresses, deflections, side-sways, inter-storey drifts, and upper and lower bounds on member sizes; they are explained in the following sub-sections.

2.1. Stress Limits

Among many design requirements for beams, Eqs.(2 & 3), that give upper bound stresses due to bending and shear, have been used. Lateral torsional buckling of beams, minimum lateral support spacing, crippling and vertical buckling of webs, etc., can also be included as additional constraints; however, they have been neglected in this study.

$$f_b = \frac{M}{S} \le F_b \qquad (2)$$

$$f_V = \frac{V}{As} \le F_V \qquad (3)$$

In these equations M is bending moment, S is section modulus, f_b and F_b are existing and allowable bending stresses. V is shear force; As is the shear area; f_v is shear stress and F_v is allowable shear stress.

Eqs.(4 & 5) are used for combined bending and axial stress constraints in columns while Eq.(3) is also used for shear stress

constraints.

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1. \qquad (4)$$

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F_e'}\right)F_b} \le 1. \qquad (5)$$

In these equations f_a is the axial stress; F_a is the allowable axial stress and it is calculated based on slenderness of column (KL/r), from Eqs.(6). F'_e is the Euler buckling stress and is given by Eq.(7). C_m is a parameter that depends on the values and sign of moments and direction of curvatures of the two ends of the column.

$$F_a = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2} \qquad (6\text{-a})$$

$$\text{if} \quad KL/R \succ C_C \quad ; \quad C_C = \sqrt{\frac{2\pi^2 E}{F_y}}$$

$$F_a = \frac{\left(1 - 0.5\beta^2\right)F_y}{\frac{5}{3} + \frac{3}{8}\beta - \frac{1}{8}\beta^3} \qquad (6\text{-b})$$

$$\text{if} \quad KL/R \prec C_C \quad ; \quad \beta = \frac{\frac{KL}{r}}{C_C}$$

$$F_e' = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2} \qquad (7)$$

2.2. Deflection Limits

There are three types of deflection limits in member and structure levels. These limits include deflection of beams, side sway limit at the roof and inter-storey drift. They are restored by using Eqs.(8-10) as design constraints respectively.

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$$\delta_m \leq \delta^U = \frac{L_m}{360} \qquad m = beam\ numbers \quad (8)$$

$$\Delta_r \leq \Delta_r^U \qquad (9)$$

$$\Delta_j - \Delta_{j-1} \leq 0.005 h_j \qquad (10)$$

2.3. Size Limits

Minimum and maximum sizes for structural members are usually imposed to design optimization problem to prevent vanishing and oversizing of the members. These limits may also be imposed to restore some design code provisions. For example a lower bound=200 on slenderness of compressive members restrict the minimum size of corresponding member to a minimum value related to its length. The size limits are often treated as side constraints and are satisfied separately during optimization process. These limits are given by Eq.(11).

$$A^L \leq A \leq A^U \qquad (11)$$

2.4. Optimization Problem

Having all design constraints introduced, it remains to integrate them in the form of a standard optimization problem. Using the standard notation xi for design variables the optimal design of seismic resistant frames can be expressed in the following form.

$$Minimize \qquad Z = \sum_{i=1}^{n} \rho L_i X_i \qquad (12\text{-}a)$$

$$S.T. \quad g(X)_j \leq 0 \qquad j = 1,...,n \quad (12\text{-}b)$$

$$X^L \leq X \leq X^U \qquad (12\text{-}c)$$

In which the objective function, (12-a), is in fact given by Eq.(1); the set of constraints (12-b) are given by Eqs.(2 - 5 and 8 - 10) and

the size limits, (12-c) are those in Eq.(11).

Solution of design problem of Eqs.(12), requires completion of two prior tasks i.e. determination of internal forces in members due to combined gravity and seismic loading and identification of potentially active constraints and their explicit expression in terms of design variables. These tasks are explained in detail in the next sections.

3. Seismic Analysis

To determine the internal forces in structural members due to seismic loading, there are three different strategies [8]. The most accurate way is to perform nonlinear dynamic analysis subject to pre-assumed ground acceleration time-history(ies). Another method is to do a response spectrum analysis. In this method the first few modes of free vibration and corresponding frequencies are obtained via a linear dynamic analysis. Then using a design spectrum, given by the building / seismic code, lateral loads corresponding to the first few modes of free vibration are obtained at each floor level. Finally the seismic loading on the structure is obtained using Square Root of Sum of Squares (SRSS) or Complete Quadratic Combination (CQC) method [7]. A more simple way is to calculate the seismic forces via the so called "Equivalent Static Loading" method. In this method the seismic loading is calculated based on the period of first mode of free vibration of the structure only. In the latter method, which has been selected for the present research work, the earthquake effect is obtained from Eq.(13). It is given as shear force at the base of structure by some percentage of the total gravity loads on the structure [9,10].

The parameter C is usually defined by the

building code provisions. Without affecting the generality of the problem formulation, in this paper the "Iranian Code of Practice for Seismic Resistant Design of Buildings" [9], which is similar to UBC97 [10] code of practice, has been considered as the design code. The parameter C is defined by Eq.(14).

$$C = \frac{A.B.I}{R} \qquad (14)$$

For a given structure, the parameters A, I and R possess constant values and are defined as follows. A= The basic design acceleration. I = A parameter related to the importance of the structure. R= Reduction factor; a parameter related to the structural system. The parameter B is calculated from Eq.(15).

$$B = 2.0 \left[\frac{T_0}{T} \right]^{2/3} \qquad (15)$$

In this equation T_0 is the governing period of the soil where the structure is to be built. T is the period of the first mode of free vibration of structure. In this way parameter B relates the seismic loading to the structure's dynamic characteristics. During the optimization process, as the properties of structural members change, the value of T is changed; therefore, the effect of earthquake on the structure varies accordingly.

After evaluation of V in Eq.(13), it has to be distributed in horizontal and vertical directions. The distribution of seismic loading in vertical direction, which depends on the geometry of the structure only, is obtained from Eq.(16). F_k gives the corresponding lateral force at floor k.

$$F_k = (V - F_t) \frac{W_k h_k}{\sum_{j=1}^{n} W_j.h_j} \qquad (16)$$

In this equation, W_i and hi are the weight and

height of kth floor respectively. F_t is the extra seismic force at the roof level. The distribution of seismic loadings in horizontal direction however, depends on the distribution of structure's stiffness in the frames parallel to the direction of earthquake. If the floors of structure in a 3-D analysis are assumed to be rigid, the seismic loading at each floor may be applied to its centre of mass. In this paper, since two-dimensional frames are considerd, there is no horizontal distribution of seismic loading.

4. Explicit Design Constraints

The constraint set of Eqs.(12-b) has implicit relation with design variables. To solve the optimization problem of Eqs.(12) they have to be expressed explicitly in terms of design variables. This may be done by using Taylor series expansion. Since the calculation of higher order derivatives requires much computational effort, a truncated first order Taylor series expansion is often used. The first order approximation of constraints require gradient calculations i.e. sensitivity analysis [11,12].

4.1. Sensitivity Analysis

4.1.1. Displacement Sensitivity Analysis

In the stiffness method of analysis calculation of displacements of free nodes is the basis for calculation of the internal forces in members. Therefore the gradient of displacement vector is derived first. The basic equation of equilibrium in the stiffness method of analysis is given by Eq.(17).

$$K.\Delta = P \qquad (17)$$

Where K is the overall stiffness matrix of the structure, Δ is the displacement vector and P is the loading vector. Differentiating Eq.(17)

with respect to any generic design variable, X_i, rearranging and solving the equation for $\partial\Delta/\partial X_i$ results in Eq.(18).

$$\frac{\partial\Delta}{\partial X_i} = K^{-1}\left[\quad\frac{\partial P}{\partial X_i} - \frac{\partial K}{\partial X_i}.\Delta\quad\right] \qquad (18)$$

Here in this study, the cross sectional areas of members are taken as design variables X. However, an exponential relation of the type $I = \alpha A^\beta$ is used to link the moment of inertia I, of any member to its cross sectional area A. Parameters α and β are obtained via a curve fitting process for the ruled sections. The same procedure may be followed to obtain the relation between section modulus S and cross section area A. Figure (1) shows the relationship of I and S versus A for European *INP* (I Normal Profile) sections. Some other linking schemes may also be found in the literature [13].

4.1.2 Sensitivity of Seismic loading

The major difference between optimal design of structures under gravity loading and seismic loading is in the $\partial P/\partial X_i$ term. In the design optimization of steel frames under gravity loading, this term is often neglected [13] because, due to a ΔX_i, except a very small change in the self-weight of the structure, no change occurs in the gravity loading vector. However, in the optimal design of structures under seismic loading the change in load vector is the change in seismic loading, $\partial V/\partial X_i$ and is not negligible. It is related to change in the first natural period of the structure and can be derived from Eqs.(13-15).

$$\frac{\partial P}{\partial X_i} = \frac{\partial V}{\partial X_i} = -\frac{2}{3}V\left(\frac{\partial T/\partial X_i}{T}\right) \qquad (19)$$

It has been shown [11,14] that provided that the mode shape φ is normalized with respect

to mass matrix, M, such that $\varphi^T M\varphi = 1$, the partial derivative $\partial T/\partial X_i$ can be obtained using Eq.(20).

$$\frac{\partial T}{\partial X_i} = -\frac{T^3}{8\pi^2}\phi^T\frac{\partial K}{\partial X_i}\phi \qquad (20)$$

Back substitution for $\partial T/\partial X_i$ in Eq.(19) gives the sensitivity of seismic loading. Moreover, substituting for $\partial P/\partial X_i = \partial V/\partial X_i$ in Eq.(18) completes the sensitivity calculation of displacement vector to change in any design variable.

4.1.3. Stress Sensitivity Analysis

The internal forces in any structural member kl (standing between joint k and l) can be obtained from the following equation:

$$f_{kl} = k^l_{kk}\cdot R_{kl}\cdot\Delta_k + k_{kl}\cdot R_{lk}\cdot\Delta_l \qquad (21)$$

in which the notations f_{kl}, k^l_{kk}, k_{kl} stand for vector of internal forces of member kl at k end, stiffness matrix at k-end, and l-end in local coordinates respectively. R_{kl} is the rotation matrix relating global coordinates to local coordinates of member kl. Δk and Δl are displacement vectors of k and l ends in global coordinates. Taking derivative of both sides with respect to design variable X_i gives a general formula for the sensitivity of internal forces to change in design variables [15].

$$\frac{\partial f_{kl}}{\partial X_i} = \left(\frac{\partial k^l_{kk}}{\partial X_i}R_{kl}\Delta_k + k^l_{kk}R_{kl}\frac{\partial\Delta_k}{\partial X_i}\right)$$
$$+ \left(\frac{\partial k_{kl}}{\partial X_i}R_{kl}\Delta_l + k_{kl}R_{kl}\frac{\partial\Delta_l}{\partial X_i}\right) \qquad (22)$$

To determine the sensitivity of any stress component to change in design variables, it is noted that stress components in structural members, not only are sensitive to the corresponding internal forces, but also are

sensitive to physical and geometrical properties of cross sections. Therefore, the sensitivities of stresses due to axial force, bending moment and shear force are derived as follows:

$$\frac{\partial f_{a_j}}{\partial X_i} = \frac{1}{A}\left(\frac{\partial N_j}{\partial X_i} - f_{aj}\right) \tag{23}$$

$$\frac{\partial f_{b_j}}{\partial X_i} = \frac{1}{S_i}\left(\frac{\partial M_j}{\partial X_i} - \frac{\partial S_j}{\partial X_i} f_{bj}\right) \tag{24}$$

$$\frac{\partial F_{v_j}}{\partial X_i} = \frac{1}{A_s}\left(\frac{\partial V_j}{\partial X_i} A_S - \frac{\partial A_S}{\partial X_i} F_{vj}\right) \tag{25}$$

Since the allowable axial stress F_a depends on cross sectional properties of individual members, its value will change if the size of corresponding member is changed. Equation (26) gives the corresponding sensitivity formula.

$$\frac{\partial F_a}{\partial X_i} = \frac{\left(-\frac{3}{8} - \frac{5}{3}\beta - \frac{3}{16}\beta^2 - \frac{1}{16}\beta^4\right)F_y}{\left(\frac{5}{3} + \frac{3}{8}\beta - \frac{1}{8}\beta^3\right)^2} \cdot \frac{\partial \beta}{\partial X_i} \tag{26}$$

The sensitivity analysis from the above equation requires much computational effort. Therefore, in this study a finite difference calculation , which requires less computational effort, has been employed. The corresponding formula is as follows:

$$\frac{\partial F_a}{\partial X_i} = \frac{F_a(X_i + \delta X_i) - F_a(X_i - \delta X_i)}{2\delta X_i} \tag{27}$$

4.2. Design Constraints in Explicit Form

In the preceding sections the sensitivity analysis of displacements and stresses were outlined. Now, the constraint equations (2-5

and 8-10) can be easily expressed explicitly in terms of design variables:

$$f_{bj} - \frac{1}{S_j}\frac{\partial S_j}{\partial X_j} f_{bj}\left(X_j - X_j^0\right) + \frac{1}{S_j}\sum_{i=1}^{n}\frac{\partial M_j}{\partial X_i}\left(X_i - X_i^0\right) \le F_b \tag{28}$$

$$f_{vj} - \frac{1}{As_i}\frac{\partial As_j}{\partial X_i} f_{vj}\left(X_j - X_j^0\right) + \frac{1}{As_i}\sum_{i=1}^{n}\frac{\partial V_j}{\partial X_i}\left(X_i - X_i^0\right) \le F_v \tag{29}$$

$$\frac{1}{F_{aj}}\left(f_{aj} - \frac{f_{aj}}{F_{aj}}\frac{\partial F_{aj}}{\partial X_j}\left(X_j - X_{0_i}\right)\right) + \frac{1}{F_b}\left(f_{bj} + \sum_{i=1}^{n}\frac{\partial f_{bj}}{\partial X_i}\left(X_i - X_i^0\right)\right) \le 1 \tag{30}$$

$$\frac{1}{F_{aj}}\left(f_{aj} - \frac{f_{aj}}{F_{aj}}\frac{\partial F_{aj}}{\partial X_j}\left(X_j - X_j^0\right) + \sum_{i=1}^{n}\frac{\partial f_{aj}}{\partial X_i}\left(X_i - X_i^0\right)\right)$$
$$+\frac{Cm}{\left(1-\frac{f_{aj}}{F_{aj}}\right)F_b}\left(f_{bj} + \sum_{i=1}^{n}\frac{\partial f_{bj}}{\partial X_i}\left(X_i - X_i^0\right)\right)$$
$$+\frac{Cmf_b}{\left(F_{aj}' - F_{aj}\right)^2 F_b}\left(\frac{\partial F_{aj}'}{\partial X_j} f_{aj}\left(X_j - X_j^0\right) - \sum_{i=1}^{n}\frac{\partial f_{aj}}{\partial X_i} F_{aj}'\left(X_i - X_i^0\right)\right) \le 1 \tag{31}$$

$$\delta_m + \sum \frac{\partial \delta_m}{\partial Xi}\left(X_i - X_i^0\right) \le \frac{L_m}{360} \tag{32}$$

$$\Delta_r + \sum \frac{\partial \Delta_r}{\partial X_i}\left(X_i - X_i^0\right) \le \Delta_r^U \tag{33}$$

$$\Delta_j - \Delta_{j-1} + \sum\left(\frac{\partial \Delta_j}{\partial Xi} - \frac{\partial \Delta_{j-1}}{\partial Xi}\right)\left(X_i - X_i^0\right) \le 0.005h_j \tag{34}$$

5. Optimization Procedure

Having the design optimization problem of Eqs.(12) explicitly expressed in terms of design variables (Eqs.(28-34)); any gradient base optimization algorithm can be used to

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solve it. In this study the Kuhn-Tucker based Optimality Criteria method which has been developed by Venkayya [16] has been modified and used.

5.1. Optimality Criteria (OC) method

Equation (35) is one of the basic Kuhn-Tucker conditions for the optimality of any point in the design space [17]. It states that, at the optimum, the negative of gradient of objective function is a linear positive combination of gradients of the active set of constraints.

$$-\nabla Z = \sum_{i=1}^{n} \lambda_j \nabla g_j \iff -\frac{\partial Z}{\partial X_.} = \sum_{i=1}^{n} \lambda_j \frac{\partial g_j}{\partial X_.}$$
$$;i=1,...,n \qquad (35)$$

Dividing Eq.(35) to $\partial Z / \partial X_i$, rearranging it to a normalized form, multiplying both sides to X' and taking its γth root and applying a first order binomial expansion results the following linear recursive relation [15]

$$x_i^{(r+1)} = x_i^{(r)} \left[1 - \frac{1}{\gamma} \left[1 + \sum_{j=1}^{m1} \lambda_j \frac{\left(\frac{\partial g_j}{\partial X_i}\right)}{\left(\frac{\partial Z}{\partial X_i}\right)} \right] \right]_{(r)} ;$$
$$i=1,...,m1 \qquad (36)$$

in which *m1* is the number of active constraints. γ is a step-size parameter that controls the convergence of recursive process. The parameter ν indicates successive iterations ($\nu=0$ corresponds to the initial design stage).

Equation (36) is the basic formula for updating the values of design variables in OC method. The independence of updating formula (36) to the number of design variables has made the OC method quite powerful and efficient.

5.2. Evaluation of Lagrange multipliers

To apply Eq.(36) for finding the new values of design variables $X_i^{\nu+1}$ the current values of Lagrange multipliers λ_j^{ν} must first be determined. To find λ values, consider the change Δg_k in the *kth* constraint due to changes ΔX_i in the design variables, in the (ν+1)th iteration:

$$\Delta g_k = g_k(X_i^v + \Delta X_i) - g_k(X_i^v) = \sum_{i=1}^{n} \frac{\partial g_k}{\partial X_i} \Delta X_i$$
$$(37)$$

To have the design remain feasible, and constraints not violated, we have to have: $g_k(X_i^\nu + \Delta X_i) \leq 0$ Arranging Eq.(37) for $g_k(X_i^\nu + \Delta X_i)$ and preserving the feasibility of the design, results in Eq.(38).

$$g_k(X_i^v + \Delta X) = g_k(X_i^{v+1}) = g_k(X_i^v) + \sum_{i=1}^{n} \frac{\partial g_k}{\partial X_i} \Delta X_i \leq 0$$
$$(38)$$

Noting from Eq.(36) that:

$$\Delta X_i = X_i^{v+1} - X_i^v = -\frac{X_i^{(v)}}{\gamma} \left[1 + \sum_{j=1}^{m1} \lambda_j \frac{\left(\frac{\partial g_j}{\partial X_i}\right)}{\left(\frac{\partial Z}{\partial X_i}\right)} \right]^{(v)}$$
$$i= 1, 2, . . . ,n \qquad (39)$$

and substituting for ΔX_i in Eq.(38) and rearranging, reduces to the inequality of (40).

$$\zeta_k \equiv \sum_{j=1}^{m1} \lambda_j \sum_{i=1}^{n} X_i \frac{\left(\frac{\partial g_k}{\partial X_i}\right)\left(\frac{\partial g_j}{\partial X_i}\right)}{\left(\frac{\partial Z}{\partial X_.}\right)} \geq \gamma g_k - \sum_{i=1}^{n} X_i \left(\frac{\partial g_k}{\partial X_i}\right)$$
$$(40)$$

This inequality may be reproduced for all potentially active constraints. Then we will have a set of inequalities (41), in which Q_{kj}

and R_k are defined in Eq.(40) as the multipliers of λ_j's and the right side of inequalities respectively.

$$[Q]\{\lambda\} \geq \{R\} \qquad (41)$$

The set of inequalities (41) establish a sub-problem for calculation of Lagrange multipliers λ, in space of dual variables in which any dual constraint ζ_k is defined in Eq.(40). This dual sub-problem, in its inequality form, has been incorporated in the Optimality Criteria method for the first time in this paper, and will be solved in a special manner.

Since the number of inequality constrains in the dual sub-problem is the same as the number of dual variables (i.e. Lagrange multipliers), the coefficient matrix Q is a square matrix. This property has led many researchers [12,15,16] to suggest the solution of this inequality in its equality form (42).

$$[Q]\{\lambda\} = \{R\} \qquad (42)$$

This means that they have initially assumed that all selected constraints in main (primal) problem become active at $X+\triangle X$. However, since it is not known a priori that which constraint is definitely active and which one is not, the solution of Eq.(42) gives incorrect results (negative values) for some of dual variables, λ. To approach the correct results, two strategies have been used in the literature.

5.2.1. Adaptive Gauss-Seidel method

Gauss-Seidel method is one of iterative algorithms for solution of simultaneous linear equations. In this method some initial values are assigned to the unknowns (λ_i^0). Then they are updated using Eq.(43) in the successive calculations.

$$\lambda_i^{v+1} = \frac{1}{Q_{ii}}(R_i - \sum_{j=1, j\neq i}^{m1} Q_{ij}\lambda_j) \qquad (43)$$

The adaptive version of this algorithm uses the same updating equation (43), except that it checks for the non-negativity of results. If any unknown λj assumes a negative value, a zero value will be assigned for it, otherwise, its value will be used for calculation of next unknown λ_{j+1} . This process is continued until a convergence criterion is met for the values of unknown λ's. Since this procedure results in non-negative λ's, it is thought that the results are correct. However, as it will be shown in section 5.2.5, results of Adaptive Gauss-Seidel method are sensitive to the order of equations. Thus it does not give the unique and correct solution.

5.2.2 Method of Sequential Reduced Linear Equations

Some researchers [1] have employed the method of Sequential Reduced Linear Equations (SRLE). In this method, for solution of linear equations (42), any closed form solution procedure such as Gauss elimination, Choleskey decomposition, etc. is used. Then the results are checked for non-negativity. If some of Lagrange multipliers assume negative values, zero values are attributed to them and corresponding rows and columns are eliminated from the set of equations. This reduces the size of simultaneous linear equations. Solution process continues with reduced linear equations until there is no negative result for λ's.

Despite robust appearance of this method, it can be shown that this method also results in false results. The reasons will be discussed in the next sub-section.

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5.2.3. Drawbacks of Previous Solution Schemes

Two main reasons may be cited for false results of methods in 5.2.1. and 5.2.2. The first reason is that in the dual sub-problem, a set of equalities, instead of inequalities, is solved for Lagrange multipliers. This means that all selected constraints in main (primal) problem are assumed to be active while we know that some of them may be inactive. The error arises from the fact that some inequalities are forced to be equality. This, of course, may lead to false results. The second reason lies on the fact that to find non-negative Lagrange multipliers, the Eq.(42) should be solved as a whole, because value of any unknown affects values of other unknowns.

In the Adaptive Gauss-Seidel method, any zero which is incorrectly assigned to an unknown, will produce wrong results for the rest of unknowns. Since during an iterative process, unknowns are computed one by one and there is no way to correct the error, the final results are not reliable.

In (SRLE) method the same error is encountered. When the set of equations is solved, some Lagrange multipliers get negative values while they should assume zero values. As was mentioned earlier, the values of other Lagrange multipliers get affected with these negative values and the reduced set of linear equations may lose some necessary equations. An example in section 5.2.5 will show these deficiencies in more detail.

5.2.4. Proposed Solution Procedure

The strategic point in the proposed method is that the Lagrange multipliers are determined such that they satisfy the inequality set of dual sub-problem (41), instead of Eq.(42). A phase-one Linear Programming algorithm is usually used for solution of inequality sets. However there are some relations between dual variables λ and dual constraints ζ (Eq. (40)) that should be satisfied, but Linear Programming algorithm cannot satisfy them. These relations are called "Complementary Conditions" and arise from complementary conditions in primal optimization problem.

The complementary condition for the *jth* constraint in the primal problem is defined as $g_j(X)\lambda_j=0$. It implies that either constraint *j* is active ($g_j(X)=0$) and corresponding Lagrange multiplier is non-zero, or $\lambda j =0$ and the corresponding constraint is inactive ($g_j(X)<0$). Since there is a direct correspondence between primal and dual constraints, any constraint in the dual sub-problem (41) follows the same status as in its primal counterpart, i.e.

$$When \quad g_j = 0. \equiv active \quad \Rightarrow$$
$$\zeta_j \ is \ active \ i.e. \ \zeta_j \equiv 0 \qquad (44)$$
$$When \quad g_j < 0. \equiv inactive \quad \Rightarrow$$
$$\zeta_j \ is \ inactive \ i.e. \ \zeta_j > 0$$

From the relationship between primal and dual constraints it can be concluded that the complementary conditions in primal optimization problem dominates the dual sub-problem in the form of Eq.(45).

$$\zeta_j \lambda_j = 0. \qquad\qquad ; \ j = 1, 2, \ldots . m1$$

$$(45)$$

Equation (45) describes a relation between constraints and variables of dual sub-problem. This relationship can not be considered in solution procedure of phase-one Linear Programming algorithm. Therefore an especial algorithm is required to implement solution of dual sub-problem (41) and satisfaction of complementary conditions (45) simultaneously. To that end, it is noticed

that for arbitrary values of the dual variables $\lambda > 0$ that satisfy the inequality constraints of sub-problem $\zeta > 0$, the value of $\zeta \cdot \lambda$ is always positive. Therefore if we sum up $\zeta_j \cdot \lambda_j$ for all constraints, a positive function will be established that we know a priory that it's minimum for correct values of λ's is equal to zero. In this paper this function ($\Sigma \ \zeta_j \cdot \lambda_j$) has been taken as the objective function of the dual sub-problem. This completes the establishment of dual sub-problem in the form of a standard Quadratic Programming (QP) problem as follows:

$$\begin{aligned} Minimize \quad & \boldsymbol{\lambda^T Q \lambda} - \boldsymbol{\lambda^T R} \\ s.t. \quad & \boldsymbol{Q\lambda \geq R} \\ & \boldsymbol{\lambda > 0} \end{aligned} \qquad (46)$$

From the solution of the above sub-problem unique values of Lagrange multipliers are obtained. The proposed method remedies both deficiencies of previous algorithms. The first drawback will be remedied because all inequalities are treated at their original form, i.e. the probability of inactiveness of the constraints is considered. The second drawback may also be eliminated because in the Quadratic-programming algorithm, all constraints are considered simultaneously and the results satisfy all constraints and non-negativity of variables. The complementary conditions are also automatically considered and satisfied in the objective function.

5.2.5. Comparison of Solution Schemes

Suppose that in the solution of an optimization problem via Optimality Criteria, a dual sub-problem like the set of inequalities in (47) is established. In this section, the three solution schemes will be used to solve the sub-problem (47) and the results will be compared.

$$\begin{bmatrix} 5 & -1 & 3 & 3 \\ -1 & 4 & -6 & -2 \\ 3 & -6 & 10 & 4 \\ 3 & -2 & 4 & 2 \end{bmatrix} \begin{bmatrix} \lambda1 \\ \lambda2 \\ \lambda3 \\ \lambda4 \end{bmatrix} \geq \begin{bmatrix} 55 \\ -32 \\ 62 \\ 40 \end{bmatrix} \qquad (47)$$

5.2.5.1. Solution via Adaptive Gauss-Seidel method

In this method, at the first step, the inequality sign in (47) is changed to equality sign. Then Eq.(43) is used to calculate and update the values of λ's. Starting with zero values for λ's, the results for the first iteration are obtained as follows:

$$\begin{aligned} \lambda_1 &= 11 & \lambda_2 &= -5.25 \ \rightarrow 0 \\ \lambda_3 &= 2.9 & \lambda_4 &= -2.3 \ \rightarrow 0 \end{aligned}$$

The arrow sign shows that negative values of λ_2 and λ_4 have been substituted with zeros. The second iteration starts with the results of first iteration. The results are as follows:

$$\begin{aligned} \lambda_1 &= 9.26 & \lambda_2 &= -1.335 \ \rightarrow 0 \\ \lambda_3 &= 3.422 & \lambda_4 &= -0.734 \ \rightarrow 0 \end{aligned}$$

If the iterative process is continued, the results after 10 iterations converge to the following values:

$$\begin{aligned} \lambda_1 &= 8.878049 & \lambda_2 &= 0 \\ \lambda_3 &= 3.536568 & \lambda_4 &= 0 \end{aligned}$$

If the third row of Eq.(47) is replaced with the first row and the same procedure is followed, the final results are changed to the following values:

$$\lambda_1 = 20.6607 \qquad \lambda_2 = \lambda_3 = \lambda_4 = 0$$

Since the results depend on the order of equations, several results may be obtained by changing the orders of rows. Then a question arises that which one is the correct and reliable solution.

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5.2.5.2. Solution via Sequential Reduced Linear Equations

In this method Eq.(47), is solved with its equality sign using any closed form solution. The results are as follows:

$$\lambda_1=10 \qquad \lambda_2=-8 \qquad \lambda_3=-2 \qquad \lambda_4=1$$

According to negative values of λ_2 and λ_3, the second and third rows and columns of Eq.(47) should be eliminated. This reduces the sub-problem to solution of Eq.(48).

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \lambda_4 \end{bmatrix}=\begin{bmatrix} 55 \\ 40 \end{bmatrix} \qquad (48)$$

Solution of Eq.(48) results in ë1= - 10 and ë4= 35. Due to negativity of ë1 , the first row and column should also be omitted. Then we will have only one equation and one unknown. The result is ë4=20. The final result will be as follows:

$$\lambda_1=\lambda_2=\lambda_3=0 \qquad \lambda_4=20$$

In this method the order of equations does not affect the results however the results are neither correct nor reliable.

5.2.5.3. Solution via the Proposed Method

In the proposed method using Eq.(46) a quadratic programming (QP) problem is defined as follows:

Minimize:

$$5\lambda_1^2+4\lambda_2^2+10\lambda_3^2+2\lambda_4^2-2\lambda_1\lambda_2+6\lambda_1\lambda_3+6\lambda_1\lambda_4$$
$$-12\lambda_2\lambda_3-4\lambda_2\lambda_4+8\lambda_3\lambda_4-55\lambda_1+22\lambda_2-62\lambda_3-40\lambda_4$$

S.t.

$$\begin{bmatrix} 5 & -1 & 3 & 3 \\ -1 & 4 & -6 & -2 \\ 3 & -6 & 10 & 4 \\ 3 & -2 & 4 & 2 \end{bmatrix}\begin{bmatrix} \lambda 1 \\ \lambda 2 \\ \lambda 3 \\ \lambda 4 \end{bmatrix} \geq \begin{bmatrix} 55 \\ -32 \\ 62 \\ 40 \end{bmatrix} \qquad ;\lambda\geq 0$$

$$(49)$$

By solution of this QP problem unique results will be obtained as follows:

$$\lambda_1=8 \qquad \lambda_2=0 \qquad \lambda_3=3 \qquad \lambda_4=2$$

These results are reliable results because they are unique, they satisfy Eq.(47), and they are not sensitive to the order of inequalities. Therefore this proposed method does not have the drawbacks of the two previous methods.

Steps of Design Optimization

Eqs.(36) and (46) form the basis for an improved Optimality Criteria method to solve the optimization problem posed by (12) for the design of a frame under seismic loading. The details of the automated optimal design procedure are outlined in the following steps.

1. Set $\nu=0$ and adopt an initial set of design variables X_i^ν ($i=1, 2, . . ., n$).

2. For the current X_i^ν calculate the first (largest) period of the structure.

3. Find the seismic lateral loading according to Eqs.(13-16). Apply all prescribed combinations of gravity and seismic loading.
4. For the current X_i^ν analyze the structure and check for the design constraints.

5. If there are major violation of constraints or there is no active constraint, scale the structure to activate some (at least one) of constraints . Scaling may be done at member level for stress constraints and structure level for displacement constraints [14,15]. Go to step 2.

6. Establish the gradient vector of the objective function $\partial Z/\partial X_i$ (i=1, 2,... ,n).

7. Select the potentially active constraints. Calculate their corresponding gradient vectors $\partial g_j/\partial X_i$ (for $i=1, 2, \ldots, n$ and $j=1, 2, \ldots, m1$).

8. For the current values of X_i^{ν} establish the dual sub-problem (46) to solve for the set of Lagrange multipliers λ^{ν}_j ($j=1, 2, \ldots, m1$).

9. With the current X_i^{ν} and λ^{ν}_j values, find the new set of design variables $X_i^{\nu+1}$.

10. Check for the convergence of the OC recursive process $\frac{\|\Delta x\|}{\|x\|} \leq 0.01$. If all $X_i^{\nu+1}=X_i^{\nu}$ and $\lambda^{\nu+1}_j=\lambda^{\nu}_j$ go to 11; otherwise set $\nu=\nu+1$ and update (46) for the current X_i^{ν} values and return to step 8.

11. Check convergence of the overall design process. If $\frac{\|\Delta x\|}{\|x\|} \leq 0.005$, it is concluded that the weight of structure from Eq.(1) is the same for two successive design cycles; terminate with the minimum weight structure; otherwise, set $\nu=0$ and return to step 2.

6. Examples

In this section, two examples have been provided. The first example is mainly provided to show the steps of design optimization and the second is provided to show the practicality of the proposed method in design practice.

7.1 Example 1

Consider the two storey one bay frame of figure 2. The gravity loading on beams, including dead and live load is considered to be *28 KN/m* and *8 KN/m* respectively. It will be designed under two load combinations, i.e. 1) *D.L+L.L* and 2) *3/4(D.L+0.2L.L+E)*. The governing period of the soil T0 is assumed to be $T_0= 0.3$ Sec.
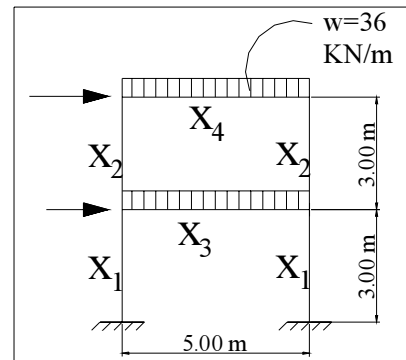

Figure 2: A two-story, one-bay frame under seismic loading.

Step 1. First the members of the frame, are grouped into four groups as shown in Fig.(2). The initial cross-sectional areas of all groups are assumed to be *70 Cm²*.

Step 2. The first natural period of the frame is obtained by Studolla method; *T=0.5027 Sec*.

Step 3. The total Base Shear and its distribution at the floor levels are obtained *V=23.46 KN, F₁=7.82 KN,   F₂=15.64 KN*

$V= 23.46\ KN,\ \ F_1= 7.82\ KN,\ \ F_2= 15.64\ KN$

Step 4. The structure is analyzed under the two load combinations. The design constraints are checked; the response ratios (RR) for all constraints are evaluated.

Based on response ratios and a proper constraint selection scheme [15], the potentially active constraints are selected. These constraints include the most critical constraint for each member group.

1 Columns in first floor, Eq.(5), RR=0.397
2 Columns in first floor, Eq.(4), RR=0.675
3 Beam in first floor, Eq.(2), RR=0.655
4 Beam in second floor, Eq.(2), RR=0.574
5 Inter-storey drift in second floor RR=0.21

Step 5. The structure is scaled to activate some of constraints. Then the process is

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Table1: Values of design variables during design optimization process.

Design Variables	Initial values	After Scaling	After 1st iteration	After 2nd iteration
A_1 (Cm^2)	70.0	39.94	39.51	39.14
A_2 (Cm^2)	70.0	55.97	55.98	55.96
A_3 (Cm^2)	70.0	54.90	54.86	54.81
A_4 (Cm^2)	70.0	50.44	50.41	50.41
Weight(Kg)	1201	860	857	855

followed from step 2. In the second constraint check, the first four constraints became active.

Step 6. The gradient of objective function (weight) $\partial Z / \partial X_i = \sum_{j=1}^{mi} \rho L_{ij}$ is obtained in this step. The index j stands for all members that are linked to X_i.

Step 7. The gradients of active constraints $\partial g_j / \partial X_i$ that have been selected in step 4, are calculated in this stage.

Step 8. The dual sub-problem is established as in Eq.(46) and solved. The matrix Q and vector R and the results for the first cycle of OC are shown below:

$$Q = \begin{bmatrix} 7.7377 & 1.002 & 0.379 & 0.221 \\ 1.002 & 10.31 & -0.042 & 0.105 \\ 0.379 & -0.042 & 12.127 & 0.124 \\ 0.221 & 0.105 & 0.124 & 11.133 \end{bmatrix} ; R = \begin{bmatrix} 4.61 \\ 5.853 \\ 5.449 \\ 4.865 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \lambda 1 \\ \lambda 2 \\ \lambda 3 \\ \lambda 4 \end{bmatrix} = \begin{bmatrix} 0.5204 \\ 0.5146 \\ 0.4305 \\ 0.417 \end{bmatrix}$$

Step 9. The new sets of design variables are updated using Eq.(43).

$X_1^1 = 39.51\ Cm^2,\quad X_2^1 = 55.98\ Cm^2,$
$X_3^1 = 54.86\ Cm^2,\quad X_4^1 = 50.41\ Cm^2.$

Step 10. The convergence criterion, is checked; since it is not met, the process continues again with step 8.

Step 8. The new results of λ's are calculated as follows $\lambda = \{0.5419,\ 0.5134,\ 0.4314,\ 0.4164\}$

Step 9. The new design variables are obtained almost the same as previous ones $X_i^1 = X_i^2$

Step 10. The convergence criterion for the OC method has met in the first iteration $\frac{\|\Delta x\|}{\|x\|} \leq 0.01$ therefore, the process continues with the next step.

Step 11. The overall convergence is checked. Since it is not converged to minimum yet, the process continues from step 2.

The above process is continued for two other iterations. Values of design variables after design optimization cycles are reported in Table 1.

Figure 3, shows the trend of reduction in the weight of the structure during optimization process.

7.2. Example 2
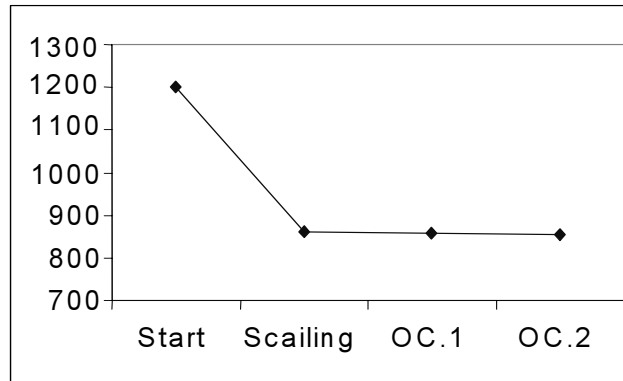
In this example an eight-storey, three bay

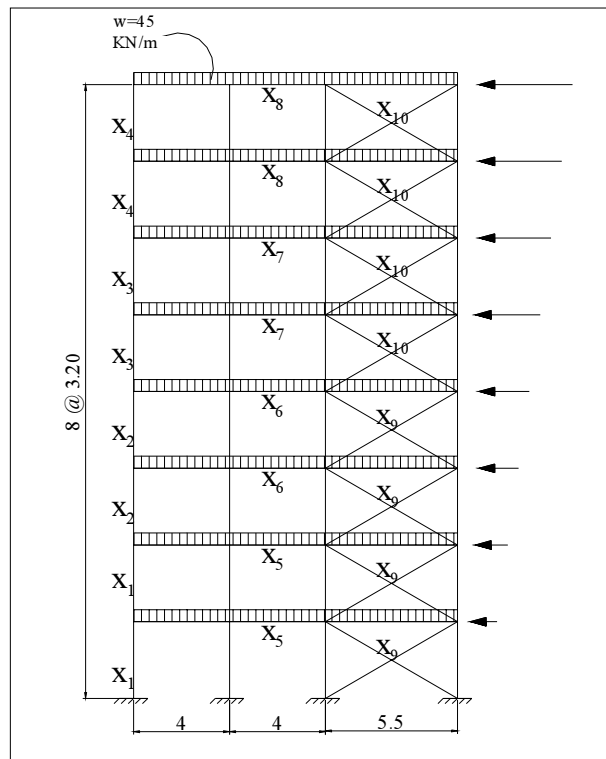Figure 3: Reduction in the weight of frame of example 1 during optimization process.

Figure 4: The eight-story, steel braced frame of example 2 under seismic loading.
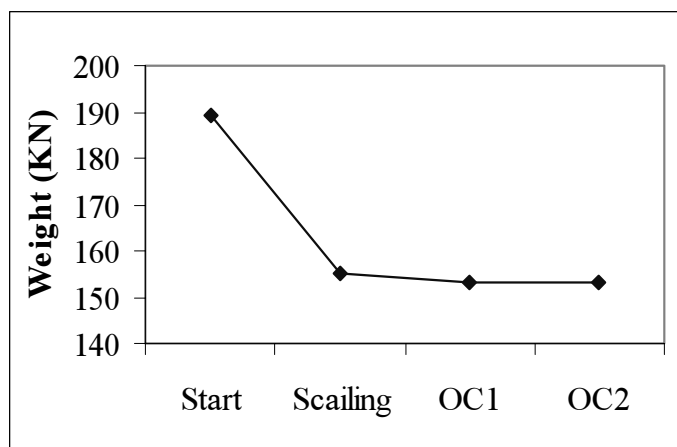
Figure 5: Reduction in the weight of frame of example 2 during optimization process.
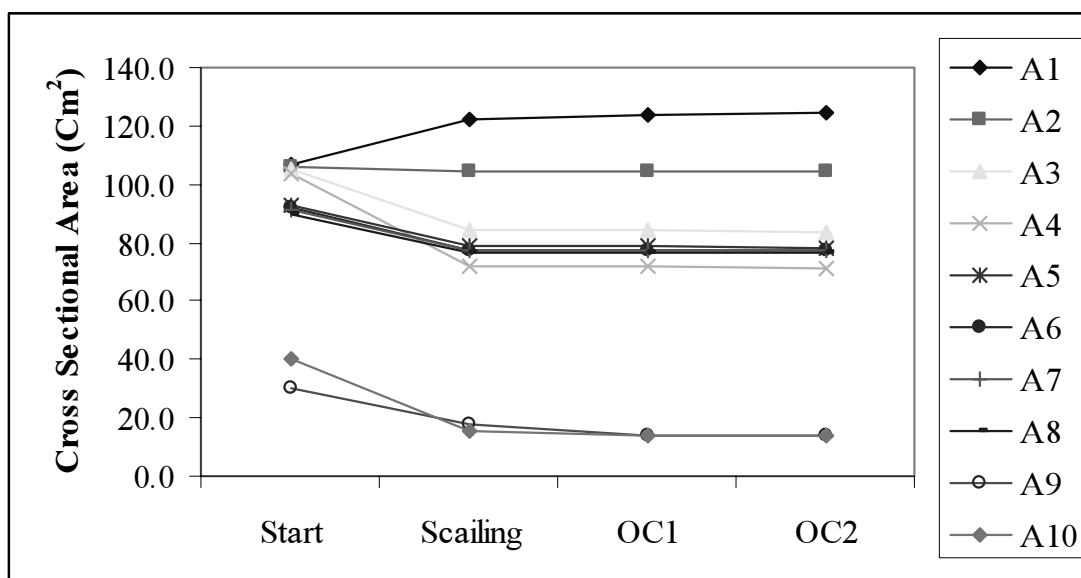

Figure 6: Variation of cross-sectional areas of example 2 during optimization process.

steel frame as shown in Fig. (4), is considered. The gravity loading includes dead load =35 KN/m and live load =10 KN/m. It is assumed that the governing period of the soil is T0 =0.3 Sec. Ten groups of structural elements, as shown in Fig.(4), have been considered. The optimal design of structure is sought under three load combinations. The constraints of this example are typically the same as in Example 1.

The same optimization procedure, as outlined in example 1, is performed. As shown in Fig.(5), the optimal design has been found after three iterations. This takes a few minutes of CPU time in a Pentium IV personal computer.

Fig.(6) shows the variation of design variables during optimization process.

An interesting result arises from the comparison of variation of first natural period of the frame and variation of total lateral seismic loading (i.e. base shear). It can be observed from Table (2) that during design optimization the first natural period of the structure increases and it turns in reduction of the lateral seismic loading. This in turn, combined with optimal proportionality of the structural members, results in doubly reduction in the weight of the structure.

To inspect what would happen if the Response Spectrum Analysis (RSA) method had been used instead of Equivalent Static Loading (ESL), the structural characteristics and base shear via RSA have been obtained at the end of the three optimization stages. The results have been plotted in Fig.(7).

It can be seen from this Figure that: during optimization process the proportionality of structural members is modified such that the

fist few natural periods of the structure, that have governing influence on the total base shear, increase and this results in reduction of total base shear. The total base shear is obtained using Complete Quadratic Combination (CQC) method. Although the total base shear using RSA method is less than that of ESL method, but it follows the same trend in reduction of base shear.

7. Conclusion

The automated design of steel frames under seismic loading was formulated in the form of a standard optimization problem. The Equivalent Static Loading scheme, was briefly described and used for determination of lateral seismic loading. The direct correspondence of the lateral loading to the dynamic characteristics of the frame and the highly iterative nature of the design process were outlined. The design constraints including stress, displacement and size constraints were explained. The implicit form of constraints was inverted to explicit form via first order Taylor series expansion. The Optimality Criteria method of optimization was briefly described and the drawbacks of the existing solution schemes for calculation of Lagrange multipliers were outlined. A modification/improvement in OC method, related to the calculation of Lagrange multipliers was formulated. It was shown that the proposed method remedies the existing drawbacks of the algorithm and results in reliable, unique solution and fast convergence. The detailed design optimization was outlined. Two examples were solved to exhibit the solution procedure and the applicability of the proposed method to optimal design of real size structures under seismic loading. It was found that the proposed method is fast, reliable and robust. Example 2, clearly showed that the proposed

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Table 2: Variation of first natural period, base shear and the weight of structure in Example 2

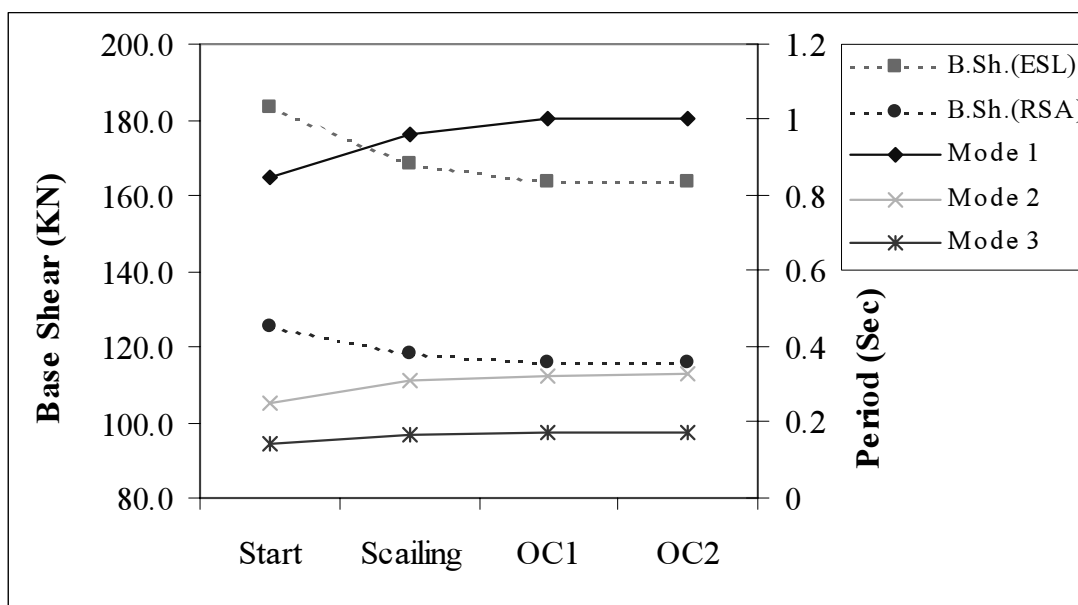|	Start	Scaling	OC1	OC2
T (Sec)	0.84704	0.96394	1.00342	1.00336
Base Shear (KN)	183.308	168.171	163.730	163.736
Weight (KN)	189.139	155.329	153.188	153.021

Figure 7: Variation of the first 3 natural periods and base shears (ESL and RSA methods) of the frame of Example 2.

algorithm tries to increase the natural periods of the frame, decrease the lateral forces and doubly reduce the weight of structure. It was shown that if Response Spectrum Analysis method had been used for determining the seismic effects, similar results to that of Equivalent Static Loading method would have been obtained. Therefore, this method is strongly recommended for the optimization of highly iterative and complicated design optimization of structures under seismic loadings.

8. References

[1].

Cheng, F.Y., "Development in Optimal Seismic Structural Design", in the Proceeding of P.R. China -U.S.A.-Japan Trilateral Symposium/ Workshop on Engineering for Multiple natural Hazard Mitigation, Beijing, China, PP. E10-1-E10-24, 1985.

[2].

Kramer, G. J. E. and Grierson, D. E., "Computer-Automated Design of Structures under Dynamic Loads", Journal of Computers and Structures, Vol. 32, No. 2, pp 313-325, 1989.

[3].

McGee, O.G. and Phan, K.F., "A Robust Optimality Criteria Procedure for Cross-Sectional Optimization of Frame Structures with Multiple Frequency Limits", Computers & Structures, Vol. 38, No. 5/6, pp. 485-500, 1991.

[4].

McGee, O.G. and Phan, K.F., "Adaptable Optimality Criterion Techniques for Large-Scale Space Frames with Multiple Frequency Constraints", Computers & Structures,

Vol. 42, No.2 PP. 197-210, 1992.

[5]. Memari, A.M. and Madhkhan, M., "Optimal Design of Steel Frames Subject to Gravity and Seismic Codes' Prescribed Lateral Forces", Structural Optimization Vol. 18, PP. 56-66, 1999.

[6]. Papadrakakis, M, Lagaros, N.D., Plevris, V., "Optimum Design of Structures under Seismic Loading', in the proceeding of the European Congress on Computational Methods in applied Sciences and Engineering, ECCOMAS 2000, Barcelona, 11-14 September 2000.

[7]. Clough, R.W. and Penzien, J., "Structural Dynamics", McGraw Hill, New York, 1993.

[8]. "Regulations for Seismic Design, A World List",Tenth edition, International Association for Earthquake Engineering, Tokyo, Japan,1996.

[9]. Iranian Code for Seismic Resistant Design of Buildings", Building and Housing Research Centre, Tehran,Iran, 1988.

[10]. Uniform Building Code, Vol. 2, " Structural Engineering Design Provisions", International Conference of Building Officials, Whittier, California, USA, 1994.

[11]. Haftka, R.T. and Kamat¸.P., "Elements of Structural Optimization", Martinus Nijhoff Publishers, Dordrecht, Netherlands, 1985

[12]. Haug, E.J., Arora, J.S. and Feng, T.T., "Sensitivity Analysis and Optimization

of Structures for Dynamic Response", ASME Journal of Mechanical Design, Vol. 101, pp. 309-314, 1979.

[13]. Chan, C.M., "An Automated Resizing Technique for Design of Tall Building Frameworks", Ph.D. thesis, University of Waterloo, Waterloo, Canada, 1993.

[14]. Alavinasab, S.A., "Optimization of 2-D Steel Frameworks Subject to Earthquake Effect", MSc. thesis, Tarbiat Modarres University, Tehran, Iran, 1999.

[15]. Moharrami, H., and Grierson, D.E.,

"Computer Automated Design of Reinforced Concrete Frameworks", ASCE, Journal of Structural Engineering, Vol. 119, No. 7, PP.2036-2058, 1993.

[16]. Venkayya, V.B., "Optimality Criteria: a Basis for multidisciplinary Design Optimization", Computational Mechanics, Vol. 5, pp. 1-21, 1989.

[17]. Arora, J.S., "Introduction to Optimum Design", McGraw-Hill inc., New York, 1989.