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Portfolio Optimization with Conditional Drawdown at Risk for the Automotive Industry

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Portfolio optimization is the process of distributing a specific amount of wealth across various available assets, with the aim of achieving the highest possible returns while simultaneously mitigating investment risks. While numerous studies have investigated portfolio optimization across various domains, there is a notable gap in the literature regarding its application specifically within the automotive industry as one of the largest manufacturing sectors in the global economy. Since the economic activity of this industry has a coherent pattern with that of the global economy, the automotive industry is very sensitive to the booms and busts of business cycles. Due to the volatile global economic environment and significant inter-industry implications, providing an appropriate approach to investing in this sector is essential. Thus, this paper aims to address this need by proposing a suitable investment methodology in the aforementioned sector. In this study, an extended Conditional Drawdown at Risk (CDaR) model with cardinality and threshold constraints for portfolio optimization problems is proposed, which is highly beneficial in practical portfolio management. The feature of this risk management technique is that it admits the formulation of a portfolio optimization model as a linear programming problem. The CDaR risk functions family also enables a risk manager to control the worst $(1-\alpha)\times 100\%$ drawdowns. In order to demonstrate the effectiveness of the proposed model, a real-world empirical case study from the annual financial statements of automotive companies and their suppliers in the Tehran Stock Exchange (TSE) database is utilized. The empirical results of this study may appeal to investors and risk managers for advanced portfolio management.

1. Introduction

Investment plays a major role in a country's financial sector and has an extraordinary impact on economic growth. By investing in the capital markets, investors are allowed to profit from their current wealth while simultaneously protecting themselves against the losses caused by inflation [1]–[3]. In order to allocate their wealth to the capital markets, investors can choose from various strategies. One option is to select companies for which they intend to invest. In such cases, they will be able to select their own companies that perform fundamental analysis [4] and control for the appropriate diversification [5] or implement technical analysis [6]. Another option is portfolio optimization, which has been one of the most frequently used investment strategies since it was first introduced by Markowitz in 1952 [7]. In this pioneering work, Markowitz introduced the mean-

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variance model, which became a new paradigm for investors to optimize portfolios. Over the past few decades, portfolio optimization has become a field of interest for researchers and practitioners, and due to technological advancements, it is still a developing topic [8], [9]. Numerous studies have been conducted on portfolio optimization across diverse domains such as hedge funds [10], cryptocurrencies [11], energy [12], technology [13], and etc. Nevertheless, none of these studies have specifically concentrated on the automotive industry, which holds a significant position as one of the largest manufacturing sectors in the global economy. As a result, this research chose to include stocks from the automotive industry to form the portfolio. The automotive industry is one of the most important industries in the world, which generates enormous benefits for the global economy. In particular, this industry impacts a wide range of international concerns such as energy consumption, emissions, and trade. A large number of today's developing countries have recognized the importance of the automotive industry and have underpinning appropriate strategies to improve it. The Iranian automotive industry was established in 1959 and has since proven that it has achieved important results despite the volatility of this industry in terms of decisions, economic and political crises, cynicism, etc. Since the economic activity of this industry has a coherent pattern with that of the global economy, the automotive industry is very sensitive to the booms and busts of business cycles [14]. In view of this problem, investors need to be provided with a suitable approach to investment in this sector.

Thus, in this study, we propose an extended Conditional Drawdown at Risk (CDaR) portfolio optimization model with cardinality and threshold (quantity) constraints and attempt to construct an optimal portfolio for investment in the Iranian automotive industry. For this purpose, a real-world case study of automotive industry stocks on the Tehran Stock Exchange (TSE) is examined.

The remainder of this paper is organized as follows. Section 2 presents a real-world case study of the automotive industry stocks on the TSE to demonstrate the effectiveness of the proposed model. Section 3 is devoted to the extended CDaR portfolio optimization model. Section 4 presents the experiments performed and the computational results, and Section 5 concludes the paper and provides suggestions for possible future research.

2. Data

In order to demonstrate the effectiveness of the proposed model, a real-world empirical case study is presented in this section. In this numerical study, a single year's historic data for 31 financial assets, ranging from March 2022 to March 2023 is used. We divide one year into 22 subintervals and compute the rate of returns in periods of 10 days.

The information utilized in this paper was collected from the annual financial statements of automotive companies and their suppliers published each year in the TSE database. The selected assets are listed in Table 1.

Table 1: Selected asset data

Alternative	Asset	Alternative	Asset
A_1	KHSH	A_{17}	RIIR
A_2	TAIR	A_{18}	KFAN
A_3	IKCO	A_{19}	FNAR
A_4	YASA	A_{20}	GHAT
A_5	KRIR	A_{21}	INDM
A_6	PLKK	A_{22}	KVRZ
A_7	TMKH	A_{23}	BARZ
A_8	RADI	A_{24}	BHMN
A_9	RTIR	A_{25}	GOST
A_{10}	RINM	A_{26}	LENT
A_{11}	ZMYD	A_{27}	MESI
A_{12}	SZPO	A_{28}	MHKM
A_{13}	AZIN	A_{29}	MNSR
A_{14}	SIPA	A_{30}	MSTI
A_{15}	RENA	A_{31}	NMOH
A_{16}	SHND		

3. Methodology

In this section, the definition of CDaR and the additional practical constraints in our portfolio optimization model is introduced first. Then, the final model is proposed in detail.

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3.1. Definition of CDaR

Drawdown, defined as the decrease in portfolio value from the previous peak, is a permanent concern for investors and is frequently used in evaluating the performance of a portfolio. The drawdown measure aids investors construct portfolios that allow them to not lose more than a certain percentage of the maximum value of their assets accrued to that time. To address these concerns, Chekhlov et al. [15], [16] proposed a drawdown measure called CDaR, which is the average of a certain percentage of the largest drawdowns over the investment horizon. CDaR risk measures illustrate relatively new developments in risk management. In fact, CDaR possesses all the properties of a deviation measure, such as convexity, positive homogeneity, and non-negativity. The application of this risk measure is similar to Conditional Value-at-Risk (CVaR) studied by Rockafellar and Uryasev [17], [18], and can be considered as a modification of CVaR for the case when the loss function is defined as a drawdown. Krokhmal et al. [19] compared the CVaR and CDaR approaches for minimum-risk portfolios of individual hedge funds.

The CDaR model can be expressed as follows; Let $w(x,t)$ be the uncompounded portfolio value at time t and suppose that $x=(x_1,x_2,\dots,x_n)$ is the weights of assets in portfolio, thus the drawdown function at time t is defined by:

$$f(x,t)=\max_{0\le\tau\le t}\{w(x,\tau)\}-w(x,t) \tag{1}$$

Suppose that r_{it} is the rate of return of i-th asset in j-th trading period. The uncompounded portfolio value at time j equals:

$$w(x,j)=\sum_{i=1}^{n}\left(1+\sum_{t=1}^{j}r_{it}\right)x_i \tag{2}$$

Then, the drawdown function at time j can be expressed as below.

$$f(x,j)=\max_{1\le k\le j}\left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k}r_{it}\right)x_i\right\}-\sum_{i=1}^{n}\left(\sum_{t=1}^{j}r_{it}\right)x_i \tag{3}$$

Considering that CDaR is the average of the worst-case drawdowns observed in the considered sample path, we can define CDaR as follows:

$$CDaR_\alpha(x,\eta)=\eta_\alpha+(1-\alpha)^{-1}\sum_{j=1}^{J}[f(x,j)-\eta_\alpha]^{+} \tag{4}$$

where η represents the threshold drawdown level which is exceeded by $(1-\alpha)J$ drawdowns, and $\alpha\in[0,1]$ denotes the confidence level. The CDaR model can also be represented as:

$$CDaR_\alpha(x,\eta)=\eta_\alpha+\frac{1}{(1-\alpha)J}\sum_{j=1}^{J}max\left\{0,\max_{1\le k\le j}\left[\sum_{i=1}^{n}\left(\sum_{t=1}^{k}r_{it}\right)x_i\right]-\sum_{i=1}^{n}\left(\sum_{t=1}^{j}r_{it}\right)x_i-\eta_\alpha\right\} \tag{5}$$

If $(1-\alpha)J$ was not integer, then the CDaR function is the solution of Equation (6).

$$CDaR_\alpha(x,\eta)=\min_{\eta}\left\{\eta+\frac{1}{(1-\alpha)}\frac{1}{J}\sum_{j=1}^{J}max\left\{0,\max_{1\le k\le j}\left[\sum_{i=1}^{n}\left(\sum_{t=1}^{k}r_{it}\right)x_i\right]-\sum_{i=1}^{n}\left(\sum_{t=1}^{j}r_{it}\right)x_i-\eta\right\}\right\} \tag{6}$$

The linear specification of the portfolio optimization model is demonstrated by Equations (7) to (12), as shown below:

$$min\ \eta+\frac{1}{(1-\alpha)}\frac{1}{J}\sum_{j=1}^{J}(y_j) \tag{7}$$

Subjected to

$$\sum_{i=1}^{n}\mu_i x_i=\mu_p \tag{8}$$

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$$y_j \geq \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right\} - \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i\right\} \quad (9)$$

$$y_j \geq 0 \quad (10)$$

$$\sum_{i=1}^{n} x_i = 1 \quad (11)$$

$$x_j \geq 0, \quad i = 1,2,\ldots,n \quad (12)$$

3.2. Constraints

In practical portfolio optimization, additional constraints are usually included in the model to make the portfolio optimization more realistic [20]. Therefore, this study examines the following most commonly studied additional practical constraints on the proposed portfolio optimization model.

3.2.1. Cardinality Constraints

The cardinality constraint caps the number of assets held in a portfolio. This capability helps restrict the number of positions in the optimal allocation, thus reducing operating costs. The status of asset selection in this constraint is indicated by the binary variable Z_i; thus, the cardinality constraint can be determined as follows:

$$\sum_{i=1}^{N} Z_i = K \quad (13)$$

$$Z_i \in \{0,1\}, \quad i = 1,2,\ldots,n \quad (14)$$

3.2.2. Threshold Constraints

Threshold constraints (quantity constraints), also known as floor and ceiling constraints, specify the minimum and maximum amounts of investment for each asset in a portfolio. Threshold constraints can be expressed as follows:

$$l_i Z_i \leq x_i \leq u_i Z_i, \quad i = 1,2,\ldots,n \quad (15)$$

$$0 \leq l_i \leq u_i \leq 1 \quad (16)$$

3.3. The Proposed Portfolio Optimization Model

The proposed portfolio optimization model with cardinality and threshold constraints is formulated as below:

$$\min \eta + \frac{1}{(1-\alpha)}\frac{1}{J}\sum_{j=1}^{J}(y_j) \quad (17)$$

Subjected to

$$\sum_{i=1}^{n} \mu_i x_i = \mu_p \quad (18)$$

$$y_j \geq \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right\} - \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i\right\} \quad (19)$$

$$y_j \geq 0 \quad (20)$$

$$\sum_{i=1}^{N} Z_i = K \quad (21)$$

$$l_i Z_i \leq x_i \leq u_i Z_i \quad (22)$$

$$Z_i \in \{0,1\} \quad (23)$$

$$\sum_{i=1}^{n} x_i = 1 \quad (24)$$

$$x_j \geq 0, \quad i = 1,2,\ldots,n \quad (25)$$

4. Computational Results

In this section, the computational results of the extended CDaR model with cardinality and threshold constraints for investment in the Iranian automotive industry are demonstrated.

The descriptive statistics of the selected assets are presented in Table 2.

Table 2: Descriptive statistics of the selected assets

	Mean	Variance	SD	Max	Min
KHSH	0.0031	0.0009	0.0292	0.0498	-0.0487
TAIR	0.0015	0.0008	0.0287	0.1306	-0.0653
IKCO	0.0014	0.0006	0.0237	0.05	-0.0489
YASA	0.0037	0.0005	0.0222	0.05	-0.0489
KRIR	0.0039	0.0011	0.033	0.1422	-0.0675
PLKK	0.002	0.0008	0.0288	0.0496	-0.0487

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	Mean	Variance	SD	Max	Min
TMKH	0.0034	0.0008	0.0276	0.0499	-0.0493
RADI	0.0037	0.0009	0.0307	0.1006	-0.0598
RTIR	0.0046	0.0008	0.028	0.05	-0.0487
RINM	0.002	0.0008	0.0277	0.0499	-0.0483
ZMYD	0.0056	0.0009	0.0298	0.0499	-0.0586
SZPO	0.0021	0.0006	0.025	0.0499	-0.0499
AZIN	0.0024	0.0007	0.0272	0.05	-0.0487
SIPA	0.001	0.0006	0.0254	0.0497	-0.0486
RENA	0.0036	0.0013	0.0354	0.0689	-0.0665
SHND	0.0015	0.0004	0.02	0.0499	-0.0489
RIIR	0.0021	0.0006	0.0237	0.0499	-0.0478
KFAN	0.0043	0.0009	0.03	0.0681	-0.0666
FNAR	0.0029	0.0011	0.033	0.1713	-0.0672
GHAT	0.0034	0.0011	0.0326	0.0694	-0.0662
INDM	0.0012	0.0007	0.0263	0.05	-0.0496
KVRZ	0.001	0.0006	0.0244	0.0698	-0.067
BARZ	0.0021	0.0005	0.0234	0.05	-0.0499
BHMN	0.0004	0.0007	0.0269	0.0691	-0.0698
GOST	0.0024	0.0011	0.0332	0.0697	-0.1339
LENT	-0.0014	0.0007	0.0266	0.0499	-0.0495
MESI	0.0053	0.0009	0.0293	0.0499	-0.0498
MHKM	0.0025	0.0009	0.0293	0.05	-0.0501
MNSR	0.0004	0.001	0.0319	0.0667	-0.0653
MSTI	0.0036	0.0008	0.0282	0.0498	-0.0484
NMOH	0.0041	0.0011	0.0329	0.0681	-0.0651

In these numerical results, the CDaR model was calculated with a reasonable level of 80% ($\alpha = 0.8$). This means that the optimization is done over 20% of the worst drawdowns. In this case study, we also set the lower bound $l_i = 0.1$ and the upper bound $u_i = 0.45$ for each asset, and the cardinality of the portfolio is set to $k = 6$.

After solving the CDaR model, the results are shown in Table 3.

Table 3: Summery of the optimal solution

Utility Function	A_4	A_9	A_{11}	A_{15}	A_{23}	A_{27}
0.04133492	0.10	0.45	0.10	0.11	0.14	0.10

The computational results indicate that YASA, RTIR, ZMYD, RENA, BARZ, and MESI are the assets assigned to the portfolio.

5. Conclusions

In this study, an extended CDaR model for portfolio optimization problems is proposed, which offers significant practical benefits in portfolio management. The CDaR risk functions family enables a risk manager to control the worst $(1-\alpha)\times 100\%$ drawdowns. By statistically averaging the drawdowns, a better prediction of the risk in the future and a more stable portfolio can be achieved.

In this paper, we developed the CDaR model by incorporating cardinality and threshold constraints and tested the performance of the model using an application for managing a portfolio of the automotive industry, which is very sensitive to the booms and busts of business cycles. We have shown that the portfolio optimization problem can be solved efficiently with CDaR risk functions and is very well suited for the automotive industry, so it can be considered for other risky assets as well.

Finally, two areas for future research are suggested: first, identifying and adding other realistic criteria and constraints that investors may face, such as liquidity, transaction costs, pre-assignment, round-lots, etc.; and second, the use of uncertainty approaches to account for the ambiguity and reliability of the problem.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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