

# Optimized Braking Force Distribution during a Braking-in-Turn Maneuver for Articulated Vehicles

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## Abstract

Improvement in braking performance and vehicle stability can be achieved through the use of braking systems whose brake force distribution is variable. Electronic braking force distribution has an important and serious role in the vehicle stopping distance and stability. In this paper a new approach will be presented to achieve the braking force distribution strategy for articulated vehicles. For this purpose, the mathematical optimization process has been implemented. This strategy, defined as an innovative braking force distribution strategy, is based on the wheel slips. The simulation results illustrate proposed strategy can significantly improve the vehicle stability in curved braking for different levels of vehicle deceleration.

**Keywords:** vehicle dynamics, braking force distribution, optimization, articulated vehicles

## 1. INTRODUCTION

In brake system design for a vehicle, the first consideration should be the braking performance. The braking system has to be able stop or reduce the vehicle speed as quickly as possible and maintain the vehicle traveling direction stable and controllable at any road conditions [1]. A key aspect of good braking performance is that braking is balanced. This occurs when each wheel brakes proportional to the weight that it carries. If this ideal is achieved then the vehicle can use all the available road friction and will have the minimum stopping distance without skidding wheels. Wheel lock-up promotes loss of directional control, particularly when it occurs on drive or trailer axles, because the locked wheel cannot provide stabilizing side forces [2].

The load sensing proportioning valve (L.S.P.V) employed to adjusting braking force among axles to the normal forces on those axles and closed actual brake force to ideal brake force distribution. These valves operate based on the static deflection of the rear suspension, but they are not able to compensate for the dynamic load transfer between the front and rear axles [3,4]. Articulated vehicles stability, much likes for cars, requires that the tractor front axle locks first, and followed by the trailer axle, with the tractor rear axle locking up last [5,6]. In the modern braking systems, such as electronic brake force distribution (EBD)

systems, using sophisticated algorithms formed based on slip control approach; the braking force distribution is done intelligently leads to higher stability and shorter stopping distance.

In this paper an innovative approach to formulate a new braking force distribution strategy for articulated vehicles is presented where, the mathematical optimization process has been included. The optimized strategy as the other modern braking force distribution strategies, works based on wheel slip control.

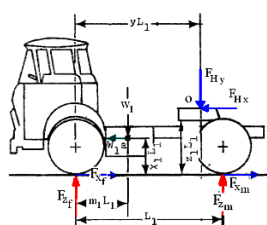
## 2. CLASSICAL BRAKING FORCE DISTRIBUTION

Classically, for straight-line braking on a level surface in the absence of any aerodynamic effects, desired braking in terms of maximum deceleration is defined as:

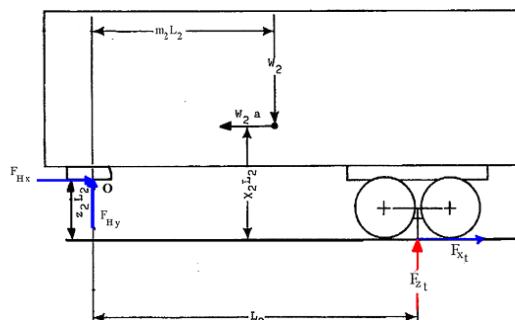
$$\mu_f = \mu_r = \mu_t = a_x \quad (1)$$

Where  $a_f$  is dimensionless vehicle deceleration (g-units) and  $\mu_f$ ,  $\mu_r$  and  $\mu_t$  are tire-road friction coefficient of respectively tractor front axle, tractor rear axle and trailer axle. For an articulated vehicle, the axle loads of the tractor unit are influenced by both the loadings and braking forces of the trailer unit [7].

Using the terminologies given in Figures (1) and (2), the equations of force and moment equilibrium



**Fig. 1.** Dynamic model of the tractor unit [7]



**Fig. 2.** Dynamic model of the trailer unit [7]

balance yield as the dynamic axle loads.

$$Fz_t = W_2.m_2 - W_2.a.(x_2 - z_2) - Fx_t.z_2 \quad (2)$$

$$Fz_r = W_1.m_1 + W_2.(1 - m_2)y + a.(-W_1.x_1 - W_2.z_1 + W_2.y.(x_2 - z_2)) + Fx_r.(z_1 + y.z_2) \quad (3)$$

$$Fz_f = W_1 \cdot (1 - m_1) + W_2(1 - m_2)(1 - y) + a(W_1 x_1 + W_2 z_1 + W_2(x_2 - z_2)(1 - y)) + Fx_i(z_2 - (z_1 + yz_2)) \quad (4)$$

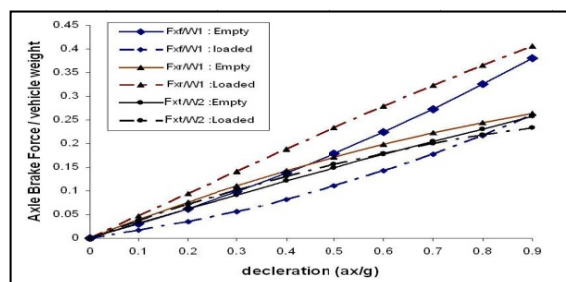
In the ideal condition, namely,  $a_x = \mu$ , all the available road friction is utilized and the brake forces are directly related to the dynamic axle loads. The ideal brake force on each axle can be computed from the following equations:

$$\frac{Fx_f}{W_1} = a_x(1 - m_1 + ax_1) + a_x \frac{W_2}{W_1}(1 - y + a_x z_1) \frac{1 - m_2 + a_x x_2}{1 + a_x z_2} \quad (5)$$

$$\frac{Fx_r}{W_1} = a_x(m_1 - a_x x_1) + a_x \frac{W_2}{W_1} (y - a_x z_1) \frac{1 - m_2 + a_x x_2}{1 + a_x z_2} \quad (6)$$

$$\frac{Fx_t}{W_t} = a_x \frac{m_2 + a_x(z_2 - x_2)}{1 + a_x z_t} \quad (7)$$

The graphical representation of Equations (5), (6) and (7) for a specific vehicle, is illustrated in figure (3) in empty and laden situation. Any point on the



**Fig. 3.** Normalized dynamic axles break force for classical distribution

classical braking force curve represents optimum braking, i.e., a condition under which the tire-road friction coefficient for front axle and rear axles are equal to dimensionless vehicle deceleration.

### 3. MODERN APPROACH TO BRAKING FORCE DISTRIBUTION

In the modern systems, known as Electronic Brake force Distribution systems (EBD), the brake forces are distributed using an active electronic control system which use wheels slip feedback to control the brake force distribution. An electronic brake force distribution utilizes the ABS hardware to function as an “intelligent brake proportioning valve.” Unlike a traditional mechanical proportioning valve which is limited by design to knee point and slope, the EBD algorithm relies on closed-loop feedback to continuously monitor wheels slip, adjusting brake line pressure to the rear wheels as appropriate.

The basic idea for developing the control algorithms for a common EBD system is that the slip difference between the front axle and the rear axles should be made minimal or theoretically equal [8].

$$S_t = S_r = S_f \quad (8)$$

Where  $S_f$  and  $S_r$  are respectively front and rear tractor wheels slip and  $S_t$  is trailer wheel slip. The combination of vehicle dynamic equation during braking, longitudinal load transfer equations and pure slip magic formula tire model [9] are considered and solved simultaneously:

$$2 \times Fx_f + 2 \times Fx_r + 2 \times Fx_t = (W1 + W2)a_r \quad (9)$$

Where

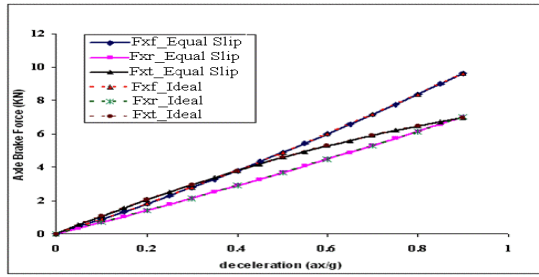


Fig. 4. Comparison of classical brake distribution and equal slips distribution

$$Fx_i = D_i \sin(C_i \tan^{-1}(B_i S_i - E_i(B_i S_i - \tan^{-1}(B_i S_i)))) = f_i(Fz_i, S_i) \quad i: f, r, t \quad (10)$$

By adding the Equation 8 to Equation 9 and solve them numerically we'll figure out that the equal slip on the tractor and trailer wheels result in a braking force distribution as same as classical distribution. It is shown in the figure (4) that the curve of "Ideal brake force distribution" lays over the curve of "equal slips strategy".

#### 4. OPTIMIZED BRAKING FORCE DISTRIBUTION

It was shown that the equal slips strategy is same as the classical braking force distribution in straight line braking. But it is well known that the braking forces in a straight line braking are different from braking forces during a turn. The Lateral tire force may decrease the tire braking force; hence braking force in braking-in-turn may be smaller than corresponding value in the straight line braking. In addition to that, there is a lateral load transfer in turning maneuver causing inequality in the braking capacity of the sides' wheels of an axle then it is notable that such a strategy may be able to provide minimum stopping distance [10] but it dose not the optimum strategy from the vehicle directional stability point of view.

To improve directional stability during braking, tractor rear wheel slip must be smaller than Trailer wheel slip and Trailer wheel slip must be smaller than tractor front wheel slip.

So, according to the above discussion we have formed a new non-equal slips, optimized strategy, based on classical constrained optimization theory.

The proposed optimization strategy maintains the braking deceleration to be unchanged when comparing to the above traditional strategy but it will control the wheels slips in such a way that they should

be lowered as much as possible. Moreover, the tractor rear wheel slips should always be kept lower than the trailer wheel slips and in order to improve the vehicle stability the trailer wheel slips should always be kept lower than the tractor front wheel slips. For this purpose, the optimized strategy for the brake in turn situation will be developed.

The constrained optimization problem could therefore, be stated as following:

$$g = \sqrt{S_{fl}^2 + S_{fr}^2 + w_1(S_{rl}^2 + S_{rr}^2) + w_2(S_{tl}^2 + S_{tr}^2)} \quad (11)$$

Where

$S_{fl}$ : Front left wheel slip;  $S_{fr}$ : Front right wheel slip  
 $S_{rl}$ : Rear left wheel slip;  $S_{rr}$ : Rear right wheel slip  
 $S_{tl}$ : Trailer left wheel slip;  $S_{tr}$ : Trailer right wheel slip

And

$$F_{xfl} + F_{xfr} + F_{xrl} + F_{xrr} + F_{xtl} + F_{xtr} - (W1 + W2)a_x = 0 \quad (12)$$

Where  $g$  is the objective function which must be minimized and equation (12) indicates the constrain equation which illustrates the unchanged deceleration condition. The weighting factors  $w_1$ , and  $w_2$  have been used to control the amount of the tractor rear wheel slips and the trailer wheel slips relative to the tractor front wheel slips.

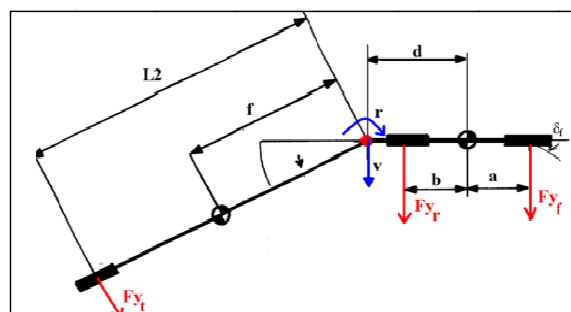
Optimization procedure could be formulated by using the Lagrange technique. Applying optimization process and simplifying leads to the following equations:

$$S_{fr} \frac{\partial F_{xfl}}{\partial S_{fl}} - S_{fl} \frac{\partial F_{xfr}}{\partial S_{fr}} = 0 \quad (13)$$

$$w_1 S_{rl} \frac{\partial F_{xfr}}{\partial S_{fr}} - S_{fr} \frac{\partial F_{xrl}}{\partial S_{rl}} = 0 \quad (14)$$

$$S_{rr} \frac{\partial F_{xrl}}{\partial S_{rl}} - S_{rl} \frac{\partial F_{xrr}}{\partial S_{rr}} = 0 \quad (15)$$

$$w_2 S_{tl} \frac{\partial F_{xrr}}{\partial S_{rr}} - w_1 S_{rr} \left( \frac{\partial F_{xfl}}{\partial S_{fl}} + \frac{\partial F_{xfr}}{\partial S_{fl}} + \frac{\partial F_{xrl}}{\partial S_{rl}} + \frac{\partial F_{xrr}}{\partial S_{rl}} + \frac{\partial F_{xtl}}{\partial S_{tl}} + \frac{\partial F_{xtr}}{\partial S_{tl}} \right) = 0 \quad (16)$$



**Fig. 5.** Linear Model of Articulated vehicle [10]

$$\begin{aligned} S_{\nu} & (\frac{\partial F_{xfl}}{\partial S_{fl}} + \frac{\partial F_{xfr}}{\partial S_{fl}} + \frac{\partial F_{xrl}}{\partial S_{fl}} + \frac{\partial F_{xrr}}{\partial S_{fl}} + \frac{\partial F_{xtl}}{\partial S_{fl}} + \frac{\partial F_{xtr}}{\partial S_{fl}}) \\ S_{ll} & (\frac{\partial F_{xfl}}{\partial S} + \frac{\partial F_{xfr}}{\partial S} + \frac{\partial F_{xrl}}{\partial S} + \frac{\partial F_{xrr}}{\partial S} + \frac{\partial F_{xtl}}{\partial S} + \frac{\partial F_{xtr}}{\partial S}) \end{aligned} \quad (17)$$

$$F_{xfl} + F_{xfr} + F_{xrl} + F_{xrr} + F_{xtl} + F_{xtr} - (W1 + W2)a_v = 0 \quad (18)$$

Based on the combined slip magic formula tire model [9], the braking force  $F_x$  is a function of longitudinal slip  $S$ , side slip angle  $\alpha$  and tire normal force  $F_z$ .

Normal force of each wheel is a summation of static vehicle weight's term and lateral/ longitudinal load transfers terms.

For calculating slip angles it is considered the steady state constant radius turn condition [11] as shown in the figure (5) and also it is assumed that the slip angles of both wheels of each axle are equal.

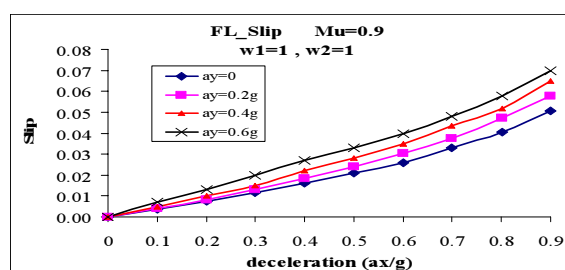
Front, rear and trailer slip angles can be expressed based on lateral acceleration  $a_y$  as following (The origin for both tractor and semi trailer has located at the common point of the fifth wheel coupling):

$$\alpha_f = -\frac{(-L_2 m_v d + d L_2 m - b L_2 m + b m_i f - d m_i f)}{C_f L_7 (a + b)} a_y \quad (19)$$

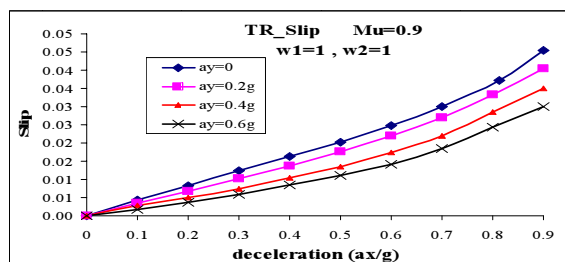
$$\alpha_r = \frac{L_2 m a + d L_2 m - m_i f a - d m_i f - L_2 m_v d}{L_2 C_v (a + b)} \cdot a_y \quad (20)$$

$$\alpha_t = \frac{L_2 ma + L_2 md - m_t fd - m_t fa - L_2 m_v d}{L_2 C_-(a+b)}. a_y \quad (21)$$

Where  $C_f$ ,  $C_r$  and  $C_t$  respectively are total lateral stiffness of front and rear axles of tractor and trailer axle besides  $m_v$ ,  $m_t$  and  $m$  are tractor mass, trailer mass and total mass respectively. Now the optimized braking force distribution for brake-in-turn situation



**Fig. 6.** Desired slip value for front left wheel



**Fig. 7.** Desired slip value for trailer right wheel

can be numerically found out.

By considering specified values for longitudinal deceleration  $a_x$  and lateral acceleration  $a_y$ , the equations (13) to (21) can be numerically solved for  $S_{fl}$ ,  $S_{fr}$ ,  $S_{rl}$ ,  $S_{rr}$ ,  $S_{rl}$ ,  $S_{lr}$  and  $S_{ll}$ . The optimized slips for a specific vehicle, which brakes on a dry bend with different longitudinal deceleration and different lateral acceleration, have been calculated and some of them have been illustrated in the figures (6) and (7).

According to the figures as a general trend, for different deceleration values, the optimized slips of left side wheels (outside wheels) are increased and the optimized slip of right side wheels (inside wheels) are decreased comparing to straight line braking values ( $a_y=0$ ). It is due to changing the normal force of each wheel as a result of longitudinal/lateral load transfer.

The unequal distribution of left/right wheels slip of each axle, can leads to much more negative yaw moment, strengthening the under steer behavior of the vehicle.

Also it is found out, by appropriate weighting factors adjustment, the desired wheel slip scheme for each individual wheel as a function of longitudinal and lateral accelerations could be achieved.

## 5. SIMULATUIN

In order to evaluate the performance of the optimized braking force distribution strategy a precise

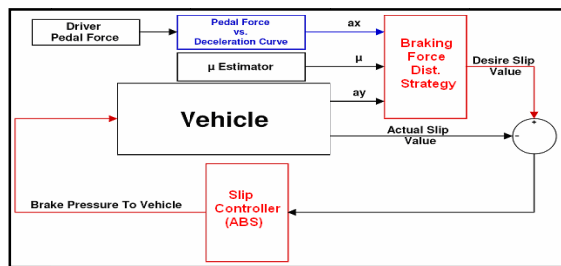


Fig. 8. Block diagram of the proposed EBD system

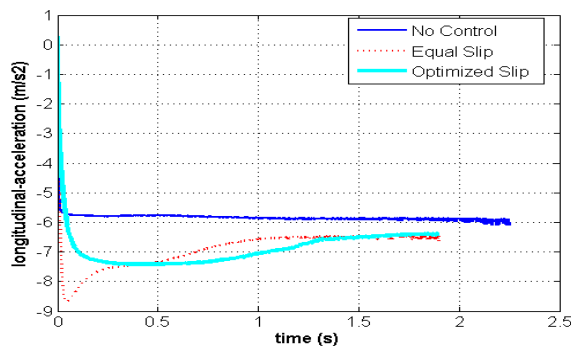


Fig. 9. Simulated longitudinal deceleration

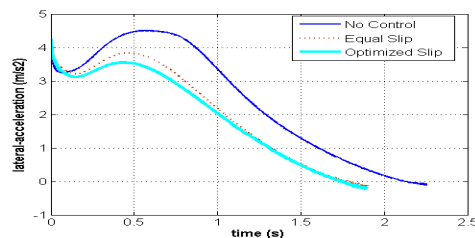


Fig. 10. Simulated lateral acceleration

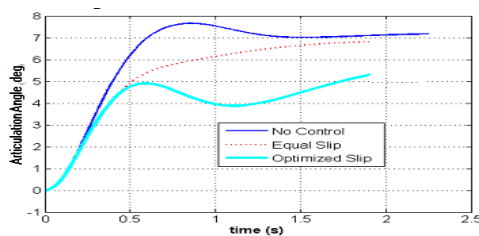


Fig. 11. Simulated articulation angle (Hitch)

computer simulation has been performed. The proposed strategy is considered as a part of a specific electronic braking force distribution (EBD) system. Figure 8 illustrates the system's block diagram. According to the figure, the system's input is driver's brake pedal force. Based on pedal force-deceleration

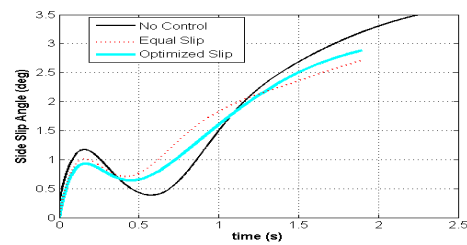


Fig. 12. Simulated side slip angle

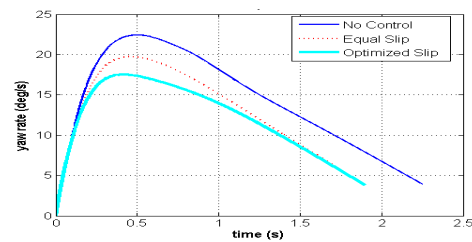


Fig. 13. Simulated tractor yaw rate

curve, driver's desired deceleration is calculated proportional to pedal force. In the higher level of the control system, based on desired deceleration's value, estimated road friction and measured lateral acceleration, desired slip of each wheel is calculated based on optimized braking force distribution strategy. The strategy is utilized in the form of pre-calculated lock-up table. In the lower layer, the desired slips are actualized by the slip controller unit, which can be a common ABS control algorithms.

To simulate the vehicle dynamic behaviors, a 10 degree of freedom vehicle dynamic model [11] with "Combined Magic Formula" non-linear tires model [9] has been used.

Also to compare the performance of different types of control strategies, four different vehicles have been considered:

- A vehicle without any kind of EBD system, nominated as 'No Control'.
- A vehicle equipped with common EBD system, nominated as 'equal slip'.
- A vehicle equipped with proposed EBD system, nominated as 'optimized Slip'.

The simulation study consists of braking during a step steering. In this study, hard braking on a dry road ( $\mu=0.9$ ) has been studied. Simultaneously, 5 degrees step steer input applies to the front wheels. The initial speed is 15 m/s.

Figures (9) to (13) show the simulation results.

According to the figure (9), the time histories of the controlled vehicles deceleration is approximately the same. But in the case of the without control vehicle due to inefficient distribution of braking force the deceleration is 15% lower, leading to longer stopping distance.

In spite of similar longitudinal dynamics behavior, thanks to optimum braking force distribution, optimized strategy can provide much better lateral dynamic behavior than without control or even equal slips strategy. Figures (10) to (13) illustrate respectively the lateral acceleration, hitch angle of the articulated vehicle, side slip angle and yaw rate of tractor unit that in all over figures, optimized strategy has better lateral dynamic behaviors which improves the vehicle stability.

## 6. CONCLUSION

It is well known that to achieve stable and safe braking, the tractor rear wheels slip must be less than trailer wheels slip and they must be less than tractor front wheels slip. According to this simple rule, common braking force distribution systems work based on a simple strategy known as equal slip strategy. In this strategy the front and rear wheels slips are theoretically set to the same value. Such a simple strategy cannot be perfect. By keeping the total braking force unchanged, the front wheels portion can be increased unlike the rear wheels portion is decreased. On the other word, the front wheels slip set to much higher value to rear wheels slip leads to more stability. The other interesting point is unequal braking capacity of left/right wheels during a braking in turn maneuver. So an intelligent braking force distribution strategy must regulate the left/right wheels slips as well as front/rear wheels slips in different values. Dramatically such an approach can lead to an unequal braking force distribution not only longitudinally but also laterally.

In this paper using optimization theory, has been tried to formulate the above mention Idea. The so-called optimized barking force distribution strategy keeps the deceleration value and hence stopping distance of the vehicle unchanged comparing to the classical braking force distribution strategy. But intelligently distributes the braking force between the six wheels unequally, based on the requested longitudinal deceleration and known lateral

acceleration.

The simulation results show much more stable behavior for the articulated vehicle using the optimized strategy during hard braking turn situation comparing to traditional equal slip strategy.

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