9.6 STRESS INTENSITY FACTORS FOR VARIOUS CONFIGURATIONS

By considering the stress distribution in the vicinity of the tip of a crack in the middle of a large plate subjected to a remote tensile stress, Relation 9.14 has been derived, namely,

$$K_I = \sigma\sqrt{\pi a} \leq K_{Ic} \tag{9.14}$$

Obviously, Relation 9.14 gives the stress intensity factor as a function of the applied stress σ and the crack size $2a$ for the particular given plate geometry of Figure 9.4. A fracture criterion more general than Relation 9.14 may be written as

$$K_I = \sigma\sqrt{\pi a}\ \text{CCF} \leq K_{Ic} \tag{9.15}$$

where "CCF" is a configuration correction factor depending on the loading and geometry of the cracked body. For instance, for a centrally cracked finite plate loaded in tension as in Figure 9.6b, CCF = $f(a/W)$ depends on the ratio of the crack half-length a to the width W. For $a/W \to 0$, CCF = $f(a/W) \to 1$ as may be seen from Equation 9.14 for an infinitely wide plate of Figure 9.6a.

For geometries and loadings other than Figure 9.6a, the CCF must be determined. Several theoretical and experimental methods exist to do this.* One exact, but rather involved, method makes

* These include boundary collocation, conformal mapping, integral transforms, and finite element method. See Parker.[3] Experimental techniques such as photoelasticity and interferometry are also used.

use of stress functions of complex variables. This analytical method solves only relatively simple geometries of cracked bodies. Other approximate methods are used for complicated geometries.

The results of such methods are presented either graphically or algebraically in the form of polynomials. Below are polynomial expressions and graphical representations for some of the most common configurations. In all cases utmost care must be taken in defining the stress appearing in Equation 9.15 as the one associated with the graphical or algebraic information used to determine the CCF.

9.6.1 Plates Under Tensile Loading

a. Crack of length $2a$, in the middle of an infinite plate, Figure 9.6a:

$$\text{CCF} = f(a/W) = 1$$

b. Crack of length $2a$ in a plate of finite width W, Figure 9.6b[11]:

$$\text{CCF} = f(a/W) = \left[\frac{W}{\pi a}\tan\frac{\pi a}{W}\right]^{1/2} \quad \text{for } a/W \le 0.25$$

or

$$= \sqrt{\sec\frac{\pi a}{W}} \quad \text{for } a/W \le 0.4$$

c. Double edge-cracks, each of length a, in a plate of finite width W, Figure 9.6c[12]:

$$\text{CCF} = f(a/W) = \left[\frac{W}{\pi a}\tan\left(\frac{\pi a}{W}\right) + \frac{0.2W}{\pi a}\sin\left(\frac{\pi a}{W}\right)\right]^{1/2}$$

or

$$= \left[1.12 + 0.43\left(\frac{a}{W}\right) - 4.79\left(\frac{a}{W}\right)^2 15.46\left(\frac{a}{W}\right)^3\right] \quad (\text{for } a/W > 0.7)$$

d. A single edge-crack of length a, in a plate of finite width W, Figure 9.6d[12]:

$$\text{CCF} = f(a/W) = \left[1.12 - 0.23\left(\frac{a}{W}\right) + 10.6\left(\frac{a}{W}\right)^2 - 21.7\left(\frac{a}{W}\right)^3 + 30.4\left(\frac{a}{W}\right)^4\right]$$

for $a/W < 0.7$

e. A circular internal crack of radius a (penny-shaped crack) embedded in an infinite solid, Figure 9.6e[5]:

$$\text{CCF} = 2/\pi$$

f. Semielliptical surface flaw of length a and width $2c$ in a plate, Figure 9.6f[13,14]:

$$\text{CCF} = f(a/2c) = 1.12/\sqrt{Q}$$

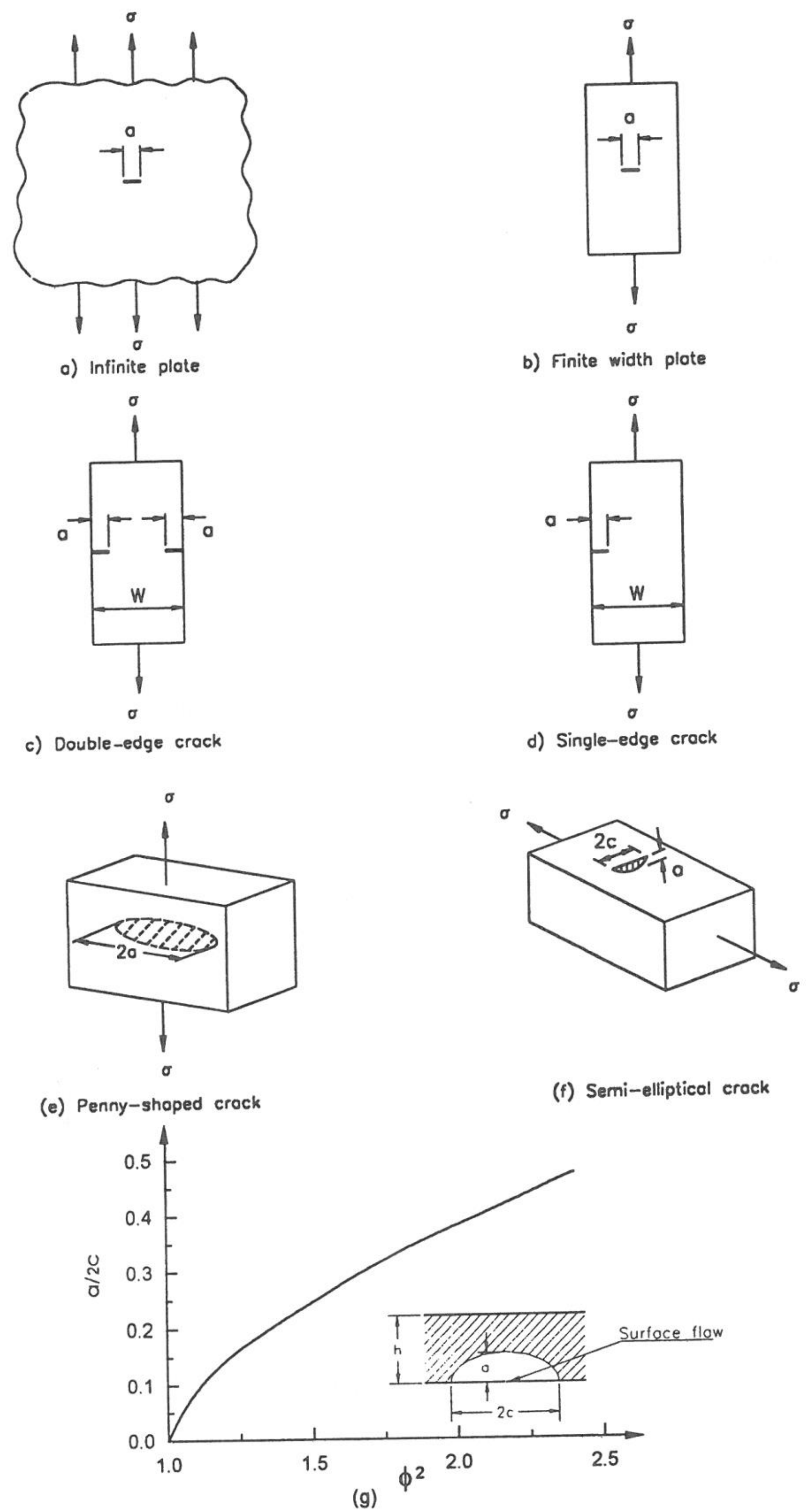

FIGURE 9.6 Configurations of various stress intensity correction factors, CCF. (From Irwin, G. R., Trans. Am. Soc. Mech. Eng., *J. Appl. Mech.*, 651–654, 1962. With permission.)

where $Q = [\phi^2 - 0.212\ (\sigma/Y)^2]$, Y being the yield strength. The parameter ϕ may be determined from Figure 9.6g or using the approximate expression*

$$\phi \cong \frac{3\pi}{8} + \frac{\pi}{2}\left(\frac{a}{2c}\right)^2$$

9.6.2 Cracks Emanating from Circular Holes in Infinite Plates

a. A single crack or a double crack at a circular hole of radius r_i in the middle of an infinite plate, Figure 9.7a and b, respectively[14]:

$$\text{CCF} = \left(\frac{r_i}{a} + \frac{1}{2}\right)^{1/2} \quad \text{for a single crack } a > 0.12r_i$$

$$\text{and CCF} = \left(\frac{r_i}{a} + 1\right)^{1/2} \quad \text{for a symmetric double crack } a > 0.12r_i$$

These are approximate CCFs, where a is the crack length measured from the edge of the hole having a radius r_i. The stress σ in Expression 9.15 designates the remote tensile stress perpendicular to the crack. Note that for very small cracks ($a/r_i \to 0$) in an exact solution the CCF = $f(a/2r_i) \to 3$, i.e., three times that for a normal central crack as expected from stress concentration around circular holes.

b. Two cracks each of length a at a circular hole of radius r_i in a plate of finite width, Figure 9.7c[15]: Here CCF is determied from Figure 9.7c, noting that the total crack length includes the hole diameter.

c. A corner (nonthrough) crack at holes, Figure 9.7d.[14,16] An approximate solution considers the hole as a part of an elliptical surface flaw of a ratio $a'/2c = a/(2\sqrt{2r_ib})$. The maximum stress intensity factor occurs at point A with CCF as:

$$\text{CCF} = \frac{1.2}{\phi}\left\{1 + \frac{a^2(2r_i - b)^2}{16r_i^2b^2}\right\}^{1/4}$$

where ϕ is defined by Figure 9.6. Expression 9.15 is used in conjunction with a crack length a.

d. A single radial crack a at a hole of radius r_i under equibiaxial tensile loading, Figure 9.7e[17]: The configuration correction factor CCF = $f(a/W)$ depends on the ratio (a/r_i) as well as the applied loading. As noted before, the determination of the stress intensity factor is affected only by the crack-opening stress.** However, for a crack emanating from a hole, the stress concentration around the hole is known to be dependent on the remote stress field: uniaxial or biaxial. Hence, as previously found in Section 6.4, for an equibiaxial stress system ($\sigma_1 = \sigma_2 = \sigma$), the stress concentration factor at the edge of hole is only 2 compared to a value of 3 for a uniaxial stress system. This fact is reflected

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* The dependence of Q on the ratio of the stress opening the crack to the yield strength of material results from consideration of plasticity at the crack tip. The above CCF has to be multiplied by a correction M_k, accounting for the proximity of the free surface in front of the crack, dependent on a/h. For shallow cracks, i.e., a/h << and $a/2c \to 0.5$; $M_k \simeq 1$. However, for $a/h = 0.6$ and $a/2c = 0.1$, M_k could attain a value of 2. See Broek,[14] pp. 90–94.

** This is true for an absolutely elastic condition. If plasticity at the crack tip is considered, a transverse stress loading affects the stress intensity factor; it is increased with a compressive stress and decreased with a tensile one.

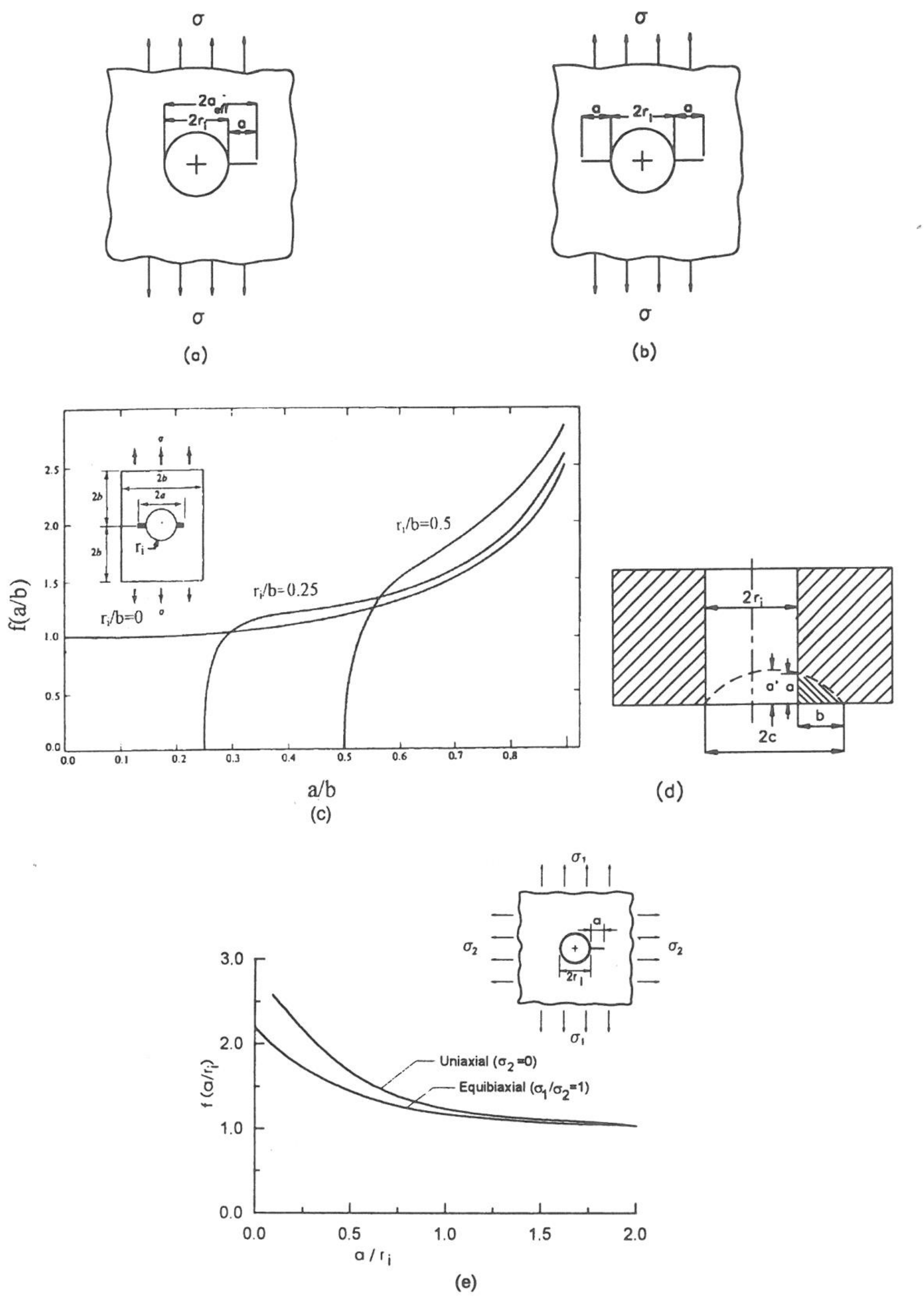


FIGURE 9.7 Graphs for various CCFs. (c, from Rooke, D. P. and Cartwright, D. J., *Compendium of Stress Intensity Factors*, The Hillingdon Press, London, 1976. With permission.)

in Figure 9.7e which indicates a smaller CCF factor for biaxial tensile loading as compared to uniaxial loading.

9.6.3 Plates Under Bending

a. Pure bending for a cracked plate of depth *W*, Figure 9.8a[18]:

$$CCF = \left(\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)\right)\frac{0.923 + 0.199\left(1 - \sin\frac{\pi a}{2W}\right)^4}{\cos\left(\frac{\pi a}{2W}\right)}$$

b. Three-point bending for a cracked plate of $L/W = 2$, Figure 9.8b[12]:

$$CCF = f(a/W) = \left[1.107 - 2.12\left(\frac{a}{W}\right) + 7.71\left(\frac{a}{W}\right)^2 - 13.6\left(\frac{a}{W}\right)^3 + 14.2\left(\frac{a}{W}\right)^4\right]$$

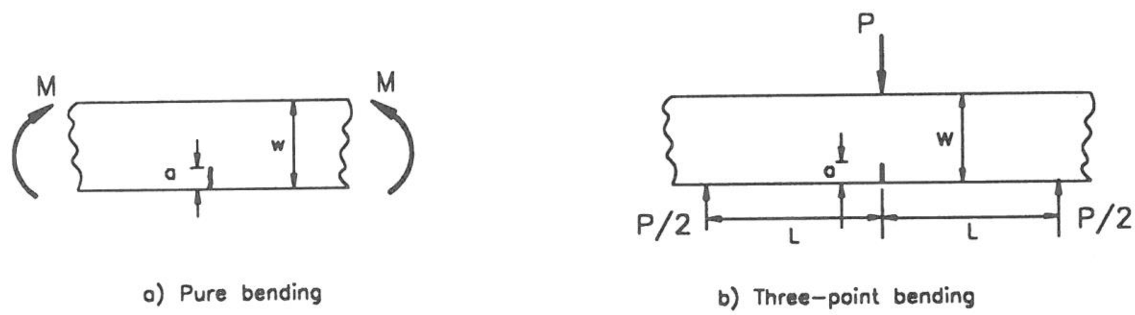


FIGURE 9.8 Cracked bars under bending: (a) pure bending, (b) three-point bending.

9.6.4 Circular Rods and Tubes

a. Central circular crack in a cylindrical bar of radius r_o, Figure 9.9a, b, and c,[19] subjected to the following:
   - Tensile load *P* causing a net average stress $\sigma = P/\pi(r_o^2 - a^2)$, hence,

$$CCF = \frac{2\sqrt{1 - a/r_o}}{\pi}\left[1 + 0.5\left(\frac{a}{r_o}\right) - 0.625\left(\frac{a}{r_o}\right)^2 + 0.421\left(\frac{a}{r_o}\right)^3\right]$$

   - Pure bending moment *M* causing a stress $\sigma = 4MPa/\pi(r_o^4 - a^4)$, hence,

$$CCF = \frac{4\sqrt{1 - a/r_o}}{3\pi}\left[1 + 0.5\left(\frac{a}{r_o}\right) + 0.375\left(\frac{a}{r_o}\right)^2 + 0.313\left(\frac{a}{r_o}\right)^3 - 0.727\left(\frac{a}{r_o}\right)^4 + 0.483\left(\frac{a}{r_o}\right)^5\right]$$

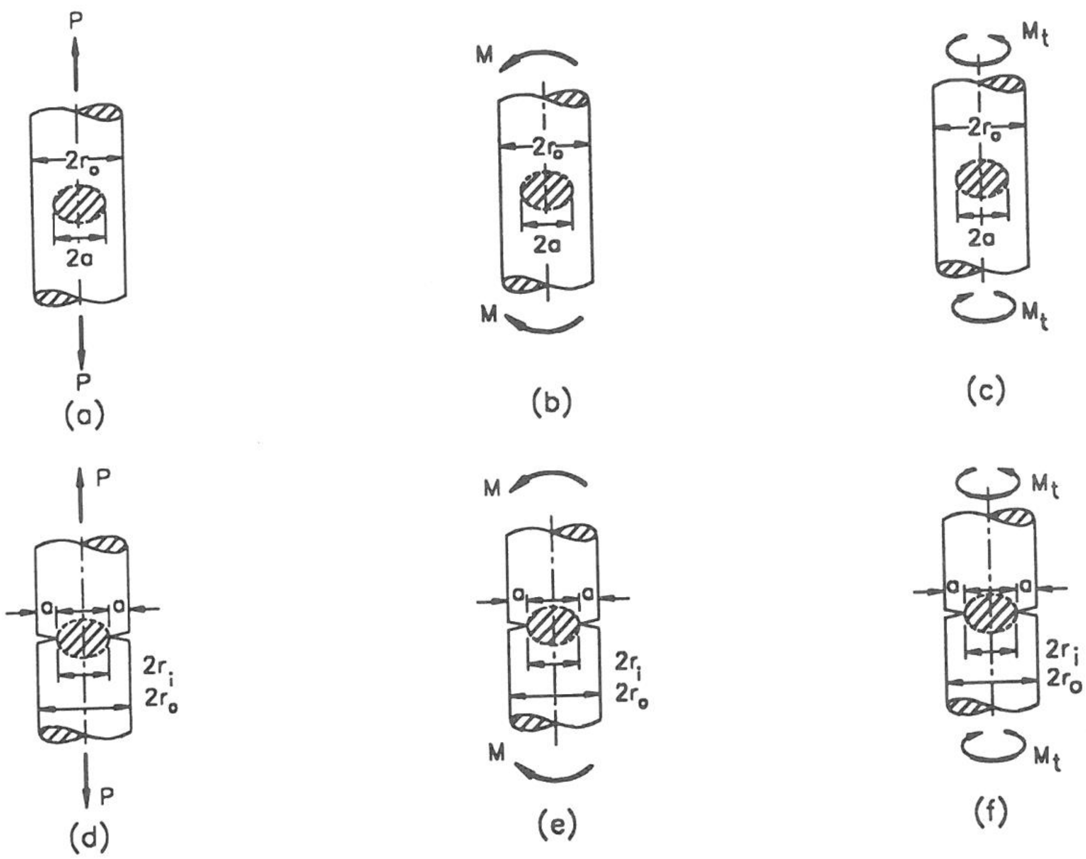


FIGURE 9.9 Cracked circular rods and tubes subjected to different loading.

- Twisting moment M_t causing a shear stress, $\tau = 2M_t\, a/\pi(r_o^4 - a^4)$, hence, CCF is approximately equal to that for pure bending for cracks of (a/r_o) up to 0.6.

b. External circumferential crack in a rod of radius r_o, Figure 9.9d, e, and f,[19] subjected to the following:

- Tensile load P causing a net tensile stress $\sigma = P/\pi r_i^2$; hence,

$$\mathrm{CCF} = \frac{\sqrt{r_i/r_o}}{2}\left[1 + 0.5\left(\frac{r_i}{r_o}\right) + 0.375\left(\frac{r_i}{r_o}\right)^2 - 0.363\left(\frac{r_i}{r_o}\right)^3 + 0.731\left(\frac{r_i}{r_o}\right)^4\right]$$

- Pure bending moment M causing a net stress $\sigma = 4M/\pi r_i^3$; hence,

$$\mathrm{CCF} = \frac{3\sqrt{r_i/r_o}}{8}\left[1 + 0.5\left(\frac{r_i}{r_o}\right) + 0.375\left(\frac{r_i}{r_o}\right)^2 + 0.313\left(\frac{r_i}{r_o}\right)^3 + 0.273\left(\frac{r_i}{r_o}\right)^4 + 0.537\left(\frac{r_i}{r_o}\right)^5\right]$$

- Twisting moment M_t causing a net shear stress $\tau = 2M_t/\pi r_i^3$: CCF is approximately equal to that for pure bending.

Note that in the above three cases the stresses are defined with respect to the net cross section, i.e., excluding the crack.

c. External circumferential crack in a long tube, Figure 9.10a, b, and c,[15] subjected to the following:
   - Uniaxial tensile stress σ applied remote from the crack, CCF is obtained from Figure 9.10a.
   - Twisting moment M_t applied, remote from the crack, about the tube axis: CCF is obtained from Figure 9.10b.

d. External radial crack in a long tube subjected to a twisting moment M_t: CCF is obtained from Figure 9.10c.

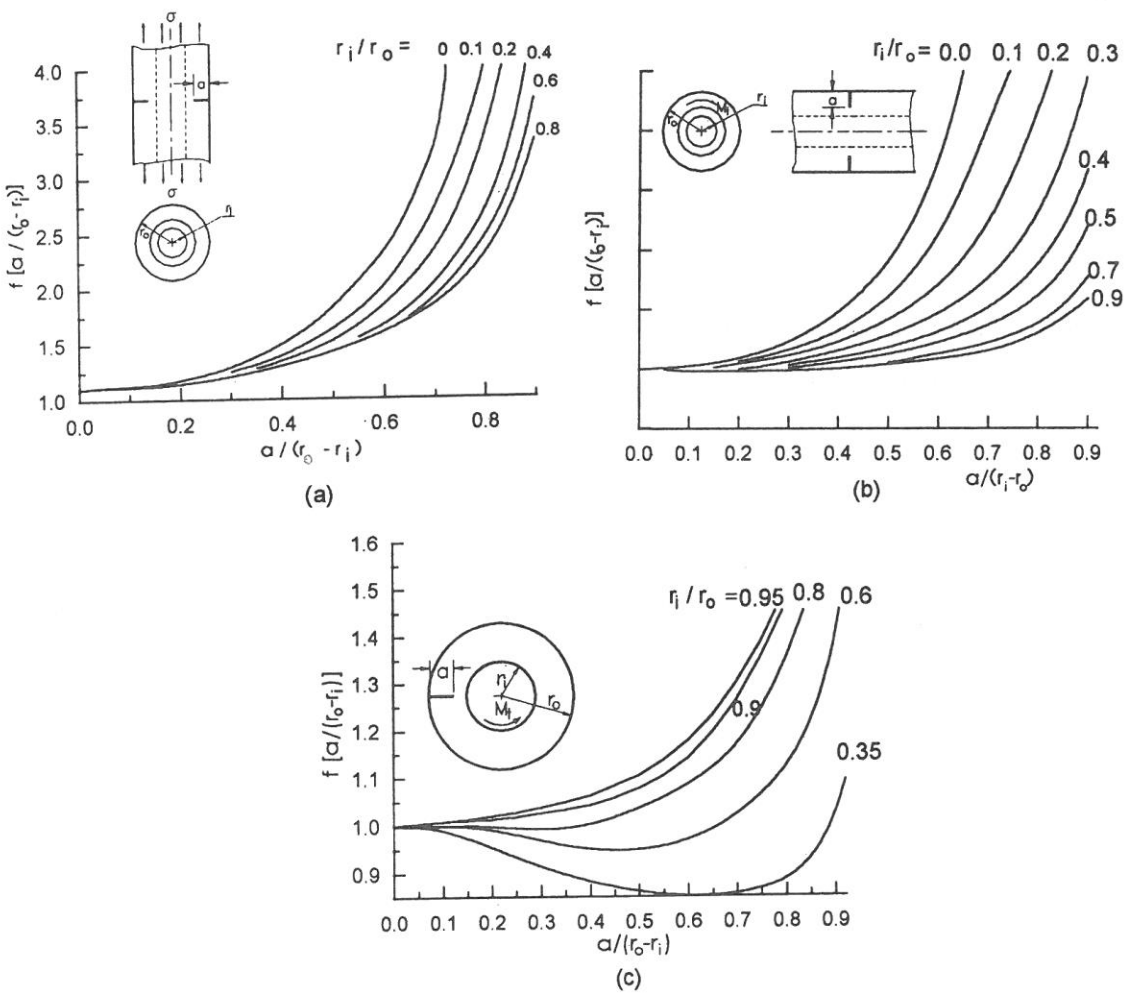


FIGURE 9.10 CCFs for loaded long tubes: (a) circumferential crack under tension, (b) circumferential crack under torsion, and (c) radial crack under torsion. (From Rooke, D. P. and Cartwright, D. J., *Compendium of Stress Intensity Factors*, The Hillingdon Press, London, 1976. With permission.)

9.6.5 Pressurized Thick-Walled Cylinders

The stress intensity factors for long, thick-walled cylinder of outside and inside radii r_o and r_i, respectively, with radial edge crack of length a, while being subjected to a uniform internal pressure is given by

$$K_I = \sigma\sqrt{\pi a}\ \text{CCF} = \sigma\sqrt{\pi a} f\left[a/(r_o - r_i)\right] \tag{9.16}$$

where $f\,[a/(r_o - r_i)]$ is the CCF as obtained from Figure 9.11a[15] for an external radial edge crack depending on the ratio r_o/r_i. Note that σ in Equation 9.16 is taken as the highest tensile hoop stress σ_θ at the location of the crack, i.e., $\sigma = (\sigma_\theta)_{r=r_o}$ as given by Equation 6.2 namely,

$$\sigma = 2p_i r_i^2/(r_o^2 - r_1^2)$$

For a pressurized long, thick-walled cylinder with internal radial crack. The configuration factor $f[a/(r_o - r_i)]$ is found from Figure 9.11b depending on the ratio r_o/r_i. In this case, the stress opening the crack comprises that due to the internal pressure $(\sigma_\theta)r{=}r_i$ according to Equations 6.2 together with the pressure as

$$\sigma = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + p_i$$

or

$$\sigma = 2p_i r_o^2/(r_o^2 - r_i^2)$$

9.6.6 Rotating Solid Disks and Drums

For a crack of length $2a$ located at the center of a rotating solid disk of radius r_o as shown in Figure 9.12(a), the stress intensity factor K_I is given by

$$K_I = \sigma\sqrt{\pi a}\ \text{CCF}$$

where σ is the crack-opening stress at the center, as given by Equation 6.23 namely,

$$\sigma = \frac{3+\nu}{8}\rho\omega^2 r_o^2 \qquad \text{for a solid disk (plane stress)}$$

$$\sigma = \frac{(3-2\nu)}{8(1-\nu)}\rho\omega^2 r_o^2 \qquad \text{for a solid drum (plane strain)}$$

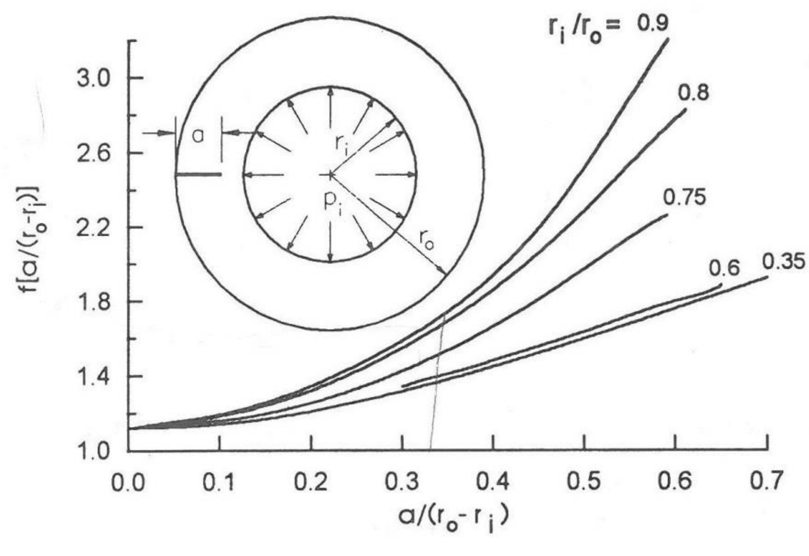
The CCF is determined according to the Expression 20:

$$\text{CCF} = 0.997 + 0.1038\left(\frac{a}{r_o}\right) + 0.6525\left(\frac{a}{r_o}\right)^2 + 0.7149\left(\frac{a}{r_o}\right)^3 \tag{9.17a}$$

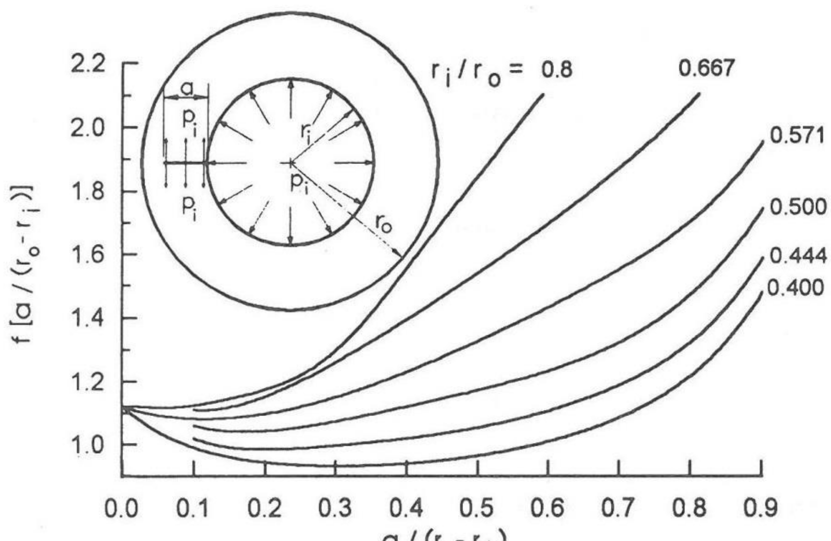
For a rotating drum with a radial external crack, as shown in Figure 9.12b, CCF may be approximated (for $\nu = 0.3$)* by[21]:

$$\text{CCF} = 1.134 + 3.465\left(\frac{a}{r_o}\right) + 2.363\left(\frac{a}{r_o}\right)^2 - 3.394\left(\frac{a}{r_o}\right)^3 + 3.848\left(\frac{a}{r_o}\right)^4 \tag{9.17b}$$

* Based on the solution of Reference 21 (in adapted form).



(a)



(b)

FIGURE 9.11 CCFs for pressurized thick-walled cylinder: (a) external radial crack and (b) internal radial crack. (From Rooke, D. P. and Cartwright, D. J., *Compendium of Stress Intensity Factors*, The Hillingdon

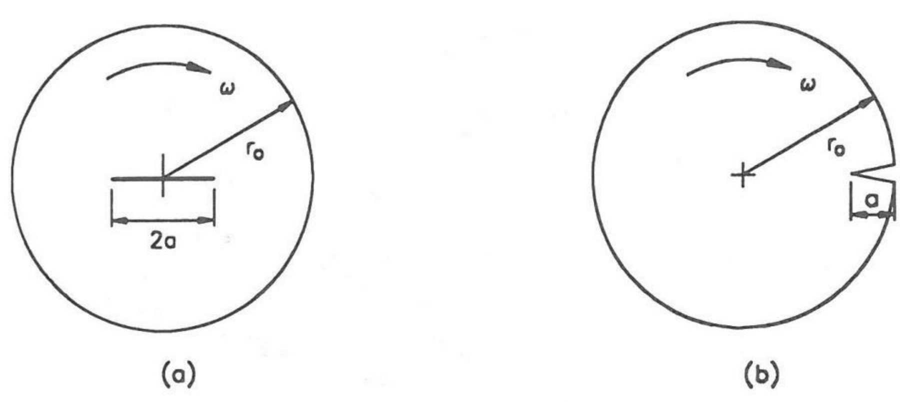


FIGURE 9.12 Cracked rotors: (a) disk with a central crack and (b) drum with a radial crack.