Analysis of Inhomogeneous Chiral Slab Using Taylor's Series Expansion

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***Abstract*—**In this paper, an analytic frequency domain method based on Taylor□s series expansion is introduced to analyze inhomogeneous planar layered chiral media for an arbitrary linear combination of TM and TE polarizations.

I. INTRODUCTION

The interaction of electromagnetic fields with chiral media has attracted many scientists and engineers over the years. Numerous studies on electromagnetic wave propagation in chiral media, such as chiral plate [1] and slab [2] and electromagnetic scattering with chiral objects [3]-[5] have been reported. The constitutive relations of an isotropic and homogeneous chiral medium assuming *exp*($j\omega t$) as time dependence are given by [6]:

$$\mathbf{D}=\varepsilon_r\varepsilon_0\mathbf{E}-j\kappa\sqrt{\varepsilon_0\mu_0}\,\mathbf{H},\qquad \mathbf{B}=j\kappa\sqrt{\varepsilon_0\mu_0}\,\mathbf{E}+\mu_r\mu_0\,\mathbf{H} \tag{1}$$

where ε_0 and μ_0 are the permittivity and permeability of vacuum, ε_r and μ_r are the relative permittivity and permeability of the chiral medium, respectively, and κ is the chirality parameter. The wave equation in a homogeneous chiral medium can be derived as following:

$$\nabla^2\mathbf{E}+2\frac{\kappa\omega}{c_0}\nabla\times\mathbf{E}+\frac{\omega^2}{c_0^2}(\mu_r\varepsilon_r-\kappa^2)\mathbf{E}=0 \tag{2}$$

where c_0 and ω are the speed of light in vacuum and the angular frequency, respectively. It can be easily seen that the RCP and LCP waves are the eigenpolarization of the wave equation in a homogeneous chiral medium [6].

The study of wave propagation in inhomogeneous chiral media which have some applications in the polarization correction of the lens and aperture antennas [7] is much more complicated than homogeneous chiral media. In the present paper, in order to analysis of inhomogeneous planar layered chiral media an analytic frequency domain method based on Taylor's series expansion is introduced.

II. THEOREY

A. Analysis of Wave Propagation in Inhomogeneous Chiral Slab

Figure 1 shows an inhomogeneous planar layered chiral medium with the thickness of t which is of infinite extent along the y-direction. Since plane wave propagating in chiral media undergoes a rotation of its polarization, TE and TM waves scattered by or transmitted through chiral media are coupled. Thus, it is assumed that a plane wave with an arbitrary polarization (an arbitrary linear combination of TM ($E_i^{\parallel}$) and TE ($E_i^{\perp}$) polarizations):

$$\mathbf{E}_i=\left[E_i^{\parallel}\left(\cos(\theta_0)\hat{x}-\sin(\theta_0)\hat{z}\right)+E_i^{\perp}\hat{y}\right]e^{-j(k_x x+k_z z)} \tag{3}$$

where $k_x = (\omega/c_0)\sin(\theta_0)$, and $k_z = (\omega/c_0)\cos(\theta_0)$, is obliquely incident with incident angle of θ_0 from free space onto the inhomogeneous chiral slab as shown in Figure 1. The chiral layer has material parameters $\varepsilon(z,\omega) = \varepsilon_0\,\varepsilon_r(z,\omega)$, $\mu(z,\omega) = \mu_0\,\mu_r(z,\omega)$, and $\kappa(z,\omega)$ which are assumed to be z and frequency dependent. In order to satisfy the boundary conditions on tangential fields at free space-chiral medium interfaces, k_x in the chiral layer must take on the same value as in free space. Thus, one can write:

$$\frac{\partial E_x}{\partial z}=\frac{\omega}{c_0}\kappa(z)\left(1-\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)E_y-j\omega\mu_0\mu_r(z)\left(1+\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)H_y \tag{4}$$

$$\frac{\partial E_y}{\partial z}=-\frac{\omega}{c_0}\kappa(z)E_x+j\omega\mu_0\mu_r(z)H_x \tag{5}$$

$$\frac{\partial H_x}{\partial z}=j\omega\varepsilon_0\varepsilon_r(z)\left(1+\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)E_y+\frac{\omega}{c_0}\kappa(z)\left(1-\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)H_y \tag{6}$$

$$\frac{\partial H_y}{\partial z}=-j\omega\varepsilon_0\varepsilon_r(z)E_x-\frac{\omega}{c_0}\kappa(z)H_x \tag{7}$$

It can be clearly seen that analytical solving of the system of differential equations describing inhomogeneous chiral layer is very difficult or maybe impossible. Thus, we discuss the use of Taylor's series expansion to solve the system of differential equations.

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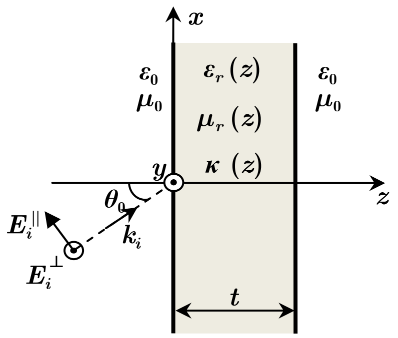

Figure 1. An inhomogeneous planar layered chiral medium illuminated by obliquely incident plane wave

B. Taylor's Series Expansion

Assuming material parameters of inhomogeneous chiral layer could be expanded by Taylor's series, one can write:

$$u(z) = \sum_{n=0}^{\infty} U_n \left(\frac{z}{t}\right)^n \tag{8}$$

where $u(z)$ can be either $j\omega\varepsilon_0\varepsilon_r(z)$, $j\omega\mu_0\mu_r(z)$, $(\omega/c_0)\ \kappa(z)$, or $1\ /\ (\kappa^2(z) - \varepsilon_r(z)\ \mu_r(z))$ and so Taylor's series coefficients U_n would be Y_n, Z_n, K_n, and A_n, respectively. Similarly, x- and y-components of electric and magnetic fields can be expressed using Taylor's series expansions with unknown coefficients Ex_n, Ey_n, Hx_n, and Hy_n, respectively. In order to determine the unknown coefficients of Taylor's series expansions of electric and magnetic fields, they should be substituted in (4) - (7), which gives:

$$Ex_{n+1} = \frac{t}{n+1}\Big[\sum_{p=0}^{n} C_{n-p} Ey_p - \sin^2(\theta_0) \sum_{p=0}^{n}\sum_{q=0}^{n-p} C_{n-q} A_q Ey_p - \sum_{p=0}^{n} Z_{n-p} Hy_p - \sin^2(\theta_0) \sum_{p=0}^{n}\sum_{q=0}^{n-p} Z_{n-q} A_q Hy_p \Big] \tag{9}$$

$$Ey_{n+1} = \frac{t}{n+1}\Big[-\sum_{p=0}^{n} C_{n-p} Ex_p + \sum_{p=0}^{n} Z_{n-p} Hx_p\Big] \tag{10}$$

$$Hx_{n+1} = \frac{t}{n+1}\Big[\sum_{p=0}^{n} Y_{n-p} Ey_p + \sin^2(\theta_0) \sum_{p=0}^{n}\sum_{q=0}^{n-p} Y_{n-q} A_q Ey_p + \sum_{p=0}^{n} C_{n-p} Hy_p - \sin^2(\theta_0) \sum_{p=0}^{n}\sum_{q=0}^{n-p} C_{n-q} A_q Hy_p \Big] \tag{11}$$

$$Hy_{n+1} = \frac{t}{n+1}\Big[-\sum_{p=0}^{n} Y_{n-p} Ex_p - \sum_{p=0}^{n} C_{n-p} Hx_p\Big] \tag{12}$$

Truncating Taylor's series expansions at the positive integer N, equations (9) - (12) for $n = 0, 1, 2, \ldots, N$-1, and boundary conditions at $z = 0$ and t, give a $(4N+4) \times (4N+4)$ system of coupled equations which can be solved either by an iterative procedure or by the inverse matrix method. Once, unknown coefficients of Taylor's series expansions were determined, the fields are obtained, and then reflection and transmission coefficients could be identified. For instance, the co-reflection coefficients can be expressed based on the Taylor's series coefficients of the electric field as the following:

$$R_{TE-TE} = \frac{Ey_0}{E_i^{\perp}} - 1, \qquad R_{TM-TM} = \frac{Ex_0}{E_i^{\parallel}\cos(\theta_0)} - 1 \tag{13}$$

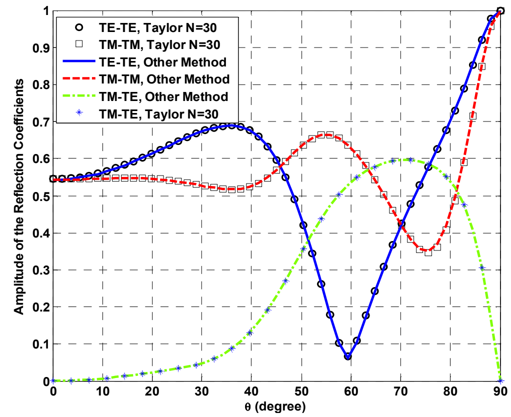

Figure 2. Co- and cross-reflection coefficients as a function of incident angle θ for the inhomogeneous chiral slab.

III. Numerical Example and Validation

In this section, scattering from an inhomogeneous chiral slab with thickness of $t = 0.2m$, $\varepsilon_r = 4$, $\mu_r = 1$, and $\kappa(z) = exp(2z)$ is considered at frequency of 1GHz. The amplitudes of the reflection coefficients obtained from the proposed method with $N = 30$ versus the angle of incidence are shown in Fig. 2. To validate the presented method, the obtained results from the method of [8] are also illustrated and compared. It is seen that the results are in an excellent agreement.

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