PEMC-Backed Perfectly Matched Layer as a Truncation Boundary

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***Abstract*— A new application for perfect electromagnetic conducting boundary, a recently introduced type of perfectly conducting boundaries creating nonreciprocal reflection is proposed. In this application, PEMC boundary is used for truncating perfectly matched layers which have been so far truncated by perfect electric/magnetic conductor in the numerical techniques based on finite methods. Due to nonreciprocal reflection from the PEMC boundary, the reflected wave can be cross-polarized with respect to the incident wave. As a result, in a 2D numerical computation, a TE/TM incident field on the truncating boundary (PEMC-backed PML) generates TM/TE reflected field, different from desired field polarization (TE/TM). Therefore the cross-polarized reflected field can be ignored in the evaluation of the desired fields.**

I. INTRODUCTION

Perfect electromagnetic conductor (PEMC) was recently introduced by Lindell and Sihvola [1]. A PEMC medium is described by the simple and peculiar relations $\overline{H} + M\overline{E} = 0$ and $\overline{D} - M\overline{B} = 0$, where M is a scalar real parameter denoting the PEMC admittance. PEMC is a nonreciprocal generalization of both the perfect electric conductor ($M \to \pm\infty$) and the perfect magnetic conductor ($M = 0$). One can easily show that field cannot convey energy into a PEMC medium and as a result PEMC is considered as an ideal boundary which has the following boundary condition [2]

$$\hat{n} \times \left(\overline{\boldsymbol{H}} + M\overline{\boldsymbol{E}} \right) = 0 . \tag{1}$$

Since PEMC boundary creates the nonreciprocal reflection, it could be used in the polarization transforming purposes and could have many rich potential applications in microwave engineering as well as antenna engineering [3]. In this paper, the PEMC boundary is proposed to be applied as truncating boundary of perfectly matched layers in the numerical techniques based on the finite methods such as finite-difference time-domain (FDTD) and finite element method (FEM). The advantage of this application is that the cross polarized reflection from PEMC-backed PML can be ignored in evaluation of the desired fields where the polarization of the desired fields is predetermined.

II. REFLECTION FROM PEMC-BACKED LAYERS

By considering a layered medium backed by a PEMC boundary at $z = z_N$, we realize that the PEMC boundary condition (1) becomes

$$\overline{\boldsymbol{E}}_{xy}(z_N) = -\frac{1}{M}\overline{\boldsymbol{H}}_{xy}(z_N). \tag{2}$$

where xy subscript is used to show transverse to z components. Assuming that the layer has an interface with free space at $z = 0$ by applying method of propagators and wave splitting technique [4] on (2), the transverse reflected field from a layered medium backed by a PEMC and illuminated by an incident plane wave is given by

$$\overline{\boldsymbol{E}}^{r}_{xy} = \overline{\overline{\mathbf{r}}}_{xy} . \overline{\boldsymbol{E}}^{i}_{xy} \tag{3}$$

In (2), $\overline{\boldsymbol{E}}^{i}_{xy}$ and $\overline{\boldsymbol{E}}^{r}_{xy}$ are the transvers incident and reflected electric field respectively, and $\overline{\overline{\mathbf{r}}}_{xy}$ is the reflection dyadic of PEMC-backed medium, given by [4]

$$\overline{\overline{\mathbf{r}}}_{xy} = \left(\overline{\overline{\mathbf{J}}} . \left(\overline{\overline{\mathbf{P}}}_{11} + \overline{\overline{\mathbf{P}}}_{12} . \overline{\overline{\mathbf{O}}}^{-1} \right) + \frac{1}{M\eta_0} \left(\overline{\overline{\mathbf{P}}}_{21} + \overline{\overline{\mathbf{P}}}_{22} . \overline{\overline{\mathbf{O}}}^{-1} \right) \right)^{-1} . \left(\overline{\overline{\mathbf{J}}} . \left(-\overline{\overline{\mathbf{P}}}_{11} + \overline{\overline{\mathbf{P}}}_{12} . \mathbf{O}^{-1} \right) + \frac{1}{M\eta_0} \left(-\overline{\overline{\mathbf{P}}}_{21} + \overline{\overline{\mathbf{P}}}_{22} . \overline{\overline{\mathbf{O}}}^{-1} \right) \right) \tag{4}$$

where $\overline{\overline{\mathbf{J}}} = -\hat{x}\hat{y} + \hat{y}\hat{x}$ is the two-dimensional rotation dyadic and $\overline{\overline{\mathbf{P}}}_{ij}$, the elements of propagator dyadic and $\overline{\overline{\mathbf{O}}}$ are 2-D dyadics, functions of the constitutive parameters of the medium, the wave number, and the incident angle given in [5].

III. PEMC-BACKED PML

Perfectly matched layers, which are lossy materials matched to free space, have been extensively used in the numerical methods as absorbing truncation boundaries. It is noted that since in the numerical techniques, the PMLs have finite depth, they cannot entirely absorb incident wave and reflections are perforce. Thus, PML regions are truncated by PEC. Here, we propose the use of PEMC boundary to truncate PML region instead of PEC. Clearly since the PEMC boundary is a perfectly conducting boundary, it reflects incident wave completely, but the reflected wave can be cross-

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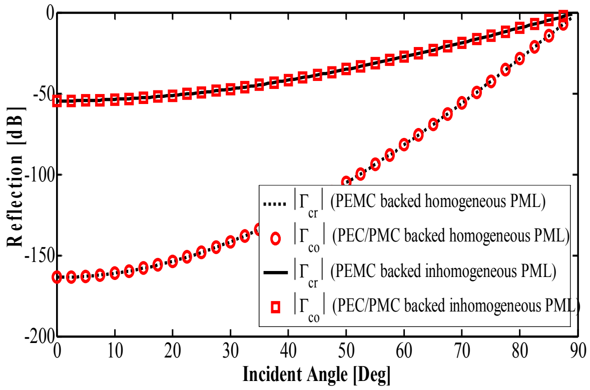

Figure. 1. Reflection coefficients from an anisotropic PML backed by a PEC, PMC, and PEMC for two cases of homogeneous and inhomogeneous PMLs.

polarized with respect to the incident wave. To explain the advantage of PEMC-backed PML, first consider a PML backed by a PEMC boundary which rotates the polarization by 90°, i.e. the normalized admittance of PEMC boundary is $M\eta_0 = \pm 1$. Now let us compare the common used PEC-backed PML with this PEMC-backed PML. The reflection of the former is co-polarized but that of the latter is cross polarized. The cross-polarization of the reflected wave can be of value if the incident and reflected waves become decoupled in numerical computations. For example in 2-D problems when the incident field is TE / TM polarized the reflection from a PEMC-backed PML will be TM / TE polarized which is completely decoupled from the incident wave.

For example, let us consider an anisotropic PML, based on the lossy uniaxial medium, introduced by Sacks et al. [6] as an alternative for Berenger's PML. It is shown that if the permittivity and permeability dyadics of a half-space medium satisfy the following relations, the PML-free space interface, normal to *z* axis, perfectly absorbs an incident plane wave with no reflection irrespective of its angle of incidence and frequency; and the transmitted wave is attenuated within the PML half-space in the direction normal to the interface, *z* direction. The relations are

$$\frac{\boldsymbol{\varepsilon}}{\varepsilon_0} = \frac{\boldsymbol{\mu}}{\mu_0} = \left(a\,\mathbf{I_t} + \frac{1}{a}\hat{z}\hat{z} \right) \tag{5}$$

where $a = 1 + j\,(\sigma/\omega\varepsilon_0)$. However, due to the finiteness of the depth of PML (backed by a conducting surface) perfect absorption does not happen and reflections are perforce. Now let us consider an anisotropic PML backed by a PEMC boundary with normalized admittance $M\eta_0 = 1$ and compare its reflection with the same PML backed by a PEC. The thickness of the PML region is considered to be 0.5 cm and its constitutive dyadics are same as (5) with $\sigma = 5$. The reflection coefficient of such a PML region backed by a PEMC boundary with $M\eta_0 = 1$ and illuminated by a TM plane wave is demonstrated in Figure 1. This figure clearly shows that the co-polarized reflected field from the PEMC-backed PML completely vanishes. Instead, the reflection is cross polarized. For the sake of comparison the reflection coefficient from PEC/PMC backed PMLs, are also calculated and illustrated in the figure. As expected, the reflection from the PEC/PMC backed PML is co-polarized. It is noted that since all perfect conductor boundaries entirely reflect electromagnetic waves, the reflections of three cases of PEC, PMC, and PEMC backed are the same in absolute; however the PEC and PMC boundaries reflect waves with co-polarization but the PEMC boundary (with $M\eta_0 = 1$) reflects waves with cross-polarization.

Usually in numerical methods, to significantly reduce numerical reflection from the PML interface, the conductivity profile of the PML is selected such that abrupt transition from one medium to another is avoided. Thus, a judicious choice for the conductivity profile of a PML region with interface at z=0 plane is [7]:

$$\sigma = \sigma_{\max} \left| \frac{z}{d} \right|^m \tag{6}$$

where $\sigma_{\max}$ is the maximum conductivity value of the anisotropic medium, *d* is the entire depth of the PML region, and *m* is the order of the spatial polynomial.

Now, let us consider an inhomogeneous PML region with conductivity profile given in (6) backed by a PEMC. In (6), $\sigma_{\max} = 5$, *m*= 2, and the thickness of PML region *d* =0.5 cm are supposed. The reflection from such a PML, backed by a PEMC are calculated and shown in Fig. 1. Furthermore, the reflection from the same PML backed by PEC and PMC are given in this figure. It is obvious that reflection from the PML interface is cross-polarized and co-polarized in the case of backing by PEMC with $M\eta_0 = 1$ and backing PEC/PMC respectively.

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