# Title: Shortcuts for Designing Finite Microstrip Reflectarrays with Non-Identical Elements in 60GHz Band

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***Abstract*** - *Microstrip Reflectarrays are attracting antennas, because not only they are constructed easily and cheaply, but also they have both reflectors' and phased arrays' properties. In order to scan the beam in phased arrays, different phase-delays are added to reflected wave, either by adding phase shifters and time delay tools to each element, or by changing the elements' size. The latter method is this paper's target because it avoids spare devices from array. In designing a finite non-identical Microstrip Reflectarray, with no existing closed formulae, required calculation time and memory is high; to lessen these parameters, infinite array analysis results are often used, but this means to bear approximations. If Microstrip Reflectarrays with large number of elements are considered, it is acceptable to gain suitable results from infinite-approximation, but what if the array size is relatively small due to its supporter's surface limitations, for example Microstrip Reflectarrays used as car radars? To find the answer, this research examines the credibility conditions of infinite-approximation for similar finite non-identical Microstrip Reflectarrays, and also presents shortcuts for designing beam stirring finite MRAs in 60GHz band. The results of sample designs are compared with HFSS analysis, at the end. The choice of 60GHz band is due to its wider application as vehicle anti-collision radars.*

**Keywords:** Microstrip, Reflector antenna, phased array, non-identical elements, beam directing.

## 1 Introduction

The use of *Microstrip Reflectarray* (MRA) antennas as reflectors has been proved to have many advantages over conventional reflector surfaces [1-2]. An MRA is a planar array of microstrip patches which reflects the illuminating wave in a desired direction. Compared with more bulky reflector antennas, such as parabolic ones, MRA has the advantage of being flat; and instead of having the depth of a parabola for concentrating the wave in desired direction, it uses phased array concept and adds different phase-delays to incident waves contacting each element.

Considering an array with identical elements, the required phase-delays can be added to each element by using phase-shifters or time-delay tools, but since the reflection and scattering coefficient phase of a patch depends on its size, this paper suggests design shortcuts for non-identical MRA, which is an MRA with elements that differ in size, and so inherently differ in reflection/scattering phase.

What makes an MRA more interesting is that it can be designed to be installed on any surface, like satellite surface, a wall or a car lateral surface, without changing the shape of its supporter, because the supporter surface curves can also be considered in calculations. That's a great advantage specially for structures whose shape and size are crucial. The other advantage is that microstrip antennas are cheaper and easier in construction.

In an array, incident electric field to each element is reflected (returned in the mirror angle of incidence) and scattered (returned in any other direction) with different amplitudes and phases, the factors showing the relation of returned fields with incident field, are called reflection and scattering coefficients shown in Eq. (1).

$$RC = \frac{\text{Re}\,flected.Electric.Field}{Incident.Electric.Field}$$
$$SC = \frac{Scattered.Electric.Field}{Incident.Electric.Field} \qquad (1)$$

If the phase-delay added to incident wave by each exclusively sized element becomes known in non-identical MRA, then the usual phase array concept can be used to concentrate reflected and scattered beams in desired direction, such as what can be found in [3-6].

Talking about phase adjustments, it becomes clear that the most important parameter in this analysis is "phase", and consequently, "Reflection coefficient phase (RCP)" and "Scattering Coefficient Phases (SCPs)", as defined in Eq. (1), will be in focus. There are no closed formulae available defining these parameters in a non-identical MRA.

The methods used for analysis of a finite MRA, introduced in many papers, e.g. [7-9], are more complicated and time/memory consuming than those of an infinite one, e.g. [10-11]; and, this complexity increases as the number of elements in the array increases.

By analyzing an infinite MRA with identical elements, using Floquet extension method, the reflection coefficient can be easily obtained. The question is that, how this result can be used for a finite non-identical MRA? To reach the answer, first, the induced current on elements of an infinite identical MRA has been calculated by using 1681 terms of Floquet extension, and current extension functions introduced in Eq. (22) of [12], then the same currents are supposed to be induced on elements of a finite identical n×n MRA illuminated by same incident wave; By adding the number of elements (n), and finding RCP of the finite MRA (using moments method), the approximate results approach the real ones for finite MRA, and when the difference between the finite and infinite results is less than a tolerable error, it can be concluded that infinite array element currents can be accepted for an finite array with this number of elements. This study has been performed for RCP in section 2. In section 3, a formula for estimating SCP and a design shortcut has been suggested. In order to approximate the induced current on each element of a non-identical finite MRA in section 3, all other elements have been assumed to have sizes identical to that one, in other words, when SCP calculations is running for an element, it is assumed that its neighbor elements have the same size as it has; this is an approximation which this paper has named it "infinite-approximation", and studied its accuracy. In section 4, a sample design has performed using suggested shortcuts.

This research is limited to square shape MRAs with rectangular elements, which is the basic structure that can be referred in the analysis of any other MRA.

## 2 Reflection Coefficient Convergence

Talking about 60GHz frequency, the inter-element distance in array must approach λ/2 which is 2.5mm for best response, so each element should have side lengths less than this value.

In Fig. 1, the $TE_z$ plane wave incidence to an array of identical 1.6×1.3mm$^2$ elements, with 2.4mm inter-element distance (in both directions; X and Y) is assumed; the permittivity and the height of substrate are assumed to be 2.22 and 0.254mm, respectively. The array is composed of square cells with side lengths of 2.4mm; each square cell contains a rectangular element. The side length of MRA ground plane, namely D, is considered slightly larger than or exactly equal to the array plane, and gets following 2 quantities when assumed to be larger than array plane.

$$D_1 = (n+2) \times 2.4mm$$
$$D_2 = (n+5) \times 2.4mm \quad (2)$$

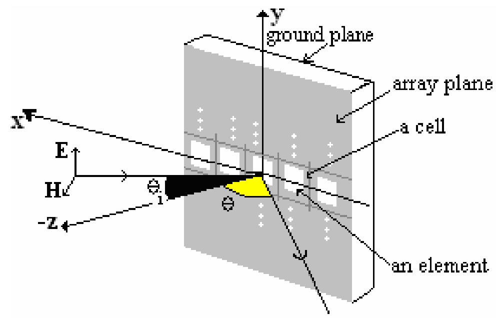



**Fig. 1** $TE_Z$ wave incidence

It is assumed that the propagation vector of the incident $TE_z$ wave is in XZ plane and makes $\Theta_i$ angle with Z axis. The choice of $TE_z$ incidence is due to two reasons, first, any optional plane wave can be described as sum of $TE_z$ and $TM_z$ components, second, the dominant current component induced on array elements is that one stimulated by $TE_z$ waves whose electric field vector is parallel to elements' larger side which is studied here.

Performing far field analysis for finite MRA and finding electric field magnitude and phase in mirror half plane, reflection coefficient and scattering coefficients can be calculated; ofcourse scattering occurs in other planes too, but in a reflector antenna, reflection plane is the most important one. θ is the angle each return beam direction makes with Z axis.

The analysis is performed for finite MRA with n (number of elements) ranging between 5 and 81 using Matlab program tool.

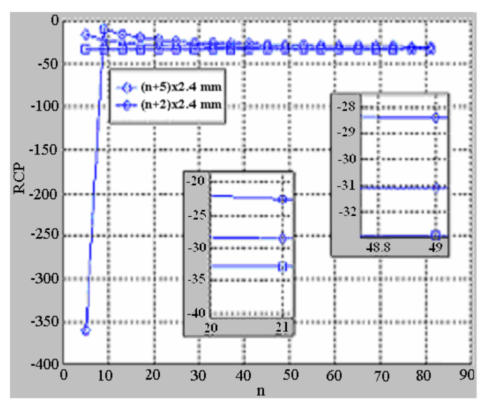



**Fig. 2** RCP convergence in normal incidence versus n

Fig. 2 shows the RCP convergence results in normal incidence condition. It can be seen that the more the array's size differs from ground plane's size, the later convergence happens. "Less than 5 degrees difference between infinite and finite results" has been considered an acceptable error; due to this, convergence happens at n=21 and n=49, for $D_1$ and $D_2$, respectively. The jumps in this curve, are just because of 360-degree phase turn, so no unacceptable change has happened.

The same process is repeated to investigate the effect of changing array parameters or incident wave parameters, on the convergence. The results have been summarized in table1.

**Table 1** Element number (n) values in which finite results converge to infinite ones per different parameter quantities

| Incidence Angle $\Theta_i$(deg) | Changed Array Parameter | n for RCP Convergence per $D_1$ | n for RCP Convergence per $D_2$ |
|---|---|---|---|
| 0 | None | 21 | 49 |
| 20 | None | 21 | 49 |
| 60 | None | 21 | 49 |
| 60 | Element Width to 1mm | 17 | 41 |
| 60 | Element Length to 2mm | 29 | 73 |
| 20 | Inter-Elements Distance to 2.5mm | 19 & 19 | 43 |
| 20 | Dielectric Thickness to 0.508mm | 57 | >81 |
| 20 | Substrate Dielectric Constant to 4.44 | 17 | 41 |

From table 1 which includes the RCP convergence conditions due to changes of different parameters in reflectarray, it can be concluded that increase in element length, element width, and dielectric thickness, delays the convergence while increase in inter-element distance and dielectric constant, speeds up the convergence. Numerical conclusions of this section have been summarizes in last section of this paper.

## 3 Scattering Coefficient Convergence

Assume an MRA whose ground plane size is exactly equal to the array plane size (D=n×2.4mm), and whose other parameters are the same as those mentioned in previous section; The incidence angle is assumed to be $20^o$ instead of previous normal one. Fig. 3 shows the SCPs for n values varying between 5 and 31. The induced current on each element has been calculated using moments method and assuming the array infinite, locally.

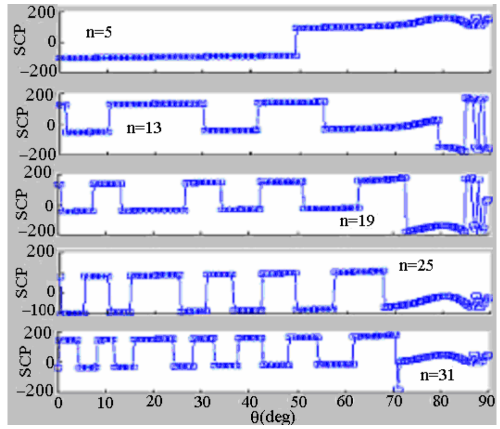



**Fig. 3** SCPs of various finite MRAs versus Θ

Note that in SCP curves related to mirror plane (relative to illumination plane), the phase related to the direction angle which is equal to incidence angle, is the same RCP; For example, in Fig. 3, the incidence angle has been assumed to be $20^o$, so the phase related to $20^o$ is the same RCP.

The semi-periodic behavior in Fig. 3, is similar to that of scattering from a finite ground plane; Since in practice, the ground plane size is almost equal to the array size, it is reasonable to see similar alternations in MRA scattering phase curves, too. As the size of the plane increases, the number of alternations (steps) increases too, thus, the step widths decrease.

For larger incidence or scattering angles, the wave path inside the dielectric layer becomes greater, and therefore, the scattering phase curve deviates from finite ground plane's curve. Chaos appeared in $70^o$-$90^o$ area of Fig. 3, are the results of this deviation.

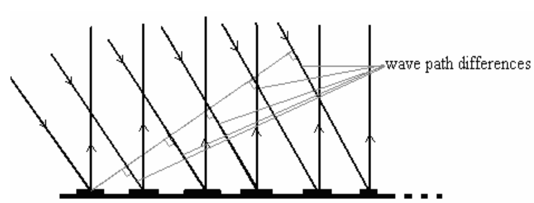



**Fig. 4** A beam stirring MRA

When a designer decides to design a non-identical MRA whose main lobe is in a special direction, like what is shown in Fig. 4, he must choose each element size so that its SCP in that direction compensates the phase-delay due to different path lengths; In this way, returned waves from all elements will add to each other in that special direction because they have no phase difference; the returned waves in other directions may even eliminate each other due to their phase differences. So the designer must be able to calculate the SCP of any element with any size, in any direction, and this is not an easy task. Fig. 3 shows approximate SCP curves for ONE special element size; now consider what a time/memory consuming work it will be to have enough curves for a design.

Having a set of curves such as Fig. 3 for finite MRA will let the designer know that in which return direction SCP is equal to RCP and in which it has a 180$^o$ difference with RCP, for each element.

Concerning the above points, this paper suggests a function that can predict the step alternations of the scattering phase curve of an MRA, and can act as a design shortcut. The function is named $\gamma$ and it can be calculated in degrees or radians.

$$\gamma_{(\theta,\varepsilon,h)} = sign\{Sinc(kD\frac{\sin\theta - \sin\theta_i}{2})\} - \{k'h(\frac{1}{\cos\theta} - \frac{1}{\cos\theta_i})\} \quad (3)$$

Where:

$$sign\{a\} = \begin{cases} 0 & a \geq 0 \\ 180 & a < 0 \end{cases}$$

k = propagation constant in air

k' = propagation constant in dielectric

h = dielectric height

$\Theta$ = scattering angle

$\Theta_i$ = incidence angle

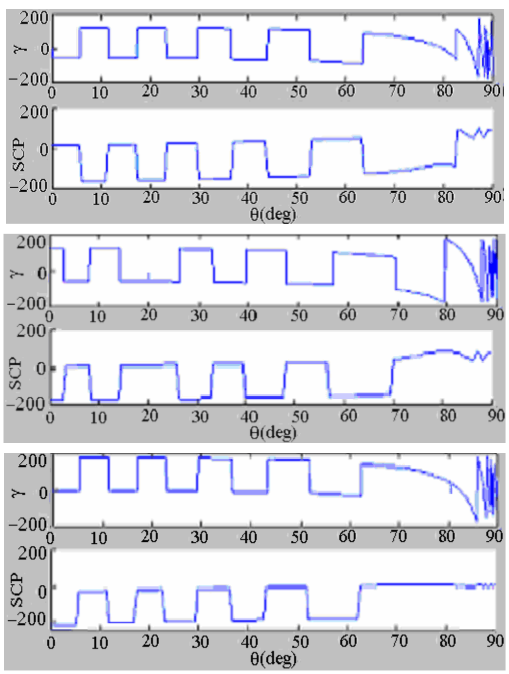



**Fig. 5** Comparison between Eq. (3) and finite MRA SCP curve for 0$^o$, 20$^o$ and 80$^o$ incidence angles, respectively, versus scattering direction $\Theta$

The first term of the above equation is scattering phase of a simple ground plane reflector which can be found in many references (e.g.[13]), and the second term shows the phase delay caused by wave path inside dielectric. Of course Eq. (3) gives approximate values, but it will be shown that this approximation generates good results. Note that $\gamma$ jumps to zero in the mirror direction of incidence angle but this jump is ignored, it has nothing to do with design process.

Fig. 5 compares the results of Eq. (3) with the SCP curves of a 21×21 MRA, for different incidence angles. Conformity is seen in curve alternation ($\pi$ jump) points, except for $\theta$s near 90$^o$. It means that simple Eq. (3) can predict the $\pi$ jump points correctly for $\theta$s less than 70$^o$.

# 4 Sample Design

To test the above described infinite-approximation, sample finite MRAs have been designed, and the analysis results of them have been compared with expected design goals.

An MRA with inter-element lengths of 2.4mm, dielectric constant of 2.22, and dielectric thickness of 0.254mm is considered at 60 GHz. The element side lengths of an n×n MRA, is calculated so that the MRA stirs the $TE_z$ oblique incident wave with $\Theta_i$=20$^o$, to normal direction in reflection. Two designs have been performed for n=9 and n=15, and the results have been compared with HFSS results which have no approximations.

The degree of freedom is more than equations, hence some parameters can be pre-selected; e.g. since the element widths have ignorable effect on phase values [15], the equal value of 1.3mm has been chosen for all element widths; also, the length of $L_0$=1.6mm is given to the central element of the array. The elements' size should not be too small; otherwise, the ground plane will alleviate the array's effect.

Fig. 6 shows the RCP (named, $\phi$) curve for an infinite MRA with constant element width values, versus element length. Similar curves can be found in other MRA papers mentioned in references [2-4]. In other words, Fig. 10 shows the RCP caused by each patch (element) size, in the case that illuminating wave makes 20$^o$ angle with array plane, calculated by method described in introduction. By this simple calculation, only RCP versus element size will be estimated.

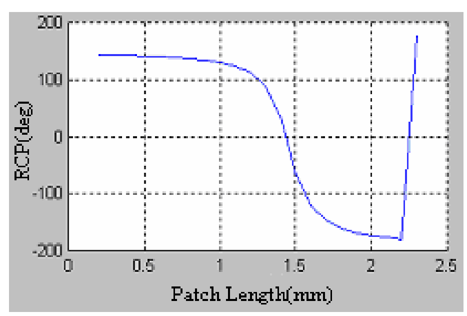



**Fig. 6** the RCP curve versus element length

Now, the simple Eq. (3) is used to find out whether RCP (shown by $\phi$) or $\phi+\pi$ can define the phase delay resulted by each element size in desired direction. Fig. 7 shows that in a 9×9 array SCP in normal direction or 0$^o$ direction (desired lobe direction) has $\pi$ radians difference with RCP (SCP in 20$^o$ which is equal to incidence angle), but for a 15×15 MRA, there is no phase difference between 0$^o$ SCP and RCP. RCP value (ϕ) for any element with any length size can be approximated by Fig. 6.

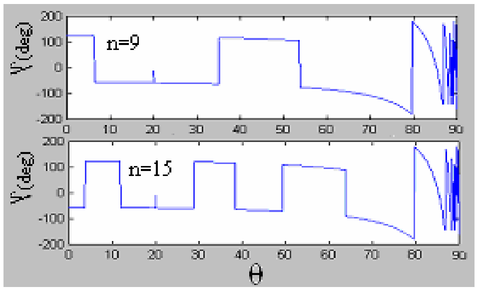



**Fig. 7** Eq. (3) for n=9 and n=15

Now that the approximate values of phase deviation caused by each element versus element length, are known; by utilizing the general phased array design process and calculating different wave path lengths, the following length side values is selected for each element in array [4]:

n=9 :

$L_1$=1.160 , $L_2$=2.380 , $L_3$=1.610 , $L_4$=1.506

$L_{-1}$=1.436 , $L_{-2}$=1.498 , $L_{-3}$=1.596 , $L_{-4}$=2.161

n=15 :

$L_1$ =1.502 , $L_2$ =1.439 , $L_3$ =1.360 , $L_4$ =1.182 , $L_5$ =2.380 , $L_6$ =1.621 , $L_7$ =1.336 ,

$L_{-1}$ =2.215, $L_{-2}$ =1.136, $L_{-3}$ =1.351, $L_{-4}$ =1.434, $L_{-5}$ = 1.496 , $L_{-6}$ = 1.591 ,$L_{-7}$ = 2.076

Positive and negative indices refer to distances from central element.

Fig. 8 shows the 15×15 designed array.

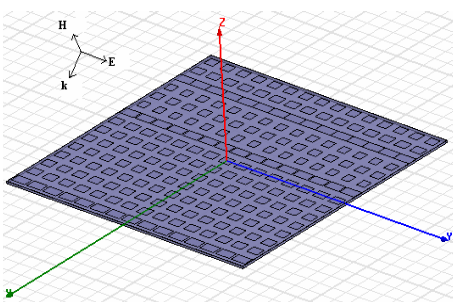



**Fig. 8** 15×15 designed MRA

In Fig. 9 the E plane pattern of designed 9×9 MRA ($MRA_1$) is compared with two similar MRAs, one composed of identical rectangular 1.66×1.3mm$^2$ elements ($MRA_2$), and the other, composed of identical square elements with side length of 1.3mm ($MRA_3$). E-plane pattern is a common measure for the performance of flat reflectors. It can be seen that the difference between electric field magnitude in 0$^o$ and 20$^o$ has reduced to 5.16dB in $MRA_1$ from 15.28dB in $MRA_3$ and 12.06dB in $MRA_2$, and just some ignorable changes have happened in other directions.

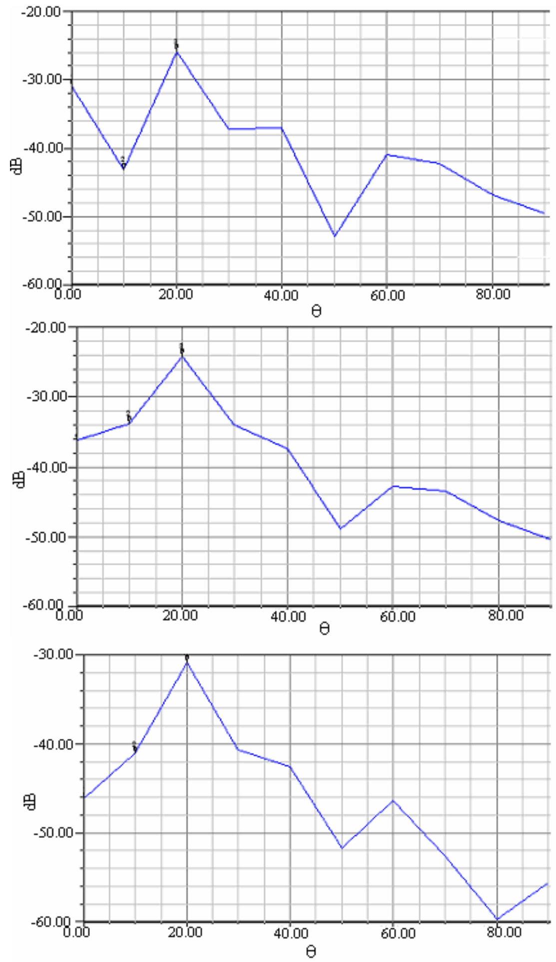
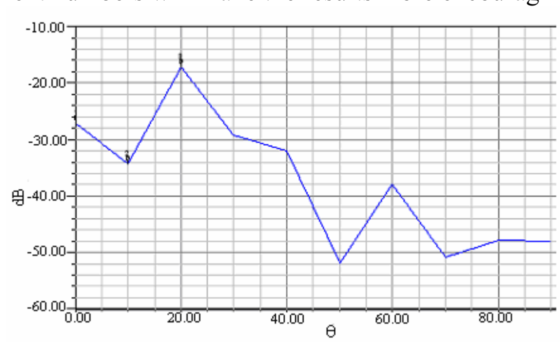



**Fig. 9** the E plane patterns of 9×9 MRAs, designed one, the one composed of identical rectangular 1.66×1.3mm$^2$ elements and the one composed of identical square elements with side length of 1.3mm, respectively

In similar way, in Fig. 10 the E plane pattern of designed 15×15 MRA ($MRA_1$) is compared with $MRA_2$, and $MRA_3$. It can be seen that the difference between electric field magnitude in 0$^o$ and 20$^o$ has reduced to 9.92dB in $MRA_1$ from 19.27dB in $MRA_3$ and 17.27dB in $MRA_2$.

Although both n=9 and n=15 are small MRAs, but the results in enhancing the beam in desired direction are encouraging, and as shown in section 1, increasing the element numbers will make the results more encouraging.



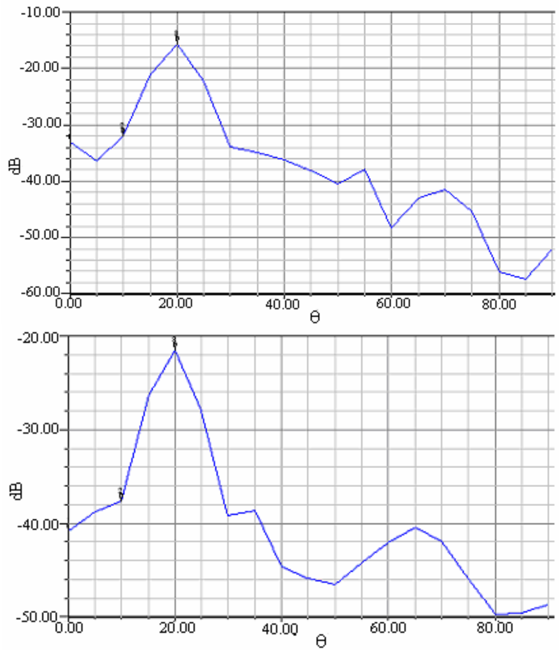



**Fig. 10** the E plane patterns of 15×15 MRAs, designed one, the one composed of identical rectangular 1.58×1.3mm$^2$ elements and the one composed of identical square elements with side length of 1.3mm, respectively

## 5 Conclusions

Although finite non-identical MRAs have proved to owe many advantages but It is a real time/memory consuming task to design them correctly. In order to represent acceptable approximations and design shortcuts to simplify this work, first, the convergence conditions of finite MRA parameters to those of an infinite one, in 60GHz band, has been investigated in this paper, and it has been shown that for MRAs with ground plane size almost equal to array size, it is enough to have at least 31×31 elements to be able to use reflection coefficient of an infinite MRA in finite MRA design with ignorable design errors. The effects of each MRA parameter on this reliability, has also been examined. So in above conditions, RCP can be calculated using infinite-approximation with ignorable error.

Second, the scattering coefficient phase curve of MRAs has been studied, and shown that it has a step-shape semi-periodic behavior, changing between $\phi$ and $\phi+\pi$. If the number of elements is large enough and the ground plane size is close to array plane size (both practical conditions), this $\phi$ is almost equal to reflection coefficient phase.

Third, in order to determine scattering coefficient phase in any direction, a simple equation is introduced which can be used to find out whether SCP for a certain direction can be approximated by $\phi$ or by $\phi+\pi$.

These approximations are reliable enough for $\Theta<70^o$ and n>31, where Θ represents both incidence and desired beam direction, and n represents array element number.

The results have been tested in small finite MRAs by comparison with HFSS results, and it was encouraging.

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