Oblique Incidence of Plane Waves on PEC, PMC or PEMC Backed Inhomogeneous Chiral Slabs

Davoud Zarifi, Homayoon Oraizi, and Mohammad Soleimani
Antenna and Microwave Research Laboratory
Department of Electrical Engineering, Iran University of Science and Technology
Tehran, Iran
zarifi@iust.ac.ir

Abstract— **An analytic method of analyzing the wave propagation and scattering problem for PEC, PMC or PEMC backed inhomogeneous planar layered chiral media based on Taylor's series expansion is introduced. The validity of the presented method is verified using two typical examples and comparing the results with the obtained results from a well-known method.**

Keywords-component: Inhomogeneous media; Chiral media; Scattering; PEMC

I. Introduction

Unlike the ordinary materials, described by electric permittivity and magnetic permeability, chiral media include a magneto-electric coupling yielding to interesting properties of the electromagnetic fields. The interaction of electromagnetic fields with chiral media has been the subject of intense research over the past decade and has lead to introduce wide applications in different microwave devices [1], [2]. The term chiral media was first used by Jaggard et al. in 1979 [3], who defined chiral media as consisting of macroscopic chiral objects randomly embedded in a dielectric. The word chiral describes something that is handed, i.e., an object whose mirror image cannot be produced solely by rotating and translating the original object and includes helices, sugar molecules and the crystal lattice of quartz. In addition to initial studies, recently, the study of electromagnetic wave propagation in chiral media is considerably developed [4]-[8]. Assuming a time harmonic field with $exp(j\omega t)$, the constitutive relations of isotropic and homogeneous chiral media are given by [1]:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \varepsilon_0 \varepsilon_r & -j\kappa\sqrt{\mu_0 \varepsilon_0} \\ j\kappa\sqrt{\mu_0 \varepsilon_0} & \mu_0 \mu_r \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \tag{1}$$

where ε_r and μ_r are the relative permittivity and permeability of chiral medium, respectively, and κ is the chirality parameter describing electromagnetic coupling.

A linearly polarized wave propagating in a chiral medium undergoes a rotation of its polarization showing that the chiral media is an optically active medium. Thus, TE and TM electromagnetic waves scattered by a chiral medium are coupled. Starting from Maxwell's equations, the wave equation in an isotropic and homogeneous chiral medium can be derived as follows:

$$\nabla^2 \mathbf{E} + 2\frac{\kappa\omega}{c_0}\nabla \times \mathbf{E} + \frac{\omega^2}{c_0^2}(\mu_r \varepsilon_r - \kappa^2)\mathbf{E} = 0, \tag{2}$$

where c_0 and ω are the speed of light in vacuum and the angular frequency, respectively. It can be seen that the right and left circularly polarized waves are the eigenpolarization of the wave equation in an isotropic and homogeneous chiral medium.

The problem of wave propagation in inhomogeneous media is important in many practical situations. In order to study wave propagation and electromagnetic scattering in inhomogeneous non-chiral media, several approach have been presented, such as Richmond method [9], solving Riccati equation [10], full-wave analysis [11], [12], use of Taylor's series expansion [13], finite difference method [14], using of Fourier series expansion [15], method of moments [16], and equivalent source method [17]. The study of wave propagation in an inhomogeneous chiral medium is much more challenging than in a homogeneous one.

This paper presents an analytic method based on the Taylor's series expansion for the frequency domain analysis of perfect electric conductor (PEC), perfect magnetic conductor (PMC) or perfect electromagnetic conductor (PEMC) backed planar layered chiral media. In Section II, the differential equations describing the inhomogeneous chiral layers are reviewed. Then the applicability of Taylor's series expansion in the analysis of inhomogeneous chiral media is presented. The accuracy of the proposed method is verified in Section III.

II. Theory and Formulation

To obtain the solution for the reflection coefficients of a PEC, PMC or PEMC backed inhomogeneous planar layered chiral media, consider the problem illustrated in Fig. 1. The chiral layer has inhomogeneous parameters $\varepsilon_r(z,\omega)$, $\mu_r(z,\omega)$, and $\kappa(z,\omega)$, which are assumed to be z and frequency dependent. Consider a plane wave with an arbitrary linear combination of TM ($E_i^{\parallel}$) and TE ($E_i^{\perp}$) polarizations:

$$\mathbf{E}_i = \left[E_i^{\parallel}\left(\cos\theta_0\,\hat{a}_x + \sin\theta_0\,\hat{a}_z\right) + E_i^{\perp}\,\hat{a}_y\right]e^{-jk_0\left(z\cos\theta_0 - x\sin\theta_0\right)} \tag{2}$$

impinging at an angle θ_0 from free space on the first interface between the left homogeneous half space and the right inhomogeneous chiral slab. Substituting the constitutive equations into Maxwell's equations, considering $\partial/\partial y = 0$

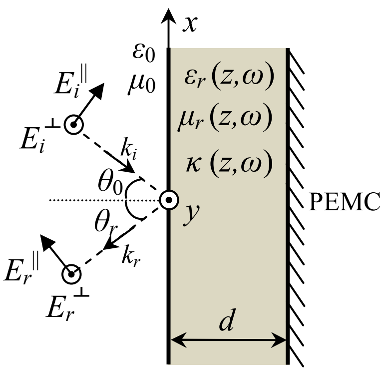

Figure 1. A typical PEC, PMC or PEMC backed inhomogeneous chiral slab exposed to an incident plane wave with an arbitrary linear combination of TM ($E_i^{\|}$) and TE ($E_i^{\perp}$) polarizations.

and $\partial/\partial x = jk_0 \sin\theta_0$, and by eliminating E_z and H_z from equations, the differential equations describing inhomogeneous chiral layer can be written in the following matrix form: right inhomogeneous chiral slab. Substituting the constitutive equations into Maxwell's equations, considering $\partial/\partial y = 0$ and $\partial/\partial x = jk_0 \sin\theta_0$, and by eliminating E_z and H_z from equations, the differential equations describing inhomogeneous chiral layer can be written in the following matrix form:

$$\frac{d}{dz}\begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{H}_x \\ \mathbf{H}_y \end{pmatrix} = \boldsymbol{\Gamma}(z)\begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{H}_x \\ \mathbf{H}_y \end{pmatrix} \tag{3}$$

where the elements of the Γ-matrix are given by:

$$\Gamma_{12} = \Gamma_{34} = \frac{\omega}{c_0}\kappa(z)(1-A) \tag{4–a}$$

$$\Gamma_{14} = -j\omega\mu_0\mu_r(z)(1+A) \tag{4–b}$$

$$\Gamma_{21} = \Gamma_{43} = -\frac{\omega}{c_0}\kappa(z) \tag{4–c}$$

$$\Gamma_{23} = j\omega\mu_0\mu_r(z) \tag{4–d}$$

$$\Gamma_{32} = j\omega\varepsilon_0\varepsilon_r(z)(1+A) \tag{4–e}$$

$$\Gamma_{41} = -j\omega\varepsilon_0\varepsilon_r(z) \tag{4–f}$$

$$\Gamma_{11} = \Gamma_{13} = \Gamma_{22} = \Gamma_{24} = \Gamma_{31} = \Gamma_{33} = \Gamma_{42} = \Gamma_{44} = 0 \tag{4–g}$$

wherein:

$$A = \frac{\sin^2(\theta_0)}{\kappa(z)^2 - \varepsilon_r(z)\mu_r(z)} \tag{5}$$

In addition, four boundary conditions at $z = 0$, d should be considered. At $z = 0$, one can write:

$$E_x(0) + \eta_0\cos(\theta_0)H_y(0) = 2E_i^{\|}\cos(\theta_0) \tag{6}$$

$$E_y(0) - \frac{\eta_0}{\cos(\theta_0)}H_x(0) = 2E_i^{\perp} \tag{7}$$

Assuming a PEMC interface at $z = d$, the boundary conditions are given by:

$$\begin{cases} H_x(d) + M\,E_x(d) = 0 \\ H_y(d) + M\,E_y(d) = 0 \end{cases}. \tag{8}$$

where M denotes the admittance of PEMC boundary. PEMC is the generalization of PEC and PMC and has been introduced by Lindell [18]. Non-reciprocity of the PEMC boundary can be demonstrated by showing that the polarization of the plane wave reflected from PEMC surface is rotated. Possibilities for the realization of a PEMC boundary are studied in terms of a layer of certain nonreciprocal materials, e.g. ferrites backed by a PEC plane [19]. Obviously, PMC and PEC correspond to $M = 0$ and $M = \infty$, respectively.

Observe that solving the aforementioned equations analytically is a hard problem. Thus, the analysis of inhomogeneous planar layered chiral media using Taylor's series expansion is presented. Assuming constitutive parameters of chiral layer and transverse components of electric and magnetic fields could be expanded using Taylor's series approach, we can write:

$$f(z) = \sum_{n=0}^{\infty} F_n\left(\frac{z}{d}\right)^n \tag{9}$$

where $f(z)$ can be either $j\omega\varepsilon_0\varepsilon_r(z)$, $j\omega\mu_0\mu_r(z)$, $(\omega/c_0)\,\kappa(z)$, or $1/(\kappa^2(z) - \varepsilon_r(z)\mu_r(z))$ with the known Taylor's series coefficients F_n, which can be either Y_n, Z_n, C_n, or A_n, respectively. Furthermore, the transverse components of electric and magnetic fields should be expressed using Taylor's series expansions with unknown coefficients Ex_n, Ey_n, Hx_n, and Hy_n, respectively. To determine the unknown coefficients of Taylor's series expansions of electric and magnetic fields, the aforementioned expansions should be substituted in (3), which gives:

$$Ex_{n+1} = \frac{d}{n+1}\Big[\sum_{p=0}^{n} C_{n-p}Ey_p - \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} C_{n-p-q}A_q Ey_p - \sum_{p=0}^{n} Z_{n-p}Hy_p - \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} Z_{n-p-q}A_q Hy_p\Big] \tag{10}$$

$$Ey_{n+1} = \frac{d}{n+1}\Big[-\sum_{p=0}^{n} C_{n-p}Ex_p + \sum_{p=0}^{n} Z_{n-p}Hx_p\Big] \tag{11}$$

$$Hx_{n+1} = \frac{d}{n+1}\Big[\sum_{p=0}^{n} Y_{n-p}Ey_p + \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} Y_{n-p-q}A_q Ey_p + \sum_{p=0}^{n} C_{n-p}Hy_p - \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} C_{n-p-q}A_q Hy_p\Big] \tag{12}$$

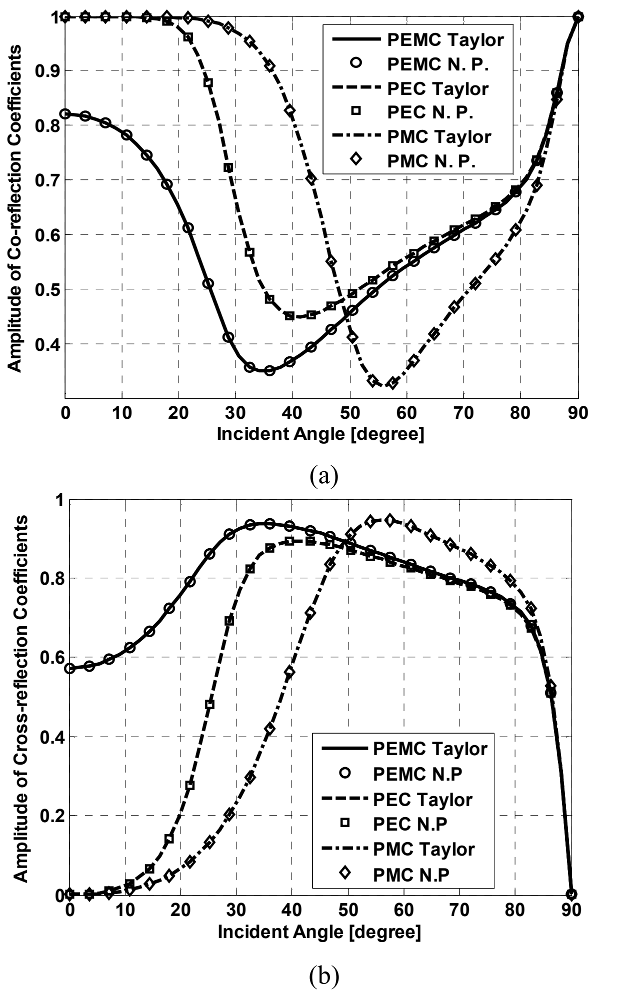

Fig. 2. Amplitudes of co- and cross reflection coefficients as a function of incident angle for PEC, PMC or PEMC backed inhomogeneous chiral slab with the constitutive parameters $\varepsilon_r(z) = 4$, $\mu_r(z) = 1$, and $\kappa(z) = 1.5 / (1 + z)$.

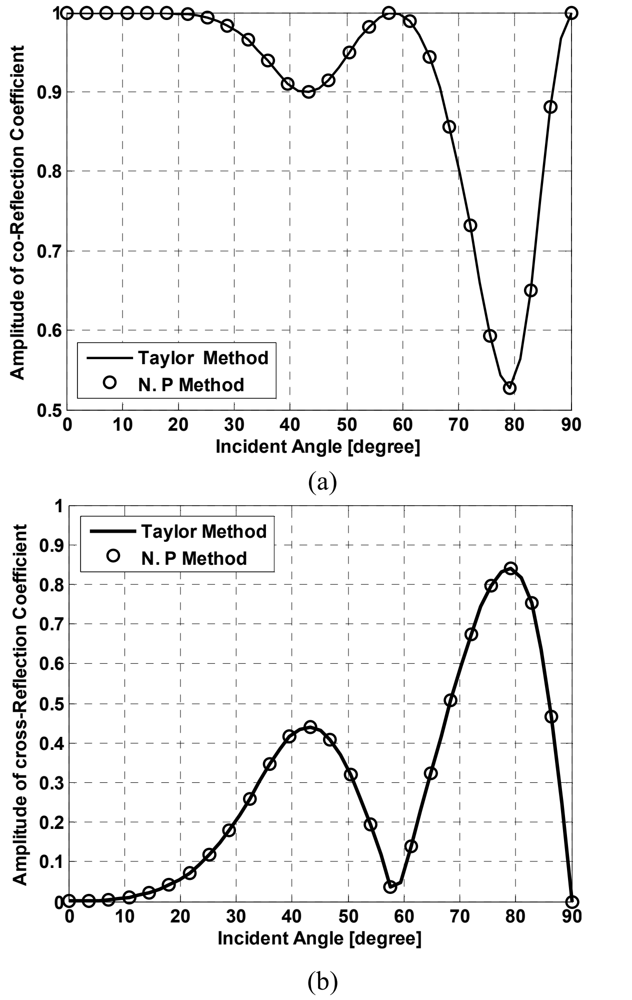

Fig. 3. Amplitudes of co- and cross reflection coefficients as a function of incident angle for PEC backed inhomogeneous chiral slab with the constitutive parameters $\varepsilon_r(z) = 4\tanh(z)$, $\mu_r(z) = 1$, and $\kappa(z) = 1.5 + z$.

$$Hy_{n+1} = \frac{d}{n+1}\Big[-\sum_{p=0}^{n} Y_{n-p} Ex_p - \sum_{p=0}^{n} C_{n-p} Hx_p \Big] . \tag{13}$$

We truncate the maximum power used in Taylor's series in (9) to N. Consequently, considering (10) – (13) for n = 0, 1, 2, …, N-1, and (6) - (9), there will be 4N+4 equations to find 4N+4 unknown coefficients. Thus, to find the unknown coefficients, a system of coupled equations should be solved either by an iterative approach or by the inverse matrix method. After finding the unknown coefficients, the electric and magnetic fields along the inhomogeneous chiral layer are determined. Also, other electromagnetic functions of the structure will be determinable. The co- and cross-reflection coefficients of the inhomogeneous chiral layer at z = 0 can be expressed as the following:

$$R_{TE-TE} = \left[\frac{E_r^{\perp}}{E_i^{\perp}}\right]_{E_i^{\parallel}=0} = \frac{Ey_0}{E_i^{\perp}} - 1 \tag{14}$$

$$R_{TM-TM} = \left[\frac{E_r^{\parallel}}{E_i^{\parallel}}\right]_{E_i^{\perp}=0} = \frac{Ex_0}{E_i^{\parallel}\cos(\theta_0)} - 1 \tag{15}$$

$$R_{TE-TM} = \left[\frac{E_r^{\perp}}{E_i^{\parallel}}\right]_{E_i^{\perp}=0} = \frac{Ey_0}{E_i^{\parallel}} \tag{16}$$

$$R_{TM-TE} = \left[\frac{E_r^{\parallel}}{E_i^{\perp}}\right]_{E_i^{\parallel}=0} = \frac{Ex_0}{E_i^{\perp}\cos(\theta_0)} \tag{17}$$

III. Numerical Examples

In this section, in order to verify the validity of the method, two special types of inhomogeneous chiral slabs are considered to analyze the presented method.

A. *Example* 1

In order to verify the accuracy of the proposed method, consider the problem of scattering from a PEC, PMC, or

PEMC (M = 0.02) backed inhomogeneous chiral slab with relative permittivity, relative permeability, chirality parameter as follows:

$$\varepsilon_r(z) = 4, \quad (18)$$

$$\mu_r(z) = 1, \quad (19)$$

$$\kappa(z) = \frac{1.5}{1+z} \quad (20)$$

and thickness d = 0.2 m. Assume a plane wave with TE polarization, unity amplitude, and the excitation frequency of 1 GHz obliquely illuminating the chiral slab. The amplitudes of co-reflection coefficients versus the angle of incidence obtained by the Taylor's series expansion method with $N = 40$ and the results obtained by the Notation of Propagator (NP) method [20] based on cascading thin linear layers, are shown in Fig. 2. Observe that there is an excellent agreement between the results from the two different methods. It is evident that, as the number of unknown coefficients increases the accuracy of the solution increases. Also, as the thickness of the slab with respect to the wavelength increases, the necessary number of unknown coefficients increases.

B. *Example* 2

As the second example, a PEC backed inhomogeneous chiral layer with thickness of 0.2 m is considered. Expressions of the relative permittivity, relative permeability, and chirality parameter used for this example are

$$\varepsilon_r(z) = 4\tanh(z), \quad (21)$$

$$\mu_r(z) = 1, \quad (22)$$

$$\kappa(z) = 1.5 + z\ . \quad (23)$$

Assume a plane wave with TE polarization, unity amplitude, and the excitation frequency of 1 GHz obliquely illuminates the slab. The amplitudes of co- and cross-reflection coefficients versus the angle of incidence obtained by the Taylor's series expansion method with $N = 40$ and the results obtained by the NP method [20] are shown in Fig. 3. Comparison between the results illustrates the good behavior of the proposed method.

The consumed time for this method is less than few seconds. Also, the proposed method is a systematic approach allowing one to simply implement it in a programming language supporting matrix manipulations.

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