Analysis of Inhomogeneous Chiral Media Using Taylor's Series Expansion

Davoud Zarifi, Mohammad Soleimani, Ali Abdolali, and Vahid Nayyeri
Antenna and Microwave Research Laboratory
Department of Electrical Engineering, Iran University of Science and Technology
Tehran, Iran
zarifi@iust.ac.ir

***Abstract*—In this paper, an analytic frequency domain method based on Taylor's series expansion is introduced to analyze inhomogeneous planar layered chiral media for an arbitrary linear combination of TM and TE polarizations.**

I. INTRODUCTION

The interaction of electromagnetic fields with chiral media has attracted many scientists and engineers over the years. Recently, there is rapid development on the study of electromagnetic wave propagation in chiral media, such as chiral plate [1], chiral slab [2], and electromagnetic scattering with chiral objects [3]-[5]. The constitutive relations of an isotropic and homogeneous chiral medium assuming $exp(j\omega t)$ as time dependence are given by [6]:

$$\mathbf{D}=\varepsilon_r\varepsilon_0\mathbf{E}-j\kappa\sqrt{\varepsilon_0\mu_0}\,\mathbf{H},\qquad \mathbf{B}=j\kappa\sqrt{\varepsilon_0\mu_0}\,\mathbf{E}+\mu_r\mu_0\,\mathbf{H} \tag{1}$$

where ε_0 and μ_0 are the permittivity and permeability of vacuum, ε_r and μ_r are the relative permittivity and permeability of the chiral medium, respectively, and κ is the chirality parameter. The wave equation in a homogeneous chiral medium can be derived as following:

$$\nabla^2\mathbf{E}+2\frac{\kappa\omega}{c_0}\nabla\times\mathbf{E}+\frac{\omega^2}{c_0^2}(\mu_r\varepsilon_r-\kappa^2)\mathbf{E}=0 \tag{2}$$

where c_0 and ω are the speed of light in vacuum and the angular frequency, respectively. It can be easily seen that the RCP and LCP waves are the eigenpolarization of the wave equation in a homogeneous chiral medium [6].

The study of wave propagation in inhomogeneous chiral media which have some applications in the polarization correction of the lens and aperture antennas [7] is much more complicated than homogeneous chiral media. In the present paper, in order to analysis of inhomogeneous planar layered chiral media an analytic frequency domain method based on Taylor's series expansion is introduced.

II. INHOMOGENEOUS CHIRAL SLAB

A. Analysis of Wave Propagation

Figure 1 shows an inhomogeneous planar layered chiral media with the thickness of t. A plane wave propagating in chiral media undergoes a rotation of its polarization, and so TE and TM waves scattered by or transmitted through chiral media are coupled. Thus, it is assumed that a plane wave with an arbitrary polarization (an arbitrary linear combination of TM ($E_i^{\parallel}$) and TE ($E_i^{\perp}$) polarizations):

$$\mathbf{E}_i=\left[E_i^{\parallel}\left(\cos(\theta_0)\hat{x}-\sin(\theta_0)\hat{z}\right)+E_i^{\perp}\hat{y}\right]e^{-j(k_x x+k_z z)} \tag{3}$$

where $k_x = (\omega/c_0)\sin(\theta_0)$, and $k_z = (\omega/c_0)\cos(\theta_0)$, is obliquely incident with incident angle of θ_0 from free space onto an inhomogeneous chiral slab. The chiral layer has material parameters $\varepsilon(z,\omega) = \varepsilon_0\,\varepsilon_r(z,\omega)$, $\mu(z,\omega) = \mu_0\,\mu_r(z,\omega)$, and $\kappa(z,\omega)$ which are assumed to be z and frequency dependent. The planar structure is of infinite extent along the y-direction, and in the inhomogeneous chiral layer k_x must take on the same value as in free space in order to satisfy the boundary conditions on tangential fields at the boundaries. Since, one can write:

$$\begin{aligned}\frac{\partial E_x}{\partial z}=&\frac{\omega}{c_0}\kappa(z)\left(1-\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)E_y\\&-j\omega\mu_0\mu_r(z)\left(1+\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)H_y\end{aligned} \tag{4}$$

$$\frac{\partial E_y}{\partial z}=-\frac{\omega}{c_0}\kappa(z)E_x+j\omega\mu_0\mu_r(z)H_x \tag{5}$$

$$\begin{aligned}\frac{\partial H_x}{\partial z}=&\,j\omega\varepsilon_0\varepsilon_r(z)\left(1+\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)E_y\\&+\frac{\omega}{c_0}\kappa(z)\left(1-\frac{\sin^2(\theta_0)}{\kappa^2(z)-\varepsilon_r(z)\mu_r(z)}\right)H_y\end{aligned} \tag{6}$$

$$\frac{\partial H_y}{\partial z}=-j\omega\varepsilon_0\varepsilon_r(z)E_x-\frac{\omega}{c_0}\kappa(z)H_x \tag{7}$$

It can be clearly seen that solving analytically the system of differential equations describing inhomogeneous chiral layer is very difficult. Thus, we discuss the use of Taylor's series expansion to solve the system of differential equations.

B. Taylor's Series Expansion

In this section, the use of Taylor's series expansion is discussed to solve the system of differential equations.

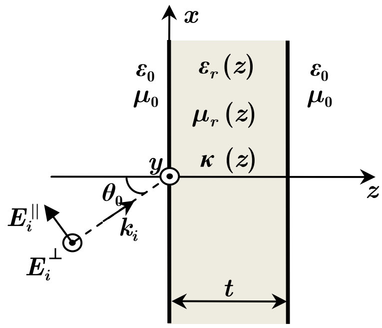

Figure 1. An inhomogeneous planar layered chiral media

Assuming material parameters of inhomogeneous chiral layer and could be expanded using Taylor's series approach, one can write:

$$u(z) = \sum_{n=0}^{\infty} U_n \left(\frac{z}{t}\right)^n \tag{8}$$

where $u(z)$ can be either $j\omega\varepsilon_0\varepsilon_r(z)$, $j\omega\mu_0\mu_r(z)$, $(\omega/c_0)\ \kappa(z)$, or $1 / (\kappa^2(z) - \varepsilon_r(z)\ \mu_r(z))$ with the known Taylor's series coefficients U_n which can be either Y_n, Z_n, K_n, and A_n, respectively. Similarly, x- and y- components of electric and magnetic fields can be expressed using Taylor's series expansions with unknown coefficients Ex_n, Ey_n, Hx_n, and Hy_n, respectively. In order to determine the unknown coefficients of Taylor's series expansions of electric and magnetic fields, they should be substituted in (4) - (7), which gives:

$$\begin{aligned} Ex_{n+1} = \frac{t}{n+1}\Big[\sum_{p=0}^{n} C_{n-p} Ey_p - \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} C_{n-q} A_q Ey_p \\ - \sum_{p=0}^{n} Z_{n-p} Hy_p - \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} Z_{n-q} A_q Hy_p \Big] \end{aligned} \tag{9}$$

$$Ey_{n+1} = \frac{t}{n+1}\Big[-\sum_{p=0}^{n} C_{n-p} Ex_p + \sum_{p=0}^{n} Z_{n-p} Hx_p\Big] \tag{10}$$

$$\begin{aligned} Hx_{n+1} = \frac{t}{n+1}\Big[\sum_{p=0}^{n} Y_{n-p} Ey_p + \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} Y_{n-q} A_q Ey_p \\ + \sum_{p=0}^{n} C_{n-p} Hy_p - \sin^2(\theta_0)\sum_{p=0}^{n}\sum_{q=0}^{n-p} C_{n-q} A_q Hy_p \Big] \end{aligned} \tag{11}$$

$$Hy_{n+1} = \frac{t}{n+1}\Big[-\sum_{p=0}^{n} Y_{n-p} Ex_p - \sum_{p=0}^{n} C_{n-p} Hx_p\Big] \tag{12}$$

Truncating Taylor's series expansions at the positive integer N, equations (9) - (12) for n = 0, 1, 2, …, N-1, and boundary conditions at z = 0 and t, give a $(4N+4) \times (4N+4)$ system of coupled equations which can be solved either by an iterative procedure or by the inverse matrix method. Once, unknown coefficients of Taylor's series expansions were determined, reflection and transmission coefficients could be identified. For instance, the co-reflection coefficients can be expressed based on the Taylor's series coefficients of the electric field as the following:

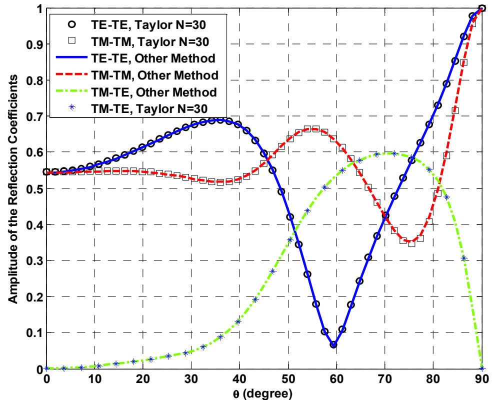

Figure 2. Co- and cross-reflection coefficients as a function of incident angle θ for homogeneous chiral slab.

$$R_{TE-TE} = \frac{Ey_0}{E_i^{\perp}} - 1, \qquad R_{TM-TM} = \frac{Ex_0}{E_i^{||}\cos(\theta_0)} - 1 \tag{13}$$

C. An Example and Results

In this section, scattering from an inhomogeneous chiral slab with thickness of t = 0.2 m, and relative permittivity, relative permeability, and chirality parameter with profiles ε_r = 4, μ_r = 1, and $\kappa(z) = exp(2z)$, respectively, at f = 1 GHz is considered. The amplitudes of the reflection coefficients obtained from the proposed method with N = 30 versus the angle of incidence are shown in Fig. 2. The obtained results from the modified method of [8] are also illustrated. It is seen that the results are in the excellent agreement.

References

[1] N. Engheta, and A. R. Mickelson, "Transition radiation caused by a chiral plate," *IEEE Trans. Antennas Propagat.*, vol. 30, pp. 1213-1216, 1982.

[2] S. Bassiri, C. H. Papas, and N. Engheta, "Electromagnetic wave propagation through a dielectric-chiral interface and through a chiral slab," *J. Opt. Soc. Am.*, vol. 5, 1450-1459, 1988.

[3] Z. Chen, W. Hong, and W. Zhang, "Electromagnetic scattering from a chiral cylinder-general case," *IEEE Trans. Antennas Propag.*, vol. 44, no. 7, pp. 912–917, 1996.

[4] Y. Shi, C. H. Chan, "Solution to electromagnetic scattering by bi-isotropic media using multilevel green's function interpolation method," *Progress In Electromagnetics Research*, PIER 97, pp. 259-274, 2009.

[5] J. Bao, D. Wang, E. K. N. Yang, "Electromagnetic scattering from an arbitrary shaped bi-isotropic body of revolution," *IEEE Trans. Antennas Propag.*, vol. 58, no. 5, pp. 1689-1698, 2010.

[6] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-isotropic Media*, Artech House, Boston, 1994.

[7] A. J. Viitanen, I. V. Lindell, "Chiral slab polarization transformer for aperture antennas," *IEEE Trans. Antennas Propag.*, vol. 46, no. 9, pp. 1395-1397, 1998.

[8] S. Rikte, G. Kristensson, M. Andersson, "Propagation in bianisotropic media - reflection and transmission," *IEE Proc.-Microw. Antennas Propag.*, vol. 148, no. 1, Febreuary 2001.