RCS Computation of Airplane Using Parabolic Equation

A. R. Mallahzadeh[1, 2], M. Soleimani[2], J. Rashed-Mohassel[3]
[1]Iran Telecommunication Research Center, Tehran, Iran
[2]Iran University of Science and Technology, Tehran, Iran
[3]University of Tehran, Tehran, Iran

***Abstract*-The parabolic equation method gives accurate results in calculation of scattering from objects with dimensions ranging from one to tens of wavelengths. Solving parabolic equation with the marching method needs limited computer storage even for scattering calculations of large targets. In this paper, first the calculation procedure of radar cross section using parabolic equation in three dimension is studied and the necessary equations are derived. In order to show the validity of the parabolic equation, the RCS of a conducting sphere is calculated and the results are compared with analytic results. The airplane RCS has been computed by using a staircase model in the parabolic equation and The results are compared with physical optics results.**

I. INTRODUCTION

Parabolic equation is an approximation of the wave equation which models energy propagating in a cone centered on a preferred direction, the paraxial direction. The parabolic equation was first introduced by Leontovich and Fock in order to study the diffraction of radiowaves around the earth [1]. By the advent of advanced computers closed form solution of the parabolic equation was replaced by numerical solutions. Since then, the parabolic equation is being applied to radar, sonar, acoustic and wave propagation. The parabolic equation has been recently used in scattering calculations in acoustics [2] and electromagnetics [3].

II. THE PARABOLIC EQUATION FRAMWORK

In this paper we concentrate on three dimensional analysis using parabolic equation. In all equations, the time dependence of the fields is assumed as $exp(-j\omega t)$. For horizontal polarization, the electric field $\vec{E}$ only has non-zero component E_z ,while for vertical polarization, the magnetic field $\vec{H}$ has the only one non-zero component H_z . The reduced function u is defined as

$$u(x,y,z) = \exp(-ikx)\psi(x,y,z) \tag{1}$$

In which $\psi(x,y,z)$ is the E_z component for horizontal polarization and H_z component for vertical polarization. The paraxial direction is assumed along the x axis. Assuming the refractive index of the medium, n , the field component ψ satisfies the following three dimensional wave equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+k^2n^2\psi = 0 \tag{2}$$

Using equations (1) and (2), the wave equation in terms of u is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+2ik\frac{\partial u}{\partial x}+k^2(n^2-1)u = 0 \tag{3}$$

Considering $Q=\sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial y^2}+\frac{1}{k^2}\frac{\partial^2}{\partial z^2}+n^2}$, (3) is reduced

$$\frac{\partial^2 u}{\partial x^2}+2ik\frac{\partial u}{\partial x}+k^2(Q^2-1)u = 0 \tag{4}$$

Which can be written as

$$\left[\frac{\partial}{\partial x}+ik(1+Q)\right]\left[\frac{\partial}{\partial x}+ik(1-Q)\right]u = 0 \tag{5}$$

Decomposing equation (5), the following pair of equations is obtained

$$\frac{\partial u}{\partial x} = -ik(1-Q)u \tag{6-1}$$

$$\frac{\partial u}{\partial x} = -ik(1+Q)u \tag{6-2}$$

The solution to (6-1) corresponds to the forward propagating waves while that of (6-2) concerns the backward propagating waves.

III. SCATTERED FIELDS CALCULATION

The simplest approximation of (6-1) is obtained using the first order expansion of Taylor series. Using this approximation, the standard parabolic equation is obtained. We assume Q as

$$Q=\sqrt{Y+Z+1} \tag{7}$$

In which

$$Y=\frac{1}{k^2}\frac{\partial^2}{\partial y^2}, Z=\frac{1}{k^2}\frac{\partial^2}{\partial z^2}+n^2-1$$

Using the Feit and Fleck approximation to decouple Y and Z [4], we will have

$$\sqrt{Y+Z+1}\sim\sqrt{Y+1}+\sqrt{Z+1}-1 \tag{8}$$

Using the first order Taylor series of each radical of (8) and Substituting in (6-1) yields

$$\frac{\partial u}{\partial x} = \frac{ik}{2}(Y + Z)u \tag{9}$$

With regard to the definition of Y and Z, equation (9) is reduced to the following form

$$\frac{\partial u}{\partial x} - \frac{i}{2k}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{ik}{2}(n^2 - 1)u = 0 \tag{10}$$

This equation is the standard parabolic equation. Equation (10) is the narrow angle approximation of parabolic equation in three dimension. Scattered field and RCS of object can be calculated by using equation (10). Integration domain is considered as a box which embraces the object. Integration domain must be truncated in the transverse plane. In order to do this we used PML in transverse plane. At first PML has been introduced as an absorbing boundary condition to solve Maxwell's equations. PML has been used as an absorbing boundary condition to solve parabolic equation by Collino [5]. Important advantage of PML is its efficiency for all incident angles by using it in a few grid points of integration domain. Integration domain with PML absorbing boundary condition has been shown in figure 1.

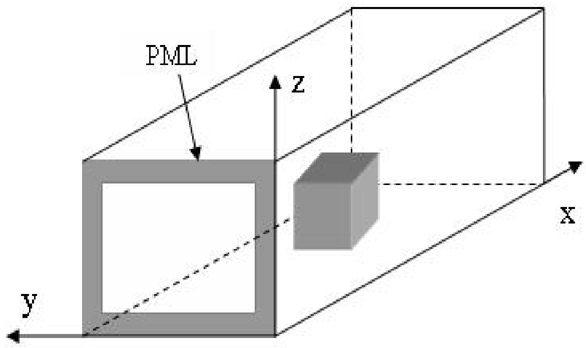

Figure 1. Integration domain with PML absorbing boundary condition.

We discretize equation (10) on rectangular grid by using finite difference method. In order to discretize parabolic equation the Crank-Nicolson scheme is usually utilized. In this paper we use another scheme which shows better stability compared to the Crank-Nicolson scheme [6]. We define region $(m\Delta x, y, z)$ as range m. Despite Crank-Nicolson's scheme, in which second order derivatives with respect to y and z are calculated by averaging between range m and range m-1, this scheme calculates second order derivatives just in range m. This work decreases the accuracy of discretizing scheme with respect to the Crank-Nicolson and requires smaller Δx. The boundary of the object must be modeled accurately in scattering problems, therefore a smaller Δx is needed. Discretizing equation (10) for free space yields

$$\frac{u^m_{i,j} - u^{m-1}_{i,j}}{\Delta x} = \frac{i}{2k}\left(\frac{u^m_{i-1,j} - 2u^m_{i,j} + u^m_{i+1,j}}{\Delta y^2} + \frac{u^m_{i,j-1} - 2u^m_{i,j} + u^m_{i,j+1}}{\Delta z^2}\right) \tag{11}$$

By using equation (11), we can calculate fields in range m versus range m-1. Positions of grid points, while can be determined regarding to equation (11), are shown in figure 2. In two dimensional analysis by parabolic equation, we have to invert a triangular matrix to obtain u at range x_m. In three dimensional case coefficient matrix is a very large sparse matrix and we can not solve resulting equations with direct inversion. In this paper we used conjugate gradient method to calculate u at range x_m [7].

In order to calculate fields in the all points of integration domain, first, the fields should be determined at range x_0. The incident field is assumed as a plane wave with unit amplitude as

$$u(x,y,z) = \exp(ik(x(\cos\theta - 1) + y\sin\theta\cos\varphi + z\sin\theta\sin\varphi)) \tag{12}$$

In which θ, φ are the angles of incident plane wave with x and y axis respectively.

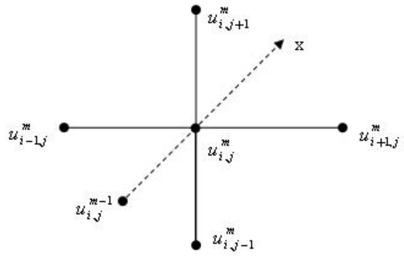

Figure 2. Positions of grid points regarding to equation (11).

IV. COMPUTATION OF RADAR CROSS SECTION

After the calculation of fields over the entire computational domain, we can compute the fields within any arbitrary domain x as a function of the fields in the domain x_0 in free space as follows [8]

$$u(x,y,z) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u(x_0,y',z')\left[ik\frac{(x-x_0)}{d(y',z')} - \frac{1}{d(y',z')}\right]\cdot \frac{e^{ikd(y',z')}}{d(y',z')}dy'dz' \tag{13}$$

In which

$$d(y',z') = \sqrt{(x_0 - x')^2 + (y - y')^2 + (z - z')^2}$$

The radar cross section is defined as

$$\sigma(\theta,\varphi) = \lim_{r\to\infty} 4\pi r^2 \left|\frac{u^s(x,y,z)}{u^i(x,y,z)}\right|^2 \quad (14)$$

In which we have

$$x = r\cos\theta \,, y = r\sin\theta\cos\varphi \,, z = r\sin\theta\sin\varphi$$

Tending (x,y,z) to infinity along a given direction in (13), and assuming a unit amplitude for the incident wave, (14) yields [8]:

$$\sigma(\theta,\psi) = \frac{k^2\cos^2\theta}{\pi}\left|\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_s(x_0,y',z')e^{-ik\sin\theta(y'\cos\varphi+z'\sin\varphi)}dy'dz'\right|^2 \quad (15)$$

In which $u_s(x,y,z)$ is the scattered field.

V. RCS RESULTS

In this section airplane RCS results which are calculated by the parabolic equation is presented. In order to show the validity of the parabolic equation solution, the RCS of a conducting sphere with radius 10λ is calculated. The incident wave is a plane wave with horizontal polarization and a wavelength equal to 1 meter (corresponding to 300MHz). The angle of incident wave is zero. RCS of the conducting sphere is illustrated in figure (3). Parabolic equation results and analytic results, where obtained from the extraction of Hankel and Legendre functions [9], are shown with solid line and circle marker respectively in figure (3). As it can be seen, there is a good agreement between the analytical results and the parabolic equation results up to angles about 10 degrees.

In order to calculate the scattered fields from the airplane, its staircase model is utilized. Dimensions of airplane and its staircase model are shown in figures (4) and (5) respectively. Integration domain size in order to calculate scattered fields from the airplane has been considered as 40λ , 30λ and 30λ in the x, y and z directions respectively. The grid spacing in the x, y and z directions are assumed $\lambda/4$, $\lambda/5$ and $\lambda/5$ respectively.

The incident wave is a plane wave with horizontal polarization and a wavelength equal to 1 meter. The angle of incident wave is zero. Near field results in planes $z = 11.3m$ and $x = 25m$ are shown in figures (6) and (7) respectively. RCS results of airplane in planes $\varphi = 0°$ and $\varphi = 90°$ are presented in figures (8) and (9) respectively. Solid lines represent parabolic equation results and diamond markers represent physical optic results.

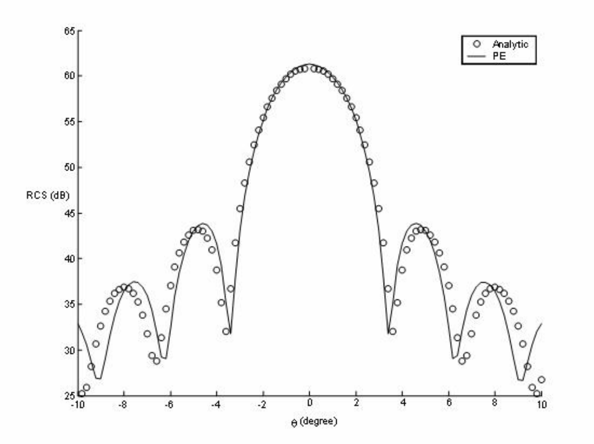

Figure 3. RCS of the conducting sphere with radius 10λ .

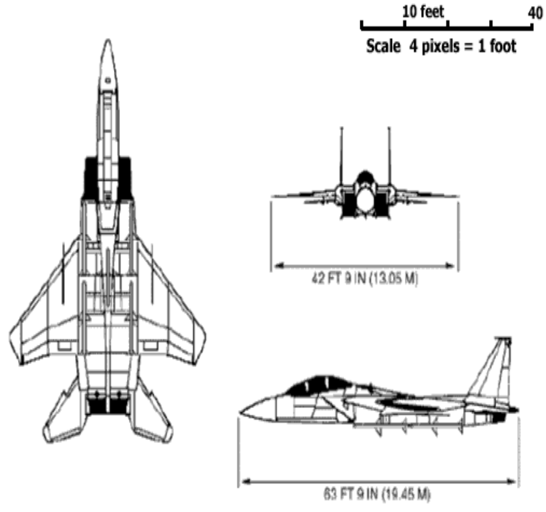

Figure 4. The geometry of the actual airplane and its dimensions.

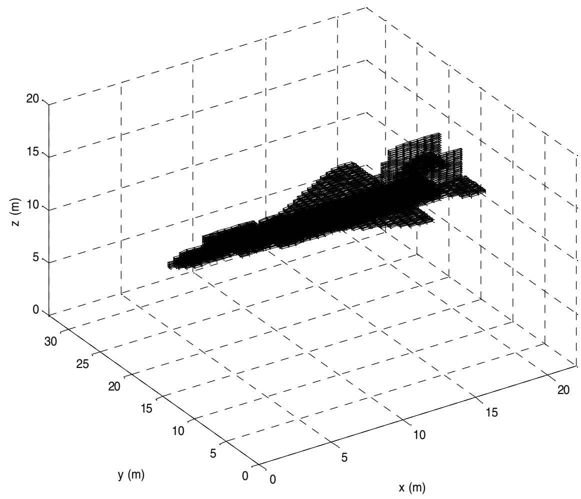

Figure 5. Airplane staircase model (dimensions are in meters)

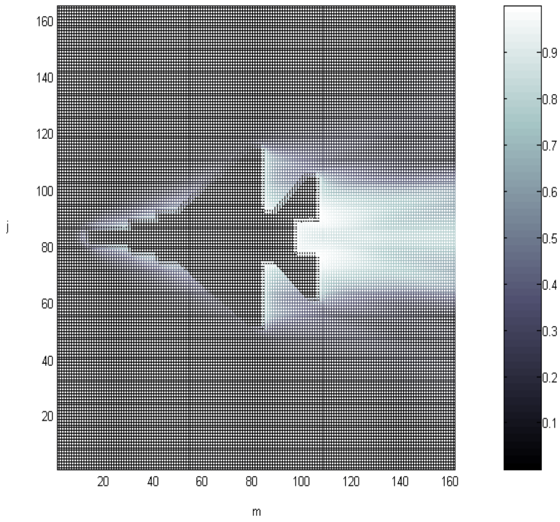

Figure 6. Amplitude of the scattered field $u_S(m,i,j)$ from the airplane in z=11.3m for 300 MHz.

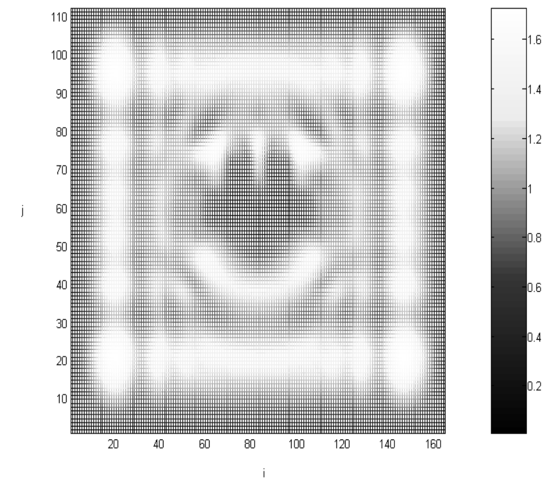

Figure 7. Amplitude of the scattered field $u_S(m,i,j)$ from the airplane in x=25m for 300 MHz.

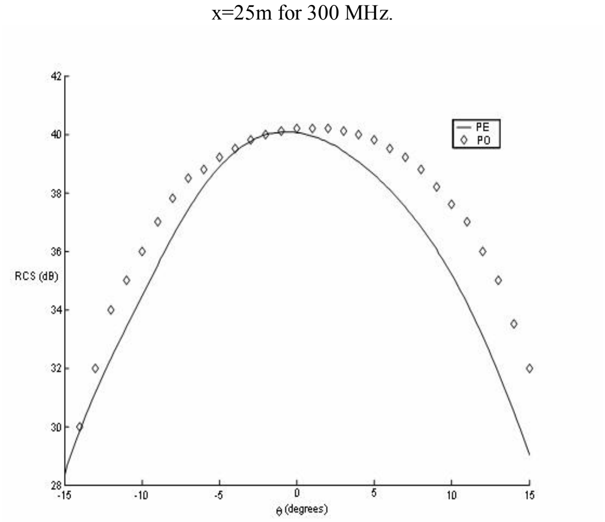

Figure 8. RCS of airplane in $\varphi = 0^\circ$ plane in 300 MHz

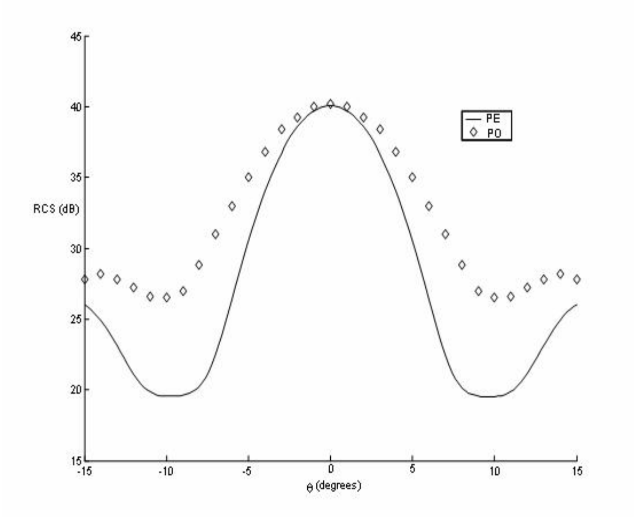

Figure 9. RCS of airplane in $\varphi = 90^\circ$ plane in 300 MHz.

VI. CONCLUSION

Parabolic equation method provides advantages in calculating scattered fields from objects with dimensions large compared to the wavelength. In this paper RCS of an airplane was calculated by three dimensional parabolic equation method. In order to show the validity of the parabolic equation, at first RCS of a conducting sphere with radius 10λ was calculated and the results were compared with analytic results. As it can be seen, there is a good agreement between the analytical results and the parabolic equation results up to angles about 10 degrees. RCS and scattered fields from the airplane has been presented by using its staircase model in 300MHz and the results were compared with physical optic results. Good agreement between two methods can be seen especially in $\varphi = 0^\circ$ plane.

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