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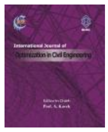

MINIMIZING HANKEL'S NORM AS DESIGN CRITERION OF MULTIPLE TUNED MASS DAMPERS

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ABSTRACT

Tuned mass damper (TMD) have been studied and installed in structures extensively to protect the structures against lateral loads. Multiple tuned mass dampers (MTMDs) which include a number of TMDs with different parameters have been proposed for improving the performance of single TMDs. When the structural system is considered as multiple degrees of freedom (MDOF) and implemented with MTMDs, there is no effective closed-form solution to determine the optimal parameters of MTMDs. On the other hand designing optimal MTMDs include a large number of variables. For optimal design of MTMDs, in this research an effective method has been proposed in which the parameters of TMDs are determined based on minimizing the Hankel's norm of structure. Since the optimization procedure includes a large number of variables, hence it has been decided to use Genetic Algorithms (GAs) for determining the variables. For numerical simulation, the method has been utilized on an eight-storey shear frame modeled as MDOF, and optimal MTMDs have been designed. The results show that using the Hankel's norm of structure as objective function has led to design effective MTMDs which could be effective in reducing the response of structure, especially the average value, under different far-field and near-field earthquakes. Also it has been found that the method is effective regarding its simplicity and convergence in solving complex optimization problem. Through extensive numerical analysis the effect of MTMDs mass ratio and TMDs number in MTMDs has been studied.

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KEY WORDS: passive control; tuned mass damper; multiple tuned mass damper; hankel's norm; optimization

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1. INTRODUCTION

To protect structures under earthquake loads, different structural control mechanisms in passive control areas have been proposed. Application of tuned mass damper (TMD) as a kind of passive control systems on linear and nonlinear structures subjected to wind and earthquake excitations has received much attention during past years [1]. Based on the aim of using TMDs different methods have been developed for determining the optimum values of TMD parameters(mass, stiffness and damping) on linear structures such as minimizing the root-mean-square (RMS) of the main structure displacement or acceleration under white noise excitation [2], maximizing the effective damping of the structure [3] and minimizing the difference between the damping of the first two modes of the structure-TMD system [4].Mainly, when using a single TMD, it is located on the top of the structure and tuned to the fundamental frequency of the structure. While single TMD could be effective in reducing the response of structure under external excitations, especially for wind-induced vibrations, but they suffer from some drawbacks such as sensitivity problem to detuning the TMD frequency, TMD damping ratio and uncertainty in dynamic properties of main structure. The mistuning of TMD, variation of TMD damping and changes in structural dynamic characteristics cause significant reduction in the effectiveness of TMD. Also in practical application of TMD on tall buildings it may require a heavy mass consequently a considerable space for its installation. For high-rise buildings which the higher modes may play a considerable role on total response, designing a single TMD tuned to the first mode of vibration may has a little effect on controlling the response of higher modes.

To overcome these shortcomings application of multiple tuned mass dampers (MTMDs) has been proposed to be used instead of a single TMD [5].They concluded that the use of MTMDs with distributed natural frequencies over a small range of a single degree of freedom (SDOF) natural frequency, could be more effective than a single TMD with the same total mass. It has been shown that the sensitivity of MTMDs to uncertainty of structural dynamic parameters is less than a single TMD [6-8]. Multiple tuned mass dampers can be used in parallel or series configurations as well as located at one floor or distributed over the floors of a building structural system. Kareem and Klein [9] investigated the characteristics and effectiveness of MTMDs with distributed nature frequencies under both wind and earthquake excitations. It was concluded that the performance of MTMDs is dependent on the total number of dampers, damping ratio, frequency range selected for designing optimal MTMDs and the distribution of TMDs on the floors.

In the previous researches for designing MTMDs different approaches have been used. In the early stages of designing MTMDs to simplify the analysis and design procedure, some design constraints such as identical masses and damping ratios have been considered for TMDs [5]. Following these constraints Yamaguchi and Harnpornchai [10] studied the effect of different parameters of MTMDs on SDOF structure under harmonic excitations. Igusa and Xu [11] to generalize the optimal design problem by relaxing the design constraints on the mass and damping ratio of TMDs and using an asymptotic analysis, designed MTMDs for a SDOF structure subjected to a wide-band force. Jangid [12] proposed a method for determining the optimum parameters of MTMDs for an undamped system subjected to harmonic excitation. The method has been based on minimizing the steady-state displacement of the main system by using a numerical searching technique. In design procedure proposed by Wu and Chen [13], an MTMDs system is divided to several groups, each corresponding to one floor and consisting of several dampers distributed on different floors. By developing a sequential procedure, MTMDs were placed optimally to minimize the acceleration of structure. Chen [14] studied the designing multiple TMDs on MDOF structures subjected to seismic loads where the MTMDs have been designed based on tuning to several modes of structure vibration where in the proposed method the number of dampers is determined based on the number of modes considered to be controlled. Li[15] studied the performance of five configurations of MTMDs which include different combinations of TMD parameters (mass, stiffness and damping) while the optimality criterion has been as the minimization of the displacement dynamic magnification factor(DDMF) and the acceleration dynamic magnification factor(ADMF) of a structure subjected to ground acceleration. Hoang and Warnitchai[16] proposed a method for designing multiple TMDs to minimize excessive vibration of MDOF linear structures by using a numerical optimizer. The method considers a gradient based nonlinear programming algorithm to find the optimal parameters of TMDs where the target response has been defined as a quadratic performance index. It has been shown that the proposed method is effective in determining a large number of TMDs parameters without imposing constraints before analysis. Distribution of TMDs vertically and in plan has been studied too [17-18]. Moon [18] investigated the effectiveness of vertically distributed MTMDs along the height of building. The vertically distributed MTMDs theory can be used for controlling not only the first mode but also the higher modes where the vertically distributed TMD zones for each mode, is determined based on its mode shape. It has been found that vertically distributing of MTMDs leads to increasing the reliability of the control system ,saving the valuable space near the top of tall buildings which is required when using a single TMD located at the top of building, easy installation of small TMDs and possessing a high potential of practical application over the conventional TMD system. Mohebbi et al. [19] proposed an effective method for designing optimal MTMDs on MDOF structures which Genetic Algorithm has been used for solving the optimization problem. In the previous researches different approaches have been selected as objective in designing optimal MTMDs where in most researches minimizing the maximum displacement or acceleration of structure has been the main objective of using MTMDs. While these methods are very effective regarding the safety and conformability of occupants criteria, but the result of numerical simulations show that using this kind of objective functions for designing MTMDs generally leads to minimize the maximum value of an specified response while the reduction in maximum or average of other responses has not been more significant [19]. Hence to overcome these shortcomings, in this paper, minimizing the Hankel's norm of structure has been considered as objective function in designing optimal MTMDs for linear MDOF structures subjected to earthquake excitation. The proposed method has been based on defining an optimization problem which considers the parameters of TMDs(including mass, stiffness and damping of an individual TMD) as design variables and minimizing the Hankel's norm of structure as objective function. By solving the optimization problem, the optimal values of TMDs parameters are determined. The optimization problem, defined for determining TMDs parameters, includes a large number of variables hence; solving it by using the traditional optimization methods will be cumbersome and needs an extensive numerical computations. On the other hand Genetic Algorithm (GA) has been extensively used for solving optimization problem in most fields of engineering [20] such as designing TMD for linear and nonlinear structures [21-23]. Therefore; it has been decided to use Distributed Genetic Algorithm (DGA) [24-25] which is an improved version of simple GA, for designing optimal MTMDs.

In the following sections first the equation of motion of structure-MTMDs will be presented in state-space form, next a brief explanation of Hankel's norm and DGA will be followed by a numerical example and conclusions.

2. STRUCTURE-MTMDS MOTION EQUATION

The equation of motion of a linear *n*-degree of freedom shear building structure subjected to an earthquake ground motion, $\ddot{x}_g(t)$, the external force, F, and equipped with *N* TMDs as shown generally in Figure 1, has the form:

$$[M]\ddot{X} + [D]\dot{X} + [K]X = [M]e\ddot{x}_g(t) + F \tag{1}$$

where [M], [D] and [K] are, respectively, the $(n+N)\times(n+N)$ mass, damping and stiffness matrices, X is the vector of displacements relative to the fixed base , e^T=[-1 -1……-1]$_{(1\times(n+N))}$ ground acceleration-mass transformation vector and F is the vector of external force.

When one or several TMDs are installed on the structure, the mass, stiffness and damping matrices in equation (1) should be developed regarding the configuration and distribution of TMDs. For example, for a four-storey shear frame equipped with four TMDs at top floor of structure as shown generally in Figure 1, the mass and stiffness matrices are as follows:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{tmd1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{tmd2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{tmd3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{tmd4} \end{bmatrix} \tag{2}$$

$$[K]=\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_4 & k_4+k_{tmd1}+k_{tmd2}+k_{tmd3}+k_{tmd4} & -k_{tmd1} & -k_{tmd2} & -k_{tmd3} & -k_{tmd4} \\ 0 & 0 & 0 & -k_{tmd1} & k_{tmd1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_{tmd2} & 0 & k_{tmd2} & 0 & 0 \\ 0 & 0 & 0 & -k_{tmd3} & 0 & 0 & k_{tmd3} & 0 \\ 0 & 0 & 0 & -k_{tmd4} & 0 & 0 & 0 & k_{tmd4} \end{bmatrix} \quad (3)$$

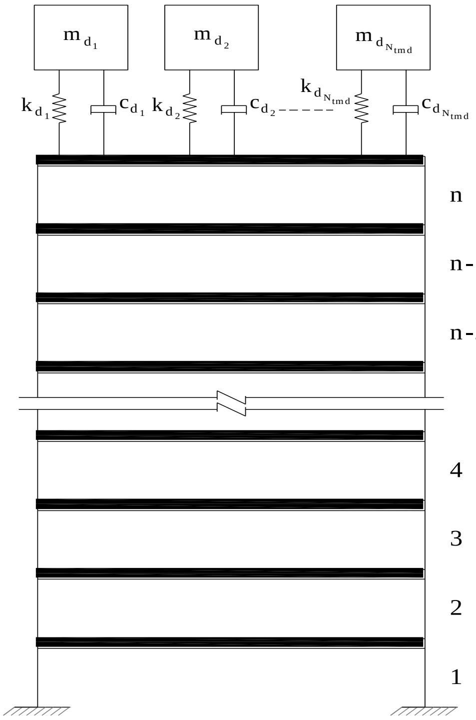

Figure 1. Multiple tuned mass dampers (MTMDs) attached to the structure [19]

The equations of motion, equation (1), can be represented in state-space form as follows [21]:

$$\dot{Z}=AZ+BF+Ew \quad (4)$$

$$y=C\,Z \quad (5)$$

$$A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix},\ B=\begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad (6)$$

$$E=\begin{Bmatrix} 0 \\ e \end{Bmatrix},\quad Z=\begin{Bmatrix} x \\ \dot{x} \end{Bmatrix},\quad w=\ddot{x}_g(t) \quad (7)$$

in which A, B, C=system matrixes where based on the type of output vector, y, the system matrix,C, is developed properly, I=identity matrix, $\ddot{x}_g(t)$=ground acceleration and 0=matrix containing zero.

In this paper by considering only earthquake loading the equation of motion defined in equation (4) can be written as:

$$\dot{Z} = AZ + Ew \tag{8}$$

3. HANKEL'S NORM

Controllability and observability are structural properties that carry useful information for structural testing and control. A structure is controllable if the installed actuators excite all its structural modes; also observability is defined as the capability of installed sensors to detect the motions of all the modes. This information, although essential in many applications, is too limited. It answers the question of mode excitation or detection in terms of yes or no. The more quantitative answer is supplied by the controllability and observability grammians, which represent a degree of controllability and observability of each mode [26].

A proper approach to check the controllability and observability of a system is by using grammians. Grammians are nonnegative matrices that express the controllability and observability properties qualitatively, and are free of the numerical difficulties of the other criteria. The controllability and observability grammians are defined as follows:

$$W_c(t) = \int_0^t \exp(A\tau)BB^T \exp(A^T\tau)d\tau \tag{9}$$

$$W_o(t) = \int_0^t \exp(A^T\tau)C^T C \exp(A\tau)d\tau \tag{10}$$

The grammians can be determined alternatively and more conveniently from the following differential equations:

$$\dot{W}_c = AW_c + W_c A^T + BB^T \tag{11}$$

$$\dot{W}_o = A^T W_o + W_o A + C^T C \tag{12}$$

The solutions $w_c(t)$ and $w_o(t)$ and are time-varying matrices. For a stable system, the stationary solutions of the above equations can be obtained by assuming $w_c= w_o=0$ where the differential equations are replaced with the algebraic following equations, called Lyapunov equations:

$$AW_c + W_c A^T + BB^T = 0 \tag{13}$$

$$A^T W_o + W_o A + C^T C = 0 \tag{14}$$

where for stable A, the obtained grammians W_c and W_o are positive definite.

The eigenvalues of the grammians are changed during the coordinate transformation. However, the eigenvalues of the grammian product are invariant where these invariants are denoted γ_i and can be obtained as follows:

$$\gamma_i = \sqrt{\lambda_i (W_c W_o)} \quad i=1, 2, ..., N \tag{15}$$

where N=number of system states and γ_i s are called the Hankel's singular values of the system. The Hankel's norm of the system is defined as the largest Hankel singular value of the system, γ_{max} .

$$\|G\|_h = \gamma_{max} \tag{16}$$

Then, the Hankel's norm of the system is the largest norm of its mode and a measure of the effect of the past input on the future output, or the amount of energy stored in and subsequently retrieved from the system. This study will integrate controllability-observability based measures, system Hankel's singular norms, into the problem of designing optimal MTMDs. The Hankel's norm is advantageous because it reflects both controllability and observability, and is invariant under linear similarity transformations.

4. OPTIMAL DESIGN OF MTMDS FOR MINIMIZING HANKEL'S NORM

In this paper for optimal design of MTMDs, an optimization problem has been defined which considers the minimization of Hankel's norm of the structure as the objective function and the parameters of TMDs(including mass, stiffness and damping of each TMD) as variables to be determined while a number of constraints are considered on TMDs response or TMDs parameters. Hence, the optimal values of TMDs parameters are determined by solving the following optimization problem:

$$\text{Find} \quad T_i= (m_{di},c_{di},k_{di}) \quad i=1,2,\ldots,N \tag{17a}$$

$$\text{Minimize} \quad \|G\|_h = \text{Hankel's norm} \tag{17b}$$

where T_i represents the vector of the TMDs parameters m_{di}, c_{di} and k_{di} which are the mass, damping and stiffness of the i^{th} TMD and N=the number of TMDs also some constraints are considered in optimization problem.

5. DISTRIBUTED GENETIC ALGORITHM (DGA)

Genetic algorithm (GA) developed by Holland[27], is a strong computational method which is inspired by natural Darwinian evolution .In GAs the design vector is considered as chromosome or individual and the variables of design vector as genes .The chromosomes evolve under a certain environment and are represented by bit strings or real-valued coding. In the early stages of string coding design variables were represented in their binary format [20, 28]. It has been shown in using the real- valued coding representations for representing the chromosomes while the optimization problem includes continuous variables, there is no need to convert chromosomes also there is greeter freedom to use different operators of GA[29-30] hence, in this research it has been decided to use the real-value coding to represent the variables which are continuous.

There are three genetic algorithm operators including selection, crossover and mutation. In every generation, a set of chromosomes is selected for mating based on their relative fitness. The fitters are given more chance of passing their genes into the next generation. This process of natural selection is operated by selection. In this paper the stochastic universal sampling method [31] has been used for selecting a number of chromosomes for mating, based on their fitness values in the current population as:

$$P(\mathbf{x}_i) = \frac{F(\mathbf{x}_i)}{\sum_{i=1}^{N_{ind}} F(\mathbf{x}_i)} \quad , i=1,2,..., N_{ind} \tag{18}$$

where $F(\mathbf{x}_i)$=fitness of chromosome $\mathbf{x}_i$ and $P(\mathbf{x}_i)$=probability of selection of x_i also N_{ind} =number of individuals.

The selected individuals are then chosen randomly through crossover to produce newborns. Crossover produces new individuals that have some parts of both parents genetic material. In this paper the method proposed by Mühlenbein and Schlierkamp-Voosen[32] for crossover has been used, where each pair of parents can produce two newborns and each newborn can get its genes from either parent with equal probability as follows:

$$O=P_1+\alpha(P_2-P_1) \tag{19}$$

where P_1 and P_2 are the parent chromosomes genes, O is the newborn gene, and α is a scaling factor chosen randomly over [-0.25, 1.25] interval typically .This method uses a new α for each pair of parents genes.

The role of mutation operator is to help the GA to escape from local minima and to provide a guarantee that the probability of searching any given string will never be zero.

In this paper the elitist strategy has been used where N_{elites} of the best chromosomes are selected as elites of the current generation to go to the next generation without modification. The rest of the chromosomes in the population are replaced by inserted newborns (N_{ins}). hence:

$$N_{elites} = N_{ind} - N_{ins} \tag{20}$$

When there is a large number of variables in an optimization problem such as optimization problem defined in this study for designing optimal MTMDs with a large number of TMDs, using the simple GA may require large number of generation to obtain the optimal answer of optimization problem. For solving this kind of problem it has been proposed to use an improved version of simple GA which called Distributed Genetic Algorithms (DGA)[24-25]. To obtain quicker convergence in optimization process, in DGA a large population is divided into smaller subpopulations, and a simple GA is executed on each subpopulation separately.

6. NUMERICAL EXAMPLE

In this paper to examine the usefulness of MTMDs in improving the performance of structures under earthquake excitation as well as to evaluate the usefulness of the proposed method in designing the optimal MTMDs, an eight-storey shear frame with uniform properties for all stories has been modeled assuming linear material behavior for structure and TMDs. The properties of each story are as follows: elastic stiffness $k = 3.404\times10^5$ kN/m, floor mass is 345.6 tons and the natural frequency of the structure is = 0.92 Hz. The linear viscous damping coefficient c is 2937 t/sec. The structure has been analyzed by considering MTMDs in parallel configuration located at the top of structure. By assuming a specified value for the total mass ratio, μ, and the numbers of TMDs, N, following the proposed method the optimal values of mass, stiffness and damping of TMDs have been determined based on minimizing the Hankel's norm of structure.

6.1. Designing Optimal MTMDs for N =10 and μ= 4%

To illustrate the proposed method for designing the optimal MTMDs, ten TMDs have been considered where the total mass of TMDs has been 110.6 tons which corresponds to μ= 4%. The optimization problem to determine the mass, stiffness and damping of each TMD, has been defined as follows:

$$\text{Find} \quad T_i = (m_{di}, c_{di}, k_{di}) \quad i=1,2,\dots,N \tag{21a}$$

$$\text{Minimize} \quad \|G\|_h = \text{Hankel's norm} \tag{21b}$$

Subject to:

$$X_{max}(\text{TMDs}) \le X_L \tag{21c}$$

$$\sum_{i=1}^{i=10} m_{di} = 110.6 \quad \text{tons} \tag{21d}$$

$$0<c_{di}<c_{d\,max},\ 0<k_{di}<k_{d\,max} \tag{21e}$$

where
$$X_{max}(TMDs) = \max\left(\left|X(TMD_i)\right|\right) \quad i=1,\ 2,\dots,\ 10 \tag{21f}$$

X_L = the maximum stroke length of TMDs, $c_{d\,max}$ and $k_{d\,max}$ = the maximum possible stiffness and damping of TMDs which should be considered by the designer.

By assuming X_L= 50 cm and k_{dmax}=1×10^4 kN/m and c_{dmax}=1000 t/sec, Distributed Genetic Algorithm (DGA) has been used for solving the optimization problem defined in equations(21a-f). To this end, first the optimization problem has been changed to an unconstraint problem by considering the constraints as penalty function; next DGA has been used for solving an unconstraint problem. In this paper to simplify the procedure of designing optimal MTMDs, uniform distribution for TMDs mass has been considered. Hence, in this case the optimization problem has 20 variables which should be determined. For solving the optimization problem, the following parameters have been considered for DGA:

Number of subpopulations=5, Number of individuals in each subpopulation=10, Number of elites in each generation =2, Number of the newborns in each generation=10, Mutation rate=0.05, Migration interval=10 and Migration rate=0.20.

For solving the optimization problem defined in equations(21a-f) by using DGA, five subpopulations each with 10 randomly generated vectors(chromosomes) of MTMDs parameters ($T_i(m_{di}, c_{di}, k_{di})$, i=*1,2,…,10*) were generated as the initial population which each chromosome includes 20 variables(genes). For the generated values of MTMDs parameters, the Hankel's norm of the structure and the maximum displacement of TMDs were recorded. Iteratively, the subpopulations were modified according to the DGA so that new generations were generated until convergence was achieved. By monitoring the Hankel's norm value of the structure for all MTMDs parameters in every generation, the fittest individual of that generation was identified. To guarantee the accuracy of optimization procedure, different runs have been done in DGA for N=10. Figure 2 shows the convergence behavior of DGA, including the variation of the best fitness during generations of DGA, for four runs. Results show that different runs have ended with the same optimum answer but with different convergence speed. Noticing the results, it can be concluded that the proposed method has been effective in designing optimal MTMDs regarding the simplicity and convergence behavior. To compare the convergence behavior of DGA with simple GA, the optimization problem has been solved again by using simple GA and the convergence of procedure toward optimum answer for 500 generation has been shown in Figure 3 for DGA and GA. Results show that by using DGA the optimum answer has been achieved after 250 generations while in simple GA after 500 generations the final optimal answer has not obtained, yet.

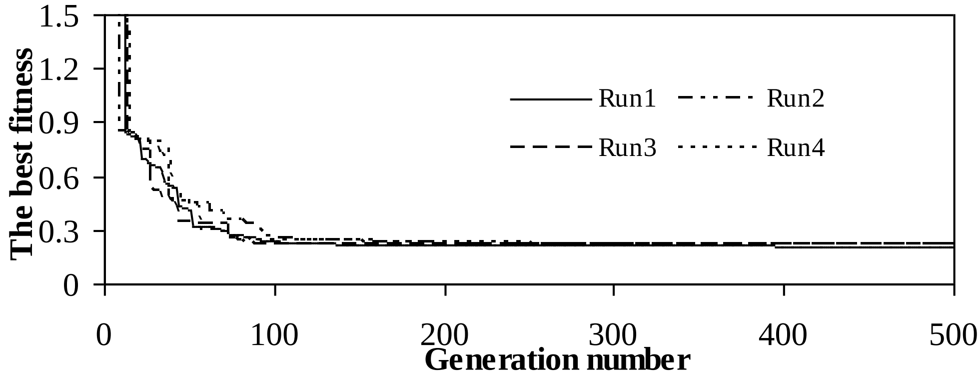

Figure 2. Convergence behavior of DGA including variation of the best fitness for four runs

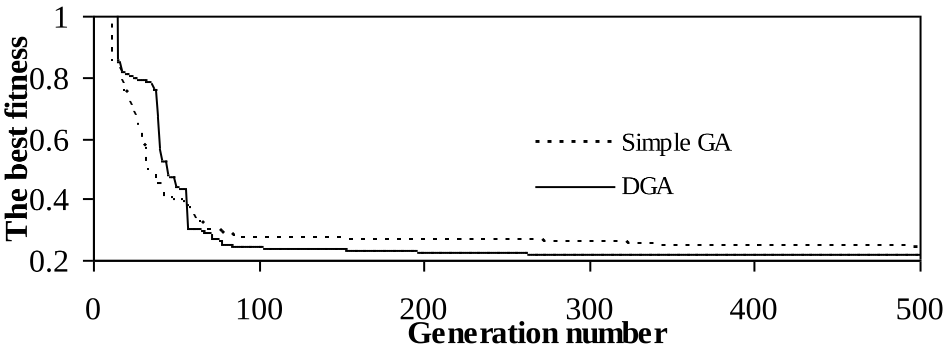

Figure 3. Comparing the convergence of DGA and simple GA

6.2. Performance of optimal MTMDs under Different Earthquakes

To assess the effectiveness of designed MTMDs for N =10 and μ= 4% in mitigating the response of structure under different real earthquakes which are different in peak ground acceleration and frequency content, the uncontrolled structure and controlled structure equipped with optimal MTMDs have been tested under El-Centro (1940, PGA=0.34g), and Hachinohe (1968, PGA=0.23g) records as far-field earthquakes as well as Northridge (1994, 0.84g) and Kobe (1995, 0.83g) records as near-field earthquakes. The maximum displacements and acceleration of uncontrolled and controlled structures have been reported in Figures 4-7, also root-mean-square (RMS) of displacement and RMS of acceleration under El-centro and Northridge excitations, as samples, have been shown in Figures 8-9 where the maximum root-mean-square (RMS) of displacement as well as acceleration of an n storey frame have been calculated according to equations (22(a,b)).

$$\mathrm{RMS}(\mathrm{v(i)}) = \left(\frac{\sum_{k=1}^{k_{max}} \mathrm{v(i)}_k^2}{k_{max}} \right)^{1/2} \qquad i=1,2,\dots,n \tag{22a}$$

$$\mathrm{RMS}_{max}(\mathrm{v}) = \max\left|\mathrm{RMS}(\mathrm{v(i)})\right| \qquad i=1,2,\dots,n \tag{22b}$$

where v(i) = displacement or acceleration of i^{th} storey and k_{max} is the total number of time steps.

According to results it can be said that (1) by using MTMDs not only the maximum values of structure response but also the response of all stories has been decreased;(2) by using Hankel's norm as objective it has been found that the maximum displacement and acceleration of structure have been reduced simultaneously while in the previous researches it has shown that when using the maximum displacement as objective function, the reduction in maximum acceleration has not been more noticeable [19], this conclusion is north worthy regarding the safety and comfort ability criteria in designing structural control system; (3)the effectiveness of MTMDs depends on the characteristics of earthquake where in this study the maximum reduction in response has been obtained when the structure subjected to El-Centro excitation; (4) the most reduction has been achieved in displacement

RMS for all earthquakes which this result is related to the objective function selected for designing MTMDs.

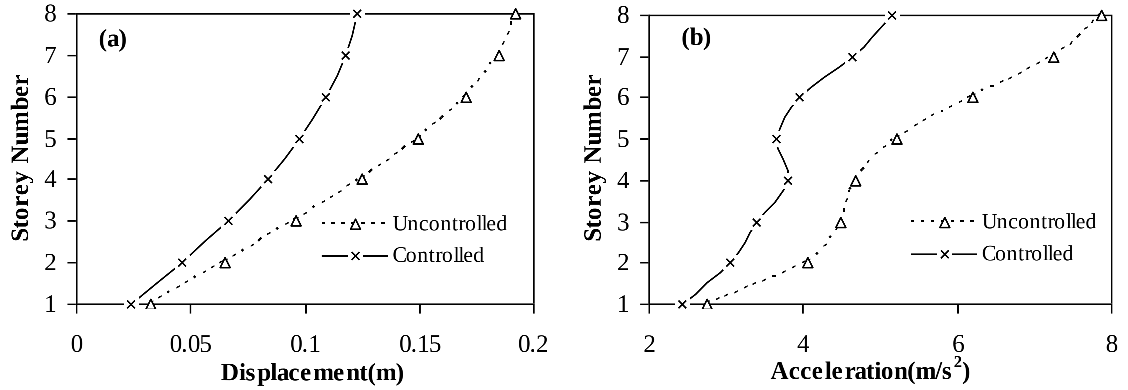

Figure 4. Uncontrolled and controlled structures maximum (a) displacement; and (b) acceleration generation for different stories under El-Centro excitation

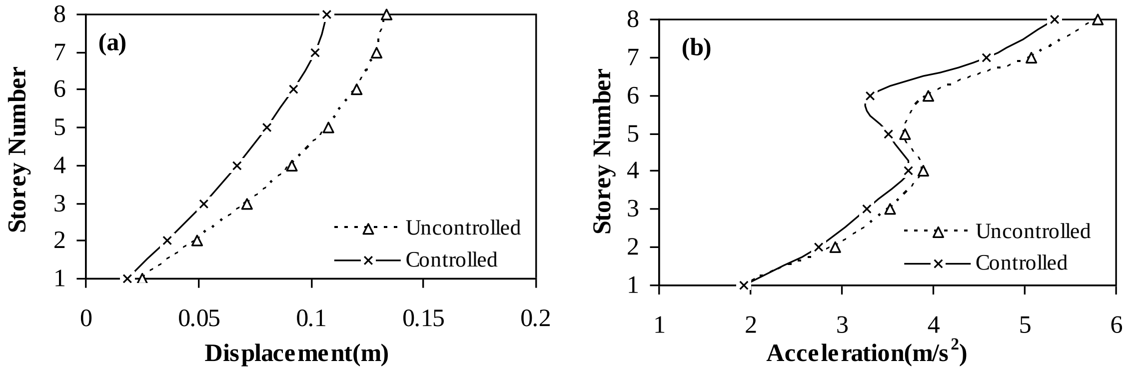

Figure 5. Uncontrolled and controlled structures maximum (a) displacement; and (b) acceleration generation for different stories under Hachinohe excitation

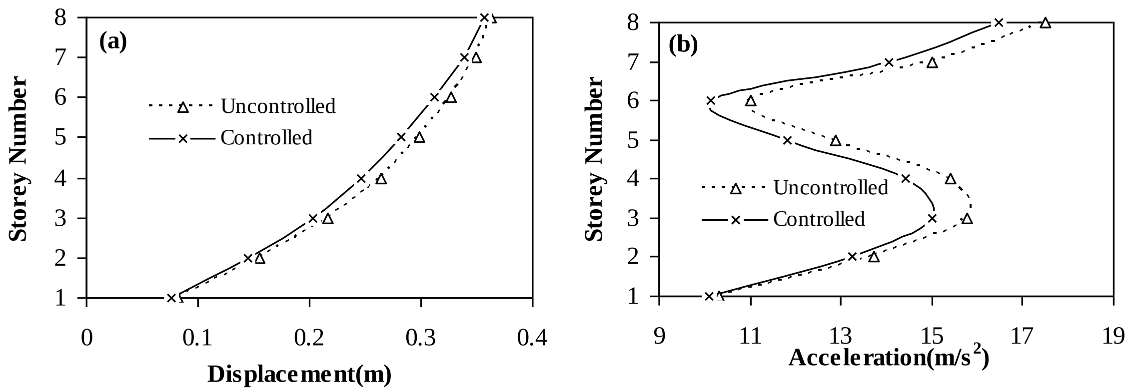

Figure 6. Uncontrolled and controlled structures maximum (a) displacement; and (b) acceleration generation for different stories under Northridge excitation

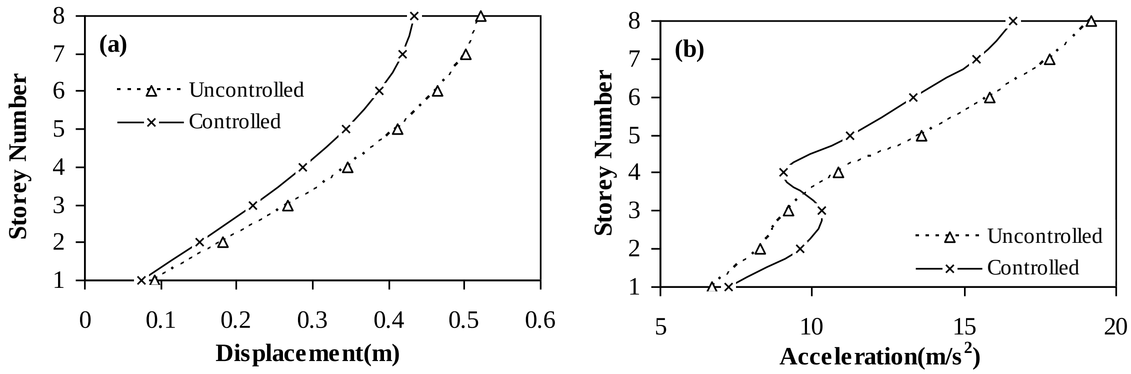

Figure 7. Uncontrolled and controlled structures maximum (a) displacement; and (b) acceleration generation for different stories under Kobe excitation

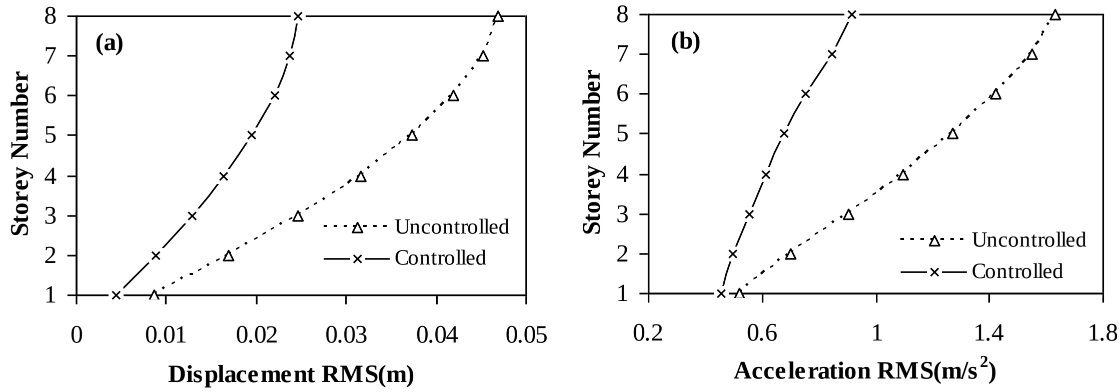

Figure 8. Uncontrolled and controlled structures (a) RMS of displacement; and (b) RMS of acceleration for different stories under El-Centro excitation

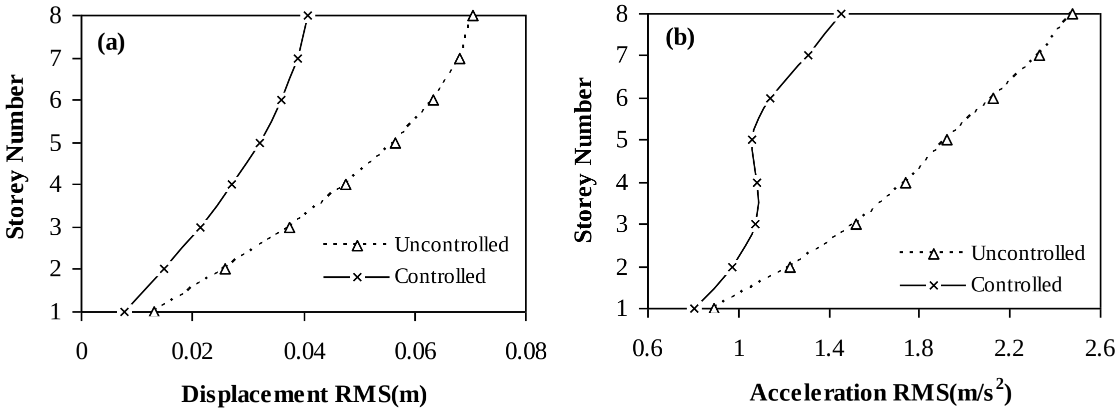

Figure 9. Uncontrolled and controlled structures (a) RMS of displacement; and (b) RMS of acceleration for different stories under Northridge excitation

6.3. Effect of TMDs Number (N) on the Performance of MTMDs

To assess the effect of TMDs number on the performance of multiple TMDs, following the same procedure applied to N_{tmd} = 10, optimal MTMDs have been designed for N_{tmd} = 1,5 and 20 by assuming μ= 4%. The uncontrolled and controlled structures have been subjected to real earthquakes and the maximum response of controlled structure has been divided to uncontrolled value and shown in Figures 10-11. Also Figure 12 shows time history of maximum displacement of uncontrolled and controlled structures for different TMDs number. Noticing the results it can be said that for a specified value of TMDs total mass, the performance of MTMDs is not sensitive to the number of TMDs. While increasing the number of TMDs has not affected the performance of MTMDs but offers a smaller size for an individual TMD which is more attractive regarding the ease of installation. Hence, for a specified total mass of TMDs, to obtain the smaller size for TMDs, the MTMDs system can be divided to large number of TMDs while the performance of MTMDs has been kept constant.

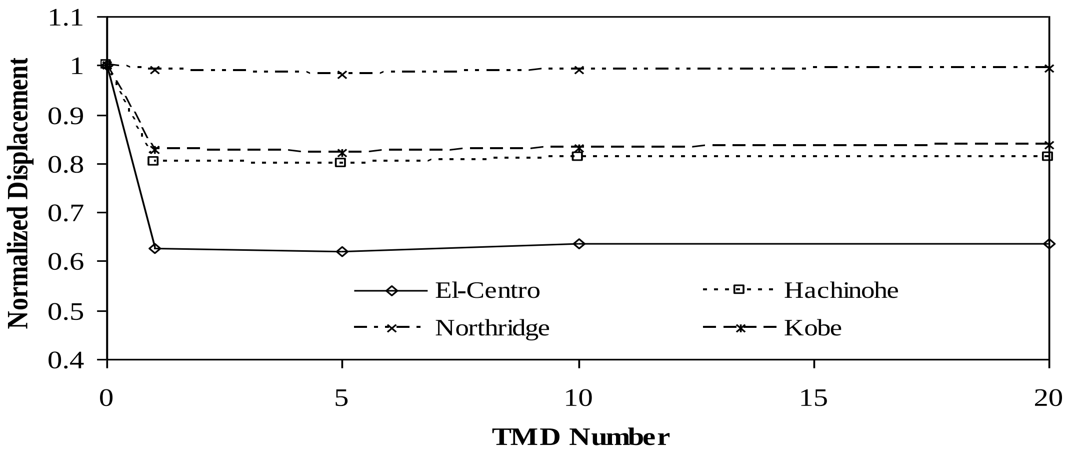

Figure 10. Normalized displacement of controlled structure under different excitations using different numbers of TMDs for μ=4%

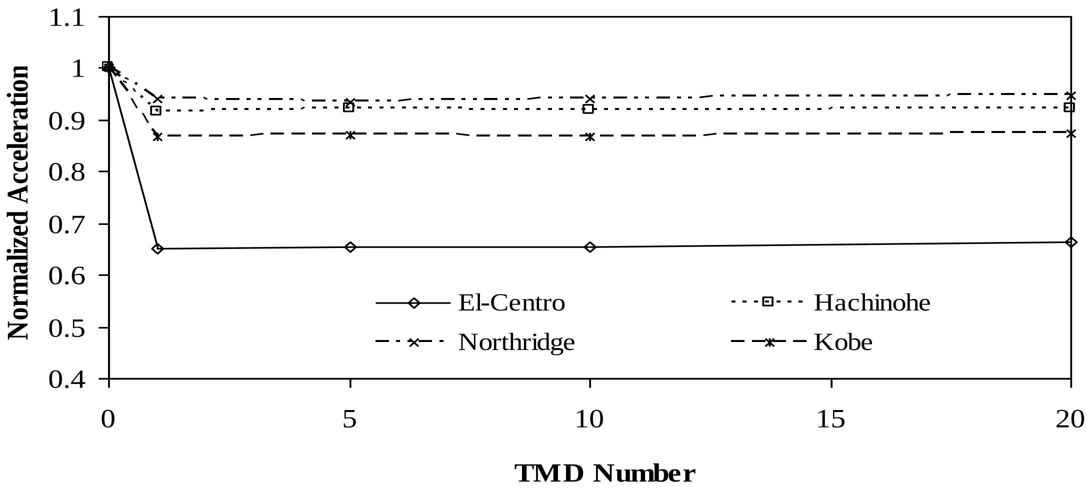

Figure 11. Normalized acceleration of controlled structure under different excitations using different numbers of TMDs for μ=4%

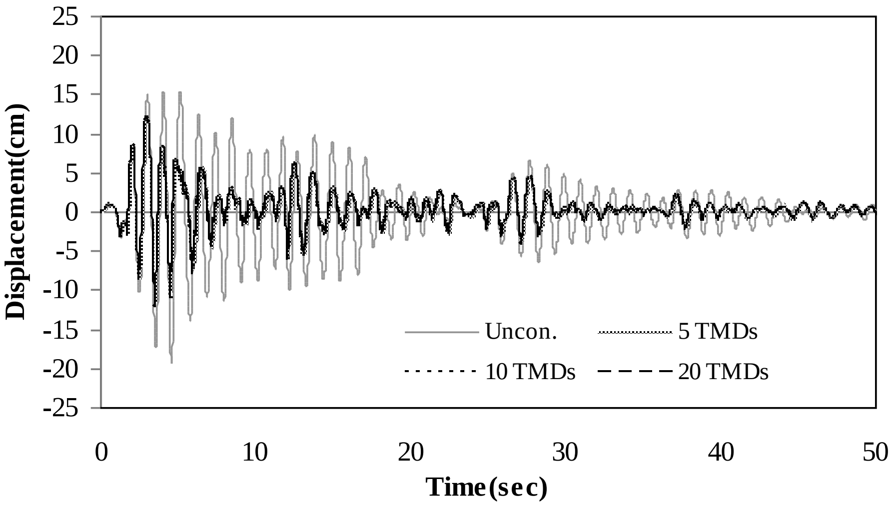

Figure 12. Maximum displacement of uncontrolled and controlled structures subjected to El-Centro earthquake when using 5, 10 and 20 TMDs

6.4. Designing Optimal MTMDs for Different Values of TMDs Total Mass

To assess the effect of TMDs total mass on the effectiveness of MTMDs, also to determine the optimal value of TMDs total mass under a specified earthquake, different MTMDs for minimizing the Hankel's norm of structure have been designed for different values of mass ratio. The maximum response of controlled frame when subjected to El-Centro excitation has been normalized to maximum uncontrolled response and shown for different values of total mass ratio, μ, in Figure 13. Based on the results, it has been concluded that the effectiveness of MTMDs depends on the value of total mass of TMDs where increasing the total mass has led to increasing the performance of MTMDs in reducing the response of structure. Also Figure 13 can be used as design guideline for determining the optimal value of TMDs total mass to mitigate the response of structure to a desired level.

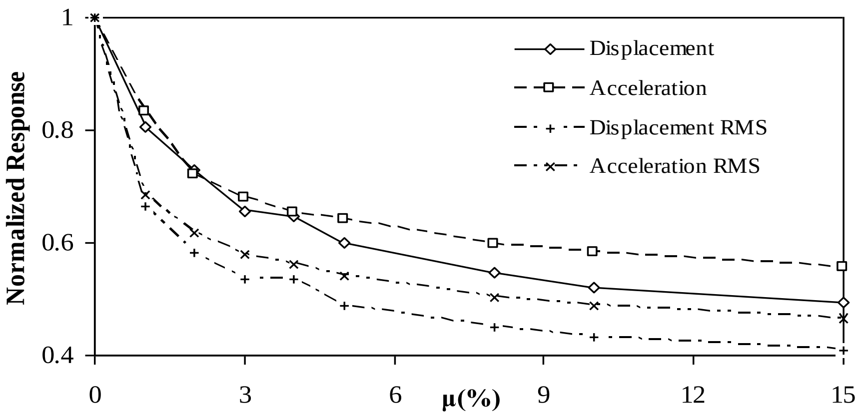

Figure 13. Normalized response of controlled structure subjected to El-Centro earthquake versus TMDs total mass ratio when using 10 TMDs

7. CONCLUSION

In this paper a method has been developed for designing optimal MTMDs to reduce the seismic response of multi degree of freedom (MDOF) shear frames. To determine the optimal parameters of tuned mass dampers (TMDs) including mass, stiffness and damping of each TMD, an optimization problem has been developed which minimizes the Hankel's norm of structure while some constraints on TMDs parameters as well as TMDs stroke length have been considered. To illustrate the procedure of design and the effectiveness of selected objective function, the method has been used for designing optimal MTMDs for an eight-storey linear shear frame. Distributed Genetic Algorithms (DGAs) has been successfully applied for solving the optimization problem which results confirm the capability of DGA as a powerful algorithm for solving a large scale MTMDs design optimization problem. According to the results, it can be concluded that the proposed method for designing optimal MTMDs has been effective regarding its simplicity and convergence behavior. Testing optimal MTMDs under far-field and near-field earthquakes which are different in frequency content and peak ground acceleration (PGA), shows that (1) using minimizing Hankel's norm as objective function has led to more reduction in RMS of displacement and acceleration, also the maximum acceleration and displacement have been reduced simultaneously which shows the effective performance of design criterion regarding safety and comfortability of occupants; (2) the performance of MTMDs in mitigating the response of structure depends on the earthquake characteristics, hence when designing an MTMDs system in a particular area, the design earthquake of that area with a proper PGA should be considered as external excitation. Assessment the effect of TMDs total mass on effectiveness of MTMDs shows that increasing the total mass of TMDs has led to increasing the performance of MTMDs in reducing the response of structure. Also it has been found that for a specified value of MTMDs mass, increasing the number of TMDs has not significantly affected the efficiency of MTMDs while it has led to obtaining the smaller size for TMDs which is attractive in practical application regarding ease installation and required space.

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