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DESIGN OF MINIMUM SEEPAGE LOSS IRRIGATION CANAL SECTIONS USING PROBABILISTIC SEARCH

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ABSTRACT

To ensure efficient performance of irrigation canals, the losses from the canals need to be minimized. In this paper a modified formulation is presented to solve the optimization model for the design of different canal geometries for minimum seepage loss, in meta-heuristic environment. The complex non-linear and non-convex optimization model for canal design is solved using a probabilistic search algorithm namely Probabilistic Global Search Lausanne (PGSL). The solutions are found to be competitive to those reported in literature while applied for different example problems. To suit for real field applications, three site specific constraints are considered and the sensitivity of solutions for the most popular trapezoidal canals is investigated. The study shows the potential of the proposed approach to perform optimal design of irrigation canals for minimum seepage loss.

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KEY WORDS: irrigation canals; optimal design; probabilistic global search lausanne; seepage loss

1. INTRODUCTION

Optimal design of irrigation canals is essential for the planning and management of irrigation projects. However, the losses from canals need to be minimized to ensure the efficient performance and effective utilization of water. Seepage loss is one of the major components of water loss from canals. A well maintained canal with 99 % perfect lining reduces seepage

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about 30-40% and the seepage cannot be controlled perfectly [1]. The proper monitoring of seepage loss from canals is essential in saving the water resource during its conveyance and such attempts are crucial in arid climatic zones. In the past, many researches attempted to quantify the amount of seepage and many of them succeeded in presenting models for the same[2-8]. Attempts were also made in the direction to incorporate the seepage loss in the canal design procedure [9-10].

Swamee *et al*., [11] proposed an optimization model for minimum seepage loss design of popular channel geometries and extended the study by adding evaporation loss [12]. Researchers like Chahar [13], Swamee and Kashyap [14] also obtained analytical solution for seepage from rectangular channels in a soil layer of finite depth and investigated the influence of the position of drainage layer. In the above studies the classical optimization procedures are followed and explicit equations are proposed after obtaining a large number of optimal sections for different design data. The non-linear optimization model (NLOM) for minimal seepage loss canal design comprises the minimization of seepage function as the objective function. Chahar [15] analyzed seepage from slit and strip channels as special cases of a polygon channel and also presented results for trapezoidal, triangular, and rectangular channels in graphical form and later on simple expressions were presented for the curvilinear channels [16].

Swamee and Kashyap [14, 17] suggested equations for minimum seepage loss non-polygon channels however there are some drawbacks in their method as highlighted by Kacimov [18]. Analytical solutions for computing the quantity of seepage from the non polygonal irrigation furrows with curved bed (CB) [19] and extensions were also made considering the variability of position of drainage layer below the canal bed [20].

The flow resistance equation for uniform flow, along with the non-negativity of the decision variables, normally constitutes the constraints of the optimization problem. The resistance equation is an equality constraint of non-linear type, and any equality constraint that is nonlinear in nature would make the optimization problem a non-convex one [21]. The presence of 'seepage function' in the model make it quite complex and practical implementation of designed canal sections may be influenced by the site specific constraints such as limitation of top width, depth or side slope (considering the stability of slopes). On incorporating such geometric constraints in the model would further complicate the non-convex non-linear optimization problem (NLOP). Such NLOPs may subject to local trapping during the search towards global optima and difficult to solve by the gradient based classical optimization methods. Such issues are addressed in past by Jain *et al*., [22], Janga Reddy and Adarsh [23, 24] for canal design problems. Therefore an efficient meta-heuristic (random search) approach may be a suitable alternative to solve the complex, non-linear and multi-modal optimization models of canal design considering seepage loss. This paper presents (a) a modified formulation to solve the model of minimal seepage loss design of irrigation canals of different geometries in a meta-heuristic environment using PGSL (2) an improvement of the model by adding additional site specific geometric constraints to equip it for real field applications and the sensitivity of solutions.

2. PROBABILISTIC GLOBAL SEARCH LAUSANNE

Probabilistic Global Search Lausanne (PGSL), proposed by Raphael and Smith [25] is a random search optimization algorithm based on global sampling of the search space using a Probability Density Function (PDF). PGSL was developed starting from the observation that optimally directed solutions can be obtained efficiently through sampling the search space without using special operators. The principal assumption is that better points are likely to be found in the neighborhood of families of 'good' points [26]. Application of PGSL on many benchmark problems with multi-variable non-linear objective functions revealed that PGSL performs better than genetic algorithms and improved variants of simulated annealing [27].

The search space is the domain of all potential solution points, which will be an N dimensional one with one axis corresponding to each variable, where N is the number of decision variables of the problem. The search space is sampled by means of assumed Probability Density Function (PDF) for all the decision variables in the form of histograms. Each axis is discretized into a fixed number of intervals (n) after defining the lower and upper bounds of the variables. After extensive experimentation, Raphael and Smith [25] suggested that 'n' can be selected as 20. To begin the search process the type of PDF can be assumed as uniform distribution. PGSL algorithm consists of four nested cycles: sampling cycle, probability updating cycle, focusing cycle, and sub-domain cycle. Each cycle serves a different purpose in search towards global optima. The sampling cycle performs a search over the entire domain (an exploration process); while the probability updating and focusing cycle refine the search in the neighbourhood of good solution (an exploitation process). Convergence is achieved in the sub-domain cycle.

In the sampling cycle number of points (say NS) is generated randomly by giving a value for each variable according to the PDF. Each point is evaluated by the objective function and the best sample (sample which give minimum objective function value for a minimization problem) is selected. In a probability updating cycle, the sampling cycle is invoked for a number of times (say NPUC). After each iteration, the PDF of each variable is modified using the probability updating algorithm [26]. The interval containing the best solution is first selected, and then the probability of that interval is multiplied by a factor greater than 1. As per the original computer code [28] presented by Raphael, a multiplication factor of 1.2 can be used to have better exploitation process. The PDF thus generated is then modified to make the area under the density function equal to unity. This ensures that the sampling frequencies in regions containing good points are increased.

In a focusing cycle, probability updating cycle is repeated for NFC (number of focusing cycle) times. After each iteration, the search is increasingly focused on the interval containing the current best point. The interval containing the best point is divided into uniform subintervals (usually 6 numbers as per [26]) keeping the fact that total intervals remains constant. A 50% probability is assigned to this interval sothat half of the points generated will be in this interval. The remaining probability is then distributed to the region outside this interval in such a way that the PDF decays exponentially from the best interval. ie., predominantly a single peak function is observed at this stage and about 3 % of the variables lie in the farthest interval [26]. Here the probabilities of region containing good solution are increased and that of less attractive solutions are decreased.

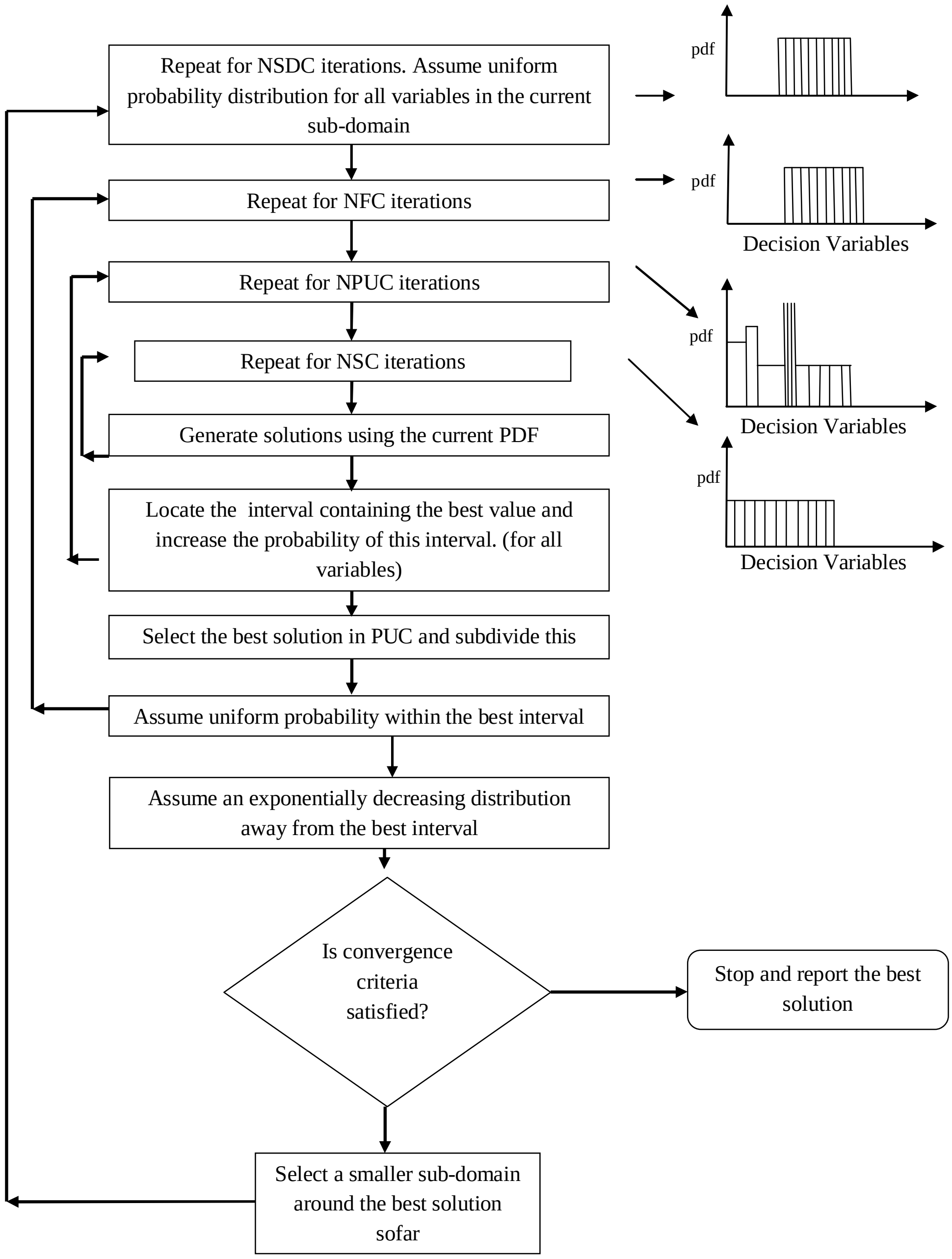


Figure 1. Flowchart of PGSL algorithm

In sub-domain cycle, the focusing cycle is repeated NSDC (number of sub domain cycles) times and at the end of each iteration, the current search space is modified. In the beginning, the entire space is searched, but in subsequent iterations a sub-domain is selected for search. Initially half width is chosen and two scale factors (SDSF1 and SDSF2) are defined to scale down the axis width. The new axis width is found out by multiplying the current width by scale factor. SDSF1 is used if there is an improvement and SDSF2 is used if there is no improvement of objective function value in the current iteration. According to Raphael and Smith [25], the value of SDSF2 is usually selected as 0.96 and SDSF1 can be fixed as SDSF1=$N^{-1/N}$ The sub-domain is selected by changing the upper and lower bound of each variable. While choosing the next sub-domain, the following precautionary measures can be used to avoid premature convergence [26]:

(i) A higher value of scale factor (reduction factor) is chosen after an iteration that does not produce a better cost
(ii) Statistical deviations of the values of the variable in previous iterations (usually 5 iterations) are considered in determining the new minimum and maximum (bounds)

The size of the sub-domain decreases gradually and the solution converges to a point.

PGSL was successfully applied for water quality modeling in recent past [29-31] and an attempt was made for the comprehensive design of trapezoidal channels by Adarsh [32]. A flowchart of the PGSL algorithm in similar lines of the one presented by [33] is given in Figure 1.

3. MODEL DESCRIPTION AND SOLUTIONS

The optimal design of canals for minimum seepage loss involves the estimation of seepage loss subject to the flow and velocity constraints. The exact analysis of seepage loss from canals is quite complex. In the present study the simplified and approximated expressions proposed by Swamee *et al.*, [11] and Chahar [16] were adopted to formulate the optimization model for minimum seepage loss design of irrigation canals.

The development of optimization model is presented below.

3.1. Seepage loss

The steady seepage loss from an unlined or a cracked lined canal in a homogeneous and isotropic porous media, when water table is at very large depth, can be expressed as

$$L_s = kyF_s \qquad (1)$$

where, L_s is the seepage discharge per unit length of the channel (m^2/s); k is the hydraulic conductivity of the porous medium (m/s); y is the flow depth (m) and F_s is the seepage function proposed as an improper integral by Vedernikov [3] for triangular and trapezoidal sections and Morel-Seytoux [4] for the rectangular section. Swamee *et al.*, [11] presented the following expressions to evaluate the seepage functions for different channel geometries considering the drainage layer is at a deeper level.

$$F_s = \left[\left(4p - p^2\right)^{1.3} + (2z)^{1.3}\right]^{0.77} \qquad (2)$$

for triangular channels applicable in the range $0 \le z \le 1000$

$$F_s = \left[\left(4p - p^2\right)^{0.77} + \left(\frac{b}{y}\right)^{0.77} \right]^{1.33} \tag{3}$$

for rectangular channels applicable in the range $0 \le \left(\frac{b}{y}\right) \le 1000$

$$F_s = \left(\left\{ \left[p\left(4 - p\right) \right]^{1.3} + (2z)^{1.3} \right\}^{\left(\frac{(0.77+0.462z)}{(1.3+0.6z)}\right)} + \left(\frac{b}{y}\right)^{\left(\frac{(1+0.6z)}{(1.3+0.6z)}\right)} \right)^{\left(\frac{(1.3+0.6z)}{(1+0.6z)}\right)} \tag{4}$$

for trapezoidal channels applicable in the range $0 \le z \le 1000$ and $0 \le \left(\frac{b}{y}\right) \le 1000$

3.2. Model description and solution

The optimization model for minimum seepage loss canal section can be expressed as follows:

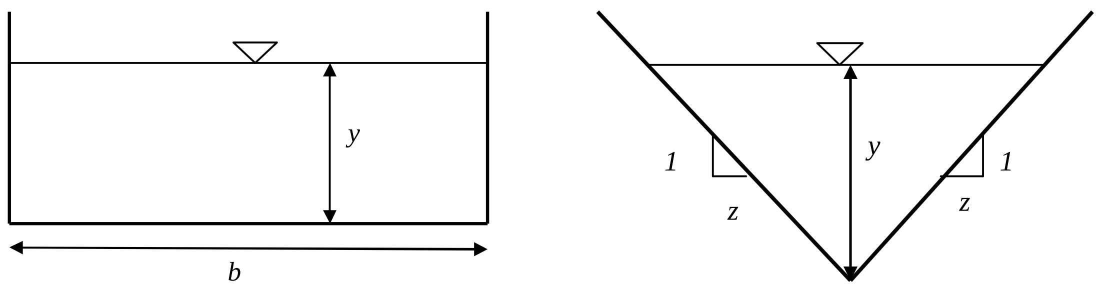
Minimize:

$$L_s(b, y, z) = kyF_s \tag{5}$$

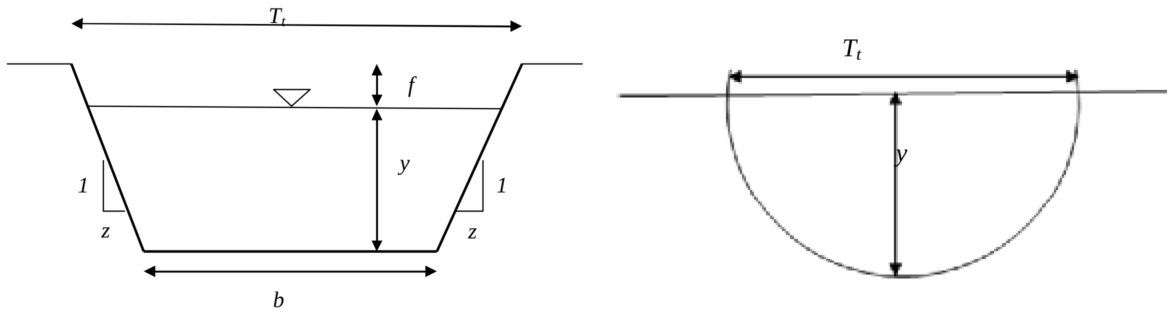
Subject to

$$Q = -2.457 A\sqrt{gRS_o}\, \ln\left(\frac{e}{12R} + \frac{0.221n}{R\sqrt{gRS_o}} \right) \tag{6}$$

Where Q is the discharge (m^3/s); A is the flow area (m^2); g is the acceleration due to gravity (m/s^2) ; S_o is the longitudinal slope; R is the hydraulic radius (m) ; ε is the average roughness height of the canal lining (m) and ν is the kinematic viscosity of water (m^2/s); b is the bed width of trapezoidal and rectangular sections and z is the side slope of triangular and trapezoidal sections. Also the irrigation furrows may be achieving a curvilinear shape in due course of time and posses advantages like easiness in construction with popular excavation equipments, quick drainage of rain water and no stress concentration at corners and many more [16]. In such canals the flow depth (y) and top width (T_t) are the design parameters. Figure 2 gives the definitions sketch of different canal geometries.



(a) Rectangular canal (b) Triangular canal



(c) Trapezoidal canal (d) Canal with curvilinear bottom

Figure 2. Definition sketches of different canal geometries

Equation (6) is the resistance equation proposed by Swamee [34]. To solve in a meta-heuristic environment, the flow resistance constraint φ_1 can be expressed as follows:

$$j_1(b,y,z) = x - \left| Q + 2.457 A\sqrt{gRS_o}\ \ln\left(\frac{e}{12R} + \frac{0.221n}{R\sqrt{gRS_o}}\right)\right| \geq 0 \qquad (7)$$

where ξ is a small positive number.

To ensure the velocity of flow within the permissible limits, the following constraints are added

$$j_2(b,y,z) = \left(V_{\max} - V\right) \geq 0 \qquad (8)$$

$$j_3(b,y,z) = \left(V - V_{\min}\right) \geq 0 \qquad (9)$$

where, V is the average flow velocity (m/s)and V_{max} and V_{min} are the maximum and minimum permissible velocities respectively.

The data for the example problem considered in this study was adopted from Swamee *et al.*, [11]. The design discharge Q =250 m^3/s, longitudinal slope = 0.0001.The canal lining has ε =1 mm. The canal lining is assumed to be cracked and having k =1x10 $^{(-6)}$ m/sec. The water temperature is 20^oC and ν =1.1x10 $^{(-6)}$ m^2/s. The permissible velocity is to be kept between 1.5 m/s to 2 m/s. The optimization model involving Equation (5) and constraints φ_1 to φ_3 (Equations (7)-(9)) is solved using PGSL.

The PGSL based solutions were obtained by solving the developed optimization models with the help of PGSL algorithm in MATLAB environment. The optimization procedure is initiated with PDF of possible solutions. The constraints are handled in such a way that if any of the generated trial solution violates the constraints, such solutions are penalized by giving a suitable penalty to the fitness. The number of intervals, number of sub-domain intervals, SDSF2 etc are found to be insensitive to the problem type and NS, NPUC and NFC can be fixed as 2,1 and 10-20N respectively [26]. In a search for the optimal solution 2 sampling cycles, 1 probability updating cycle, 60 focusing cycle and 30 sub-domain cycle are used in the present study. In the solution process a different random population gives similar solutions indicate that the global optimum has reached. The results are presented in Table 1.

Table 1. Minimum seepage cost canal dimensions

Model	Method	*b* (m)	*y* (m)	*z*	Seepage loss (m^2/s)
Trapezoidal (Basic Model)	Classical#	13.055	7.929	0.598	$4.61x10^{(-5)}$
	PGSL	12.381	8.210	0.588	$4.611x10^{(-5)}$
Rectangular (Basic Model)	PGSL	17.602	8.282	0.000	$4.911x10^{(-5)}$
Triangular (Basic Model)	PGSL	0.000	13.825	0.768	$5.051x10^{(-5)}$

The results of the model for trapezoidal canal along with that reported by Swamee *et al.* [11] are presented in Table 1. Similar solutions were obtained also for triangular and rectangular canals. The solution show that the seepage is minimum for the trapezoidal section when compared to other geometries and the result is similar to the one reported by Swamee *et al.* [11].

3.2.1. Curvilinear channels

For the irrigation furrows with curvilinear bottom the seepage loss is quantified by the equation proposed by Chahar [16].

$$L_s(T_t, y) = ky\left(\frac{T_t}{y} + \frac{p^2}{4G}\right) \tag{10}$$

where G= Catalan's constant=0.915965594 and T_t is the top width of the channel.

Equation (10) is the objective function of the optimization model and Manning's resistance equation in the following form is used as the constraint.

$$f(T_t, y) = x - \left|\frac{Qn}{\sqrt{S_o}} - \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}}\right| \geq 0 \tag{11}$$

Where P is the wetted perimeter and n is the Manning's roughness coefficient.

The design of curved channel is performed for the following data [16]: Discharge =50 m^3/s, Longitudinal slope=0.001 Manning's coefficient=0.035, hydraulic conductivity of the porous medium, k = 5 x 10^7 m/sec.

The results are presented in Table 2. In all the three cases it is found that PGSL gives solutions competent or better than that with the classical procedure.

Table 2. Optimal solution of curvilinear channels

Model	**Method**	**T_t (m)**	**y (m)**	**Seepage loss (m^2/s)**
Curvilinear bottom (Basic Model)	Classical*	3.062	7.561	7.906 x10$^{(-5)}$
	PGSL	3.062	7.553	7.905 x10$^{(-5)}$
Curvilinear bottom (restricted depth)	Classical*	13.114	2.000	9.248 x10$^{(-5)}$
	PGSL	13.091	2.000	9.241 x10$^{(-5)}$
Curvilinear bottom (restricted top width)	Classical*	6.000	3.911	8.238 x10$^{(-5)}$
	PGSL	6.000	3.867	8.213 x10$^{(-5)}$

Note : # Swamee *et al*. [11]; *Chahar [16]

3.3. Minimum seepage loss trapezoidal canals with geometric constraints

The conventional optimization formulation can be modified to make it suit for fulfilling geometrical restrictions imposed by field conditions. The maximum availability of top width, maximum allowable flow depth and fixed side slope parameter (depends on stability of soil slopes) are thee such scenarios considered independently in this study, for the most popular trapezoidal geometry.

The limitation in top width of channel can be accounted by imposing an additional constraint such as Equation (12) along with the basic optimization model presented in Section 3.2.

$$j_4(b, y, z) = (T_{t\max} - T_t) >= 0 \tag{12}$$

where, T_{tmax} is the maximum permissible top width and the top width T_t for trapezoidal channels is defined as:

$$T_t = b + (2z)(y + f) \tag{13}$$

where f is the freeboard and in this study, f is taken as 0.5.

For the best section, for b=13.055 m; y=7.929 m and z=0.598, the top width is 23.14 m. By restricting the maximum permissible top width as 20 m to 15 m in steps of 1 m, the solutions are obtained. The trend in optimal solutions is presented in Figure 3. A reduction in seepage loss with increased value of permissible top width is observed as a result of reducing trend in flow depth and increasing trend in other variables.

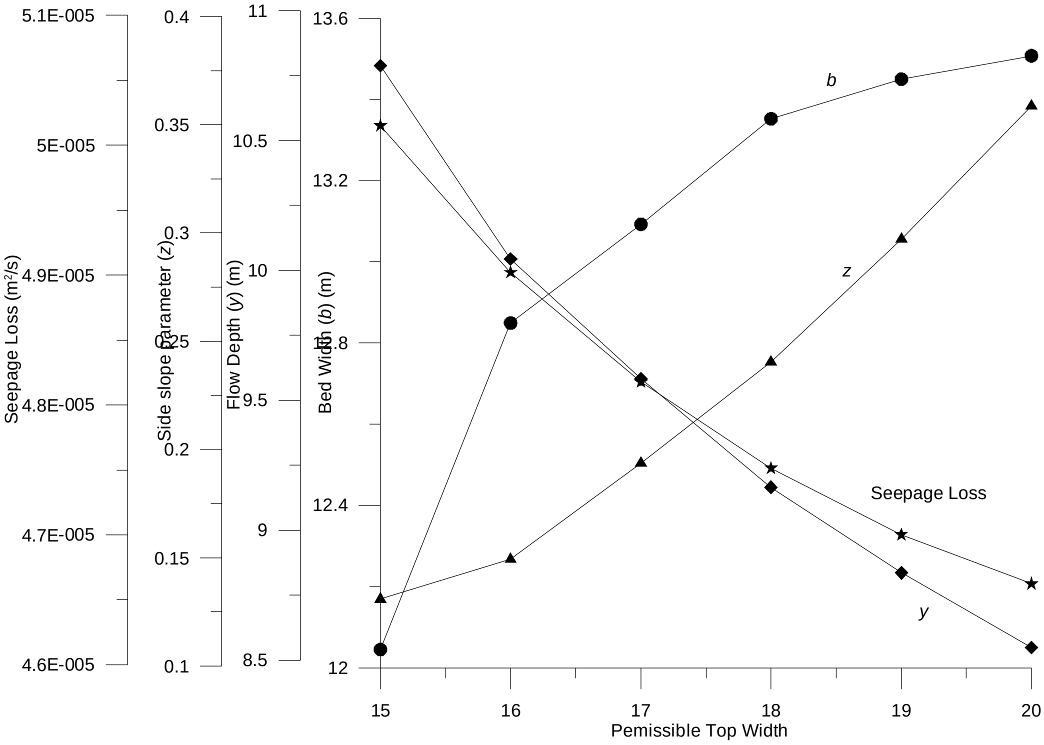


Figure 3. Sensitivity of solutions with permissible top width

The limiting depth scenario can be addressed in channel design problem by imposing Equation (14) as an additional constraint along with the basic optimization model presented in Section 3.2.

$$j_4(y) = (y_{\max} - y) \geq 0 \tag{14}$$

where, y_{max} is the maximum permissible flow depth. The solutions were obtained by fixing the flow depth as 7.5 to 5 in steps of 0.5 m and results are presented in Figure 4. The optimal solutions are obtained for a flow depth value of permissible flow depth and a reducing trend is observed in the value of seepage loss with an increase in flow depth. This may be because of the mathematical character of the constraint equation.

Solutions were also determined by restricting the side slope parameter in the range from 0.2 to 0.6 in steps of 0.1 and results are presented in Figure 5. Identical trend as that observed for the depth restricted case is observed. ie., for this case also, a reduction in seepage loss is observed because of a reduction in other decision variables.

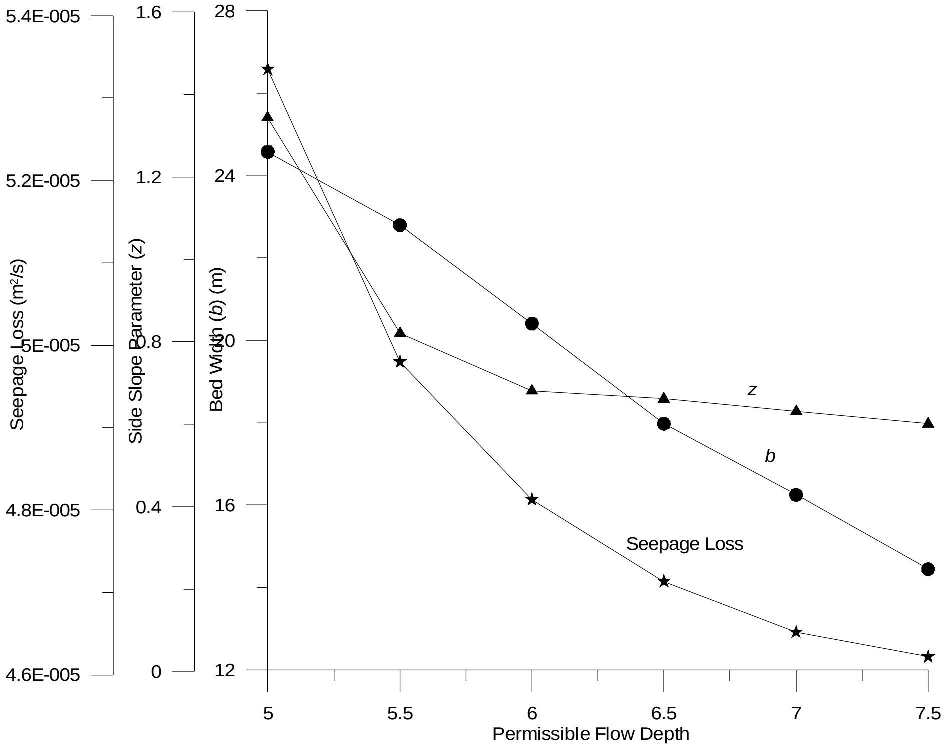


Figure 4. Sensitivity of solutions with permissible flow depth

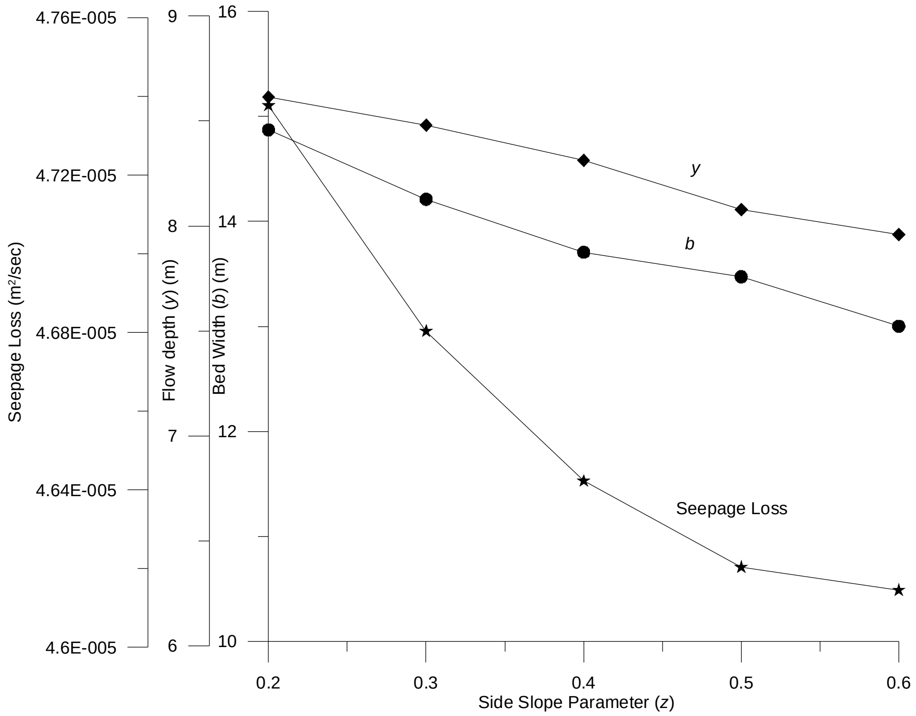


Figure 5. Sensitivity of solutions with side slope parameter

4. CONCLUSIONS

This study presents the usefulness of Probabilistic Global Search (PGSL) as efficient alternative method to perform optimal design of irrigation canals considering minimization of seepage loss as the objective function. The results show that the proposed PGSL approach performs equally well or better in some cases, than the solutions reported in the past based on classical procedures. The practical implementation of designed sections may demand the inclusion of site specific constraints in the canal design model. PGSL also found to be capable in handling the induced complexity due to such constraints, in addition to the high degree of non-linearity and non-convexity associated with the optimization model for minimum seepage loss canals. Thus PGSL approach is an efficient substitute for the optimal design of irrigation canals and can be used to perform the comprehensive design of irrigation canals considering different possible cost elements.

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