



## METAHEURISTIC-BASED SIZING AND TOPOLOGY OPTIMIZATION AND RELIABILITY ASSESSMENT OF SINGLE-LAYER LATTICE DOMES

D. Pakseresht and S. Gholizadeh<sup>\*,†</sup>

*Department of Civil Engineering, Urmia University, Urmia, Iran*

### ABSTRACT

Economy and safety are two important components in structural design process and establishing a balance between them indeed results in improved structural performance specially in large-scale structures including space lattice domes. Topology optimization of geometrically nonlinear single-layer lamella, network, and geodesic lattice domes is implemented using enhanced colliding-bodies optimization algorithm for three different spans and two different dead loading conditions. Collapse reliability index of these optimal designs is evaluated to assess the safety of the structures against overall collapse using Monte-Carlo simulation method. The numerical results of this study indicate that the reliability index of most of the optimally designed nonlinear lattice domes is low and this means that the safety of these structures against overall collapse is questionable.

**Keywords:** metaheuristic optimization; single-layer lattice dome; geometrical nonlinearity; reliability assessment; Monte-Carlo simulation.

Received: 10 September 2020; Accepted: 5 January 2021

### 1. INTRODUCTION

Space lattice domes offer one of the best structural systems to cover large spans without intermediate columns. Domes provide a completely unobstructed inner space and economy in terms of materials. They are lighter compared with the more conventional forms of structures [1]. Although dome structures are economical forms of structural systems, structural optimization techniques can be effectively utilized to design these structures for optimum weight. On the other hand, topology optimization of structures is the most challenging class of the structural optimization problems in which three types of design

---

\*Corresponding author: Department of Civil Engineering, Urmia University, Urmia, P.O. box 165, Iran

†E-mail address: s.gholizadeh@urmia.ac.ir (S. Gholizadeh)

variables with different natures, including sizing, geometry and topology are involved. The topology optimization problem has been identified as a more difficult but more important task than pure sizing and shape optimization, since potential savings in material can be far better improved than by the sizing and shape optimization procedures [2]. Despite the fact that the topological design optimization greatly improves the design, due to its complexity, this class of structural optimization problems has been investigated far less in comparison with pure sizing and shape optimization. The latticed domes are given special names depending on the form in which steel elements are connected to each other. Among the recent applications, the well-known ones are lamella, network, and geodesic domes [3]. Once diameter of a latticed dome is specified, its geometry can be defined by the total number of rings and height of crown. Therefore, in the process of topological design optimization of a lattice dome, these three parameters besides the cross-sectional areas of structural members must be taken as design variables [4]. During the recent years, some studies have been carried out on the topology optimization of domes [5-9]. In [4] a combination of firefly algorithm and particle swarm optimization and in [9] an improved electro-search algorithm have been proposed for topology optimization of single layer domes. In the current study, enhanced colliding bodies optimization (ECBO) [10] is used to topology optimization of lamella, network, and geodesic nonlinear lattice domes.

The structural designer must verify, within a prescribed safety level, the serviceability and ultimate conditions commonly expressed by the inequality of the action  $<$  the resistance. The intrinsic random nature of material properties and actions must be actually considered in the design process of structures and the probability of failure must be computed from the joint probability distribution of the random variables associated with the actions and resistances [11]. Theory and methods for structural reliability are actually useful tools for evaluating the safety of complex structures. Recent developments allow anticipating that their application will gradually increase, even in the case of common structures [12]. Monte Carlo Simulation (MCS) is a simulation method for reliability analysis. It can be applied to many practical problems allowing the direct consideration of any type of probability distribution for the random variables. It is able to compute the probability of failure with the desired precision and it is easy to implement. However, its computational burden is high as MCS requires a great number of structural analyses [13].

This study is focused on evaluating the reliability of optimally designed nonlinear lattice domes against overall collapse considering the uncertainties in demand and capacity. To achieve this, three illustrative design examples of lamella, network, and geodesic domes are presented. For each dome, three spans of 20, 30, and 40 m and two uniform external loading of 250 and 375 kg/m<sup>2</sup> are considered. Topology optimization is implemented by ECBO metaheuristic and reliability assessment are performed using MCS method. The numerical results demonstrate that the reliability index of most of the optimally designed nonlinear lamella, network, and geodesic lattice domes is low and therefore it can be concluded that these optimally designed structures are not safe against overall collapse.

**2. TOPOLOGY OPTIMIZATION OF SINGLE-LAYER DOMES**

Fig. 1 shows the plan view of lamella, network and geodesic domes. The geometry is generated by specifying the diameter  $D$ , the number of rings  $nr$ , and the height of crown  $h$ .

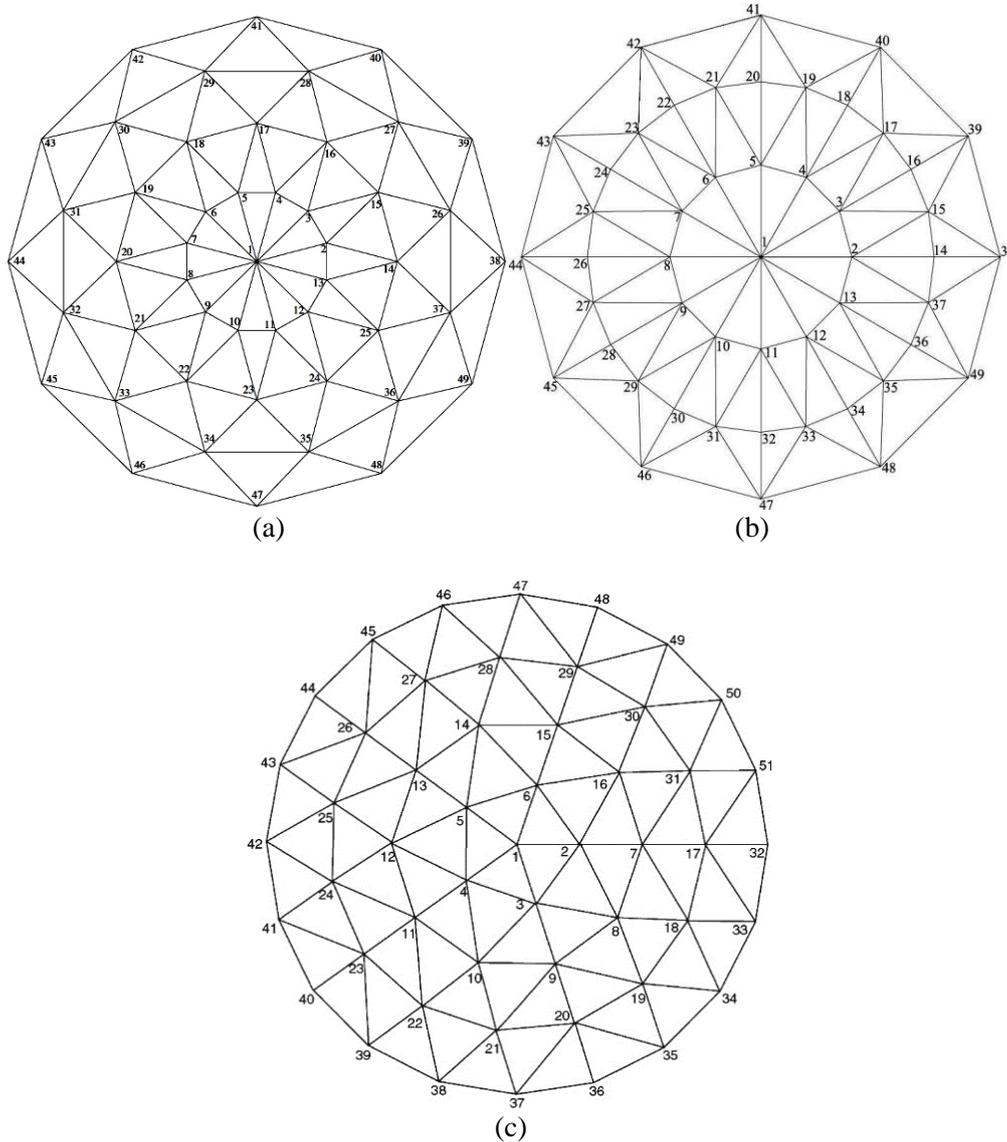


Figure 1. Plan view of (a) Lamella, (b) Network and (c) Geodesic lattice domes

In these domes, the distances between the rings on the meridian line are generally equal. Furthermore, the distances between all joints on the same ring are equal. The joint located at the crown is considered as the first joint. Further information and details of geometry generation for lamella, network and geodesic lattice domes are available in [9].

For a lattice dome consisting of  $ne$  members that are collected in  $ng$  design groups, if the variables associated with each design group are selected from a list of steel pipe sections given by AISC 360-16 code [14], a discrete topology optimization problem considering geometrical nonlinearity can be formulated as follows:

$$\text{Minimize: } f(X) = \sum_{i=1}^{ne} \rho \times A_i \times l_i \quad (1)$$

$$X = \{nr \ h \ I_1 \ I_2 \ \dots \ I_k \ \dots \ I_{ng}\}^T \quad (2)$$

Subject to:

$$g^s(X) = \det(\mathbf{K}(X)) > 0 \quad (3)$$

$$g_j^\delta(X) = \frac{\delta_j}{\delta_{all}} - 1 \leq 0 \quad , \quad j = 1, 2, \dots, nj \quad (4)$$

$$g_i^\sigma(X) = \begin{cases} \text{for } \left(\frac{P_u}{\phi_c P_n}\right)_i < 0.2 \Rightarrow \left(\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right)_i\right) - 1 \leq 0 \\ \text{for } \left(\frac{P_u}{\phi_c P_n}\right)_i \geq 0.2 \Rightarrow \left(\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right)_i\right) - 1 \leq 0 \end{cases} \quad , \quad i = 1, 2, \dots, ne \quad (5)$$

$$g_i^V(X) = \left(\frac{V_u}{\phi_v V_n}\right)_i - 1 \leq 0 \quad , \quad i = 1, 2, \dots, ne \quad (6)$$

where  $X$  is design variables vector;  $I_k$  is an integer expressing the sequence numbers of steel sections assigned to  $k$ th group;  $\rho$  is material weight density;  $A_i$  and  $l_i$  are cross-sectional area and length of the  $i$ th member, respectively;  $g^s(X)$  is stability constraint;  $\mathbf{K}$  is structural stiffness matrix;  $g_j^\delta(X)$ ,  $\delta_j$  and  $\delta_{all}$  are the displacement constraint, displacement and allowable displacement of joint  $j$ , respectively;  $g_i^\sigma(X)$  is stress constraint of  $i$ th member;  $P_u$  is the required strength;  $P_n$  is the nominal axial strength;  $M_{ux}$  and  $M_{uy}$  are the required flexural strengths in the  $x$  and  $y$  directions; respectively;  $M_{nx}$  and  $M_{ny}$  are the nominal flexural strengths in the  $x$  and  $y$  directions;  $V_u$  is the factored service load for shear;  $V_n$  is the nominal strength in shear.

The design constraints are handled by the exterior penalty function method as follows:

$$\Phi(X, r_p) = f(X) \left( 1 + \zeta \left( (\max\{0, g^s\})^2 + \sum_{j=1}^{nj} (\max\{0, g_j^\delta\})^2 + \sum_{i=1}^{ne} ((\max\{0, g_i^\sigma\})^2 + (\max\{0, g_i^V\})^2) \right) \right) \quad (7)$$

where  $\Phi$  and  $\zeta$  are the pseudo objective function and penalty parameter, respectively.

In the present work, the above mentioned topology optimization problem of lattice domes is solved by ECBO metaheuristic algorithm. The ECBO is described in the next section.

### 3. ECBO ALGORITHM

Kaveh and Ilchi Ghazaan [10] proposed enhanced colliding bodies optimization (ECBO) to improve convergence rate and reliability of colliding bodies optimization (CBO) [15] by adding a memory to save some of the best solutions during the optimization process and also utilizing a mutation operator to decrease the probability of trapping into local optima. The basic steps of ECBO are as follows [10]:

1. The initial positions of all colliding bodies (CBs) are determined randomly.
2. The value of mass for each CB is evaluated as follows:

$$m_i = \frac{1}{\Phi(X_i)} \quad (8)$$

3. Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related masses. Solution vectors in CM, are added to the population and the same number of current worst CBs are deleted. Finally, CBs are sorted according to their masses.

4. CBs are divided into two equal groups:

- (a) Stationary group;  $i_s = 1, 2, \dots, \frac{n}{2}$  and (b) Moving group;  $i_M = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n$

5. The velocities of stationary and moving bodies before collision are evaluated as follows:

$$V_{i_s} = 0, V_{i_M} = X_{i_s} - X_{i_M} \quad (9)$$

6. The velocities of stationary and moving bodies after collision are evaluated as follows:

$$V'_{i_s} = \left( \frac{(1 + \varepsilon) m_{i_M}}{m_{i_s} + m_{i_M}} \right) V_{i_M}, V'_{i_M} = \left( \frac{(m_{i_M} - \varepsilon m_{i_s})}{m_{i_s} + m_{i_M}} \right) V_{i_M} \quad (10)$$

$$\varepsilon = 1 - \frac{t}{t_{\max}} \quad (11)$$

where  $\varepsilon$  is the coefficient of restitution.

7. The new position of each CB is calculated as follows:

$$X_{i_s}^{\text{new}} = X_{i_s} + \bar{R}_{i_s} \circ V'_{i_s} \quad (12)$$

$$X_{i_M}^{\text{new}} = X_{i_M} + \bar{R}_{i_M} \circ V'_{i_M} \quad (13)$$

where  $\bar{R}_{i_s}$  and  $\bar{R}_{i_M}$  are random vectors uniformly distributed in the range of  $[-1, 1]$ .

8. A random parameter pro is introduced and it is specified whether a component of each CB must be changed or not. For each CB, pro is compared with rni ( $i=1, \dots, n$ ) which is a random number uniformly distributed within  $(0, 1)$ . If  $rni < pro$ , one dimension of the  $i$ th CB is selected randomly.

9. When a stopping criterion is satisfied, the optimization process is terminated.

#### 4. RELIABILITY ASSESSMENT

Deterministic structural optimization without considering the uncertainties in structural capacity and demands results in an unreliable design and therefore cannot provide a fine balance between cost and safety. In this case, it is not possible to ensure that the structural performance will be fulfilled during the lifetime of structures, because the uncertainty in actions and resistances affect the structural response. An appropriate framework for modeling uncertainty is probability theory which allows calculating the reliability index of structures. In the past decades, to deal with the randomness in actions and resistances, semi-probabilistic, approximate probabilistic, and exact probabilistic methods have been widely used [13]. In exact probabilistic methods the probability of failure is determined based on the joint probability distribution of the random variables. Monte Carlo simulation (MCS) is a simulation method categorized in exact probabilistic methods that allows the consideration of any probability distribution function for random variables. The major advantage of MCS is that accurate solutions can be obtained for almost every problem, however its computational cost is excessive in many cases [16].

In order to solve a reliability problem, random design variables need to be defined. For the optimally designed single-layer lattice domes the random variables taken in the present study are represented as follows:

$$Z = \{E \ f_y \ H \ Q\}^T \quad (14)$$

where  $Z$  is vector of random variables;  $E$ ,  $f_y$  and  $H$  are respectively Young's modulus, yield strength, and hardening modulus which define the plasticity model; and  $Q$  stands for external loading.

Constitutive behavior of steel materials of structural members shown in Fig. 2 is based on one-dimensional plasticity model with linear hardening.

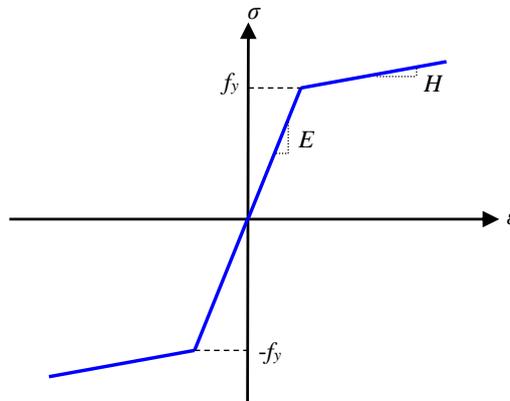


Figure 2. Material constitutive behavior of steel

A reliability problem is normally formulated using a limit state function. Limit state function for optimally designed nonlinear lattice domes is defined as follows:

$$G(Z) = Q_u(Z) - Q \quad (15)$$

where  $G$  is a limit state function;  $Q_u$  is the ultimate load that causes the dome to collapse.

The non-performance probability,  $P_f$ , is defined as a function of the defined limit state functions for the problem at hand. Estimation of the non-performance probability requires the evaluation of the multiple integral over the failure domain, i.e.  $G(Z) < 0$ , as follows:

$$P_f = \iint_{G(Z)} \dots \int F_Z(Z) dZ \quad (16)$$

where  $F_Z(Z)$  is the joint probability density function of  $Z$ .

The total exceedance probability,  $P_{f_E}$ , is defined as a series system when one of the limit state functions fails:

$$P_{f_E} = P\left(\bigcup_{i=1}^{n_l} \{G_i(Z) \leq 0\}\right) \quad (17)$$

where  $n_l$  is the number of the limit state functions.

As in the present work there is only one limit state function for each structure, Eq. (17) can be rewritten as follows:

$$P_{f_E} = P(G(Z) \leq 0) \quad (18)$$

Computation of total exceedance probability requires integration of a multi-normal distribution function. This integral can be estimated by the MCS method and it allows the determination of an estimate of  $P_{f_E}$  given by

$$P_{f_E} = \frac{1}{N} \sum_{i=1}^N I_i(Z) \quad (19)$$

$$I(Z) = \begin{cases} 1 & \text{if } G(Z) \leq 0 \\ 0 & \text{if } G(Z) > 0 \end{cases} \quad (20)$$

where  $N$  is the number of independent samples generated based on the probability distribution for each random variable.

Finally, the reliability index ( $RI$ ) for the problem at hand is determined as follows:

$$RI = 1 - P_{f_E} \quad (21)$$

## 5. METHODOLOGY

To evaluate the reliability index  $RI$  for optimally designed nonlinear lattice domes, a methodology is presented in this study. The proposed methodology is outlined as follows:

**Step1.** Topology optimization of single-layer lamella, network, and geodesic lattice domes is achieved considering geometrically nonlinearity by ECBO metaheuristic algorithm according to the formulation given by Eqs. (1) to (6). For each dome, different spans including  $S = 20, 30,$  and  $40$  m and two uniform external loading of  $Q=250$  and  $Q=375$  kg/m<sup>2</sup> are considered. This means that for each dome, 6 different optimization processes are performed and a total number of 18 optimal designs are obtained.

**Step2.** Reliability assessment of the optimally designed single-layer lamella, network, and geodesic lattice domes is carried out using MCS method. The probability density function, mean value and standard deviation of each random parameter are given in Table 1.

Table 1: Properties of the random variables for steel lattice domes

Random Variable	Probability density function	Mean value	Standard deviation
$E$	Lognormal	205 GPa	$0.05E$
$f_y$	Lognormal	250 MPa	$0.05 f_y$
$H$	Lognormal	0.03	$0.05H$
$Q$	Lognormal	250 kg/m <sup>2</sup> 375 kg/m <sup>2</sup>	$0.05Q$

The accuracy of MCS-based reliability assessment process is highly dependent to the total number of generated samples  $N$ . In order to determine the best value of  $N$ , a sensitivity analysis is performed for the first above mentioned topology optimization case by taking  $N = 5 \times 10^3, 10^4, 1.5 \times 10^4, 3 \times 10^4, 5 \times 10^4,$  and  $10^5$  and it is observed that the best value for  $N$  is  $10^4$ . The results of sensitivity analysis are depicted in Fig. 3. Therefore, in the reliability assessment of the optimally designed lattice domes, the number of samples is  $N = 10^4$ .

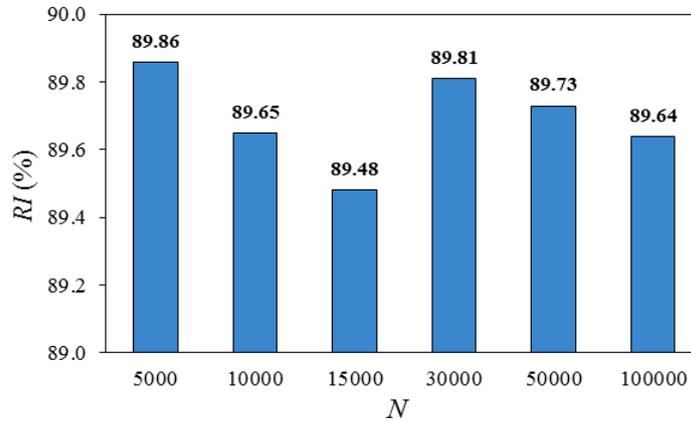


Figure 3. Results of sensitivity analysis for the first topology optimization case

## 6. NUMERICAL EXAMPLES

During the topology optimization process of single-layer lattice domes, the design variables associated with each design group are selected from the following table of steel pipe sections available in AISC 360-16 code [14].

Table 2: Available steel pipe sections [14]

No.	Pipe section	Outside Diameter (mm)	Design wall thickness (mm)	Nominal weight (kg/m)
1	1/2SCH40	21.3	2.57	1.26
2	1/2SCH80	21.3	3.48	1.62
3	3/4SCH40	26.7	2.67	1.68
4	3/4SCH80	26.7	3.63	2.20
5	1SCH40	33.4	3.15	2.50
6	1SCH80	33.5	4.22	3.23
7	1-1/4SCH40	42.2	3.30	3.38
8	1-1/2SCH40	48.3	3.43	4.04
9	1-1/4SCH80	42.2	4.52	4.46
10	1-1/2SCH80	48.3	4.72	5.40
11	2SCH40	60.3	3.63	5.44
12	2SCH80	60.3	5.18	7.48
13	2-1/2SCH40	73.0	4.80	8.62
14	3SCH40	88.9	5.11	11.3
15	2-1/2SCH80	73.0	6.53	11.4
16	2XXS	60.3	10.3	13.4
17	3-1/2SCH40	101.6	5.36	13.6
18	3SCH80	88.9	7.11	15.3
19	4SCH40	114.3	5.61	16.1
20	3-1/2SCH80	101.6	7.52	18.6
21	2-1/2XXS	73.0	13.1	20.4
22	5SCH40	141.3	6.12	21.7
23	4SCH80	114.3	8.00	22.3
24	3XXS	88.9	14.2	27.7
25	6SCH40	168.3	6.63	28.3
26	5SCH80	141.3	8.86	30.9
27	4XXS	114.3	16.0	41.0
28	8SCH40	219.1	7.62	42.5
29	6SCH80	168.3	10.2	42.5
30	5XXS	141.3	17.8	57.4
31	10SCH40	273.0	8.64	60.2
32	8SCH80	219.1	11.8	64.5
33	12STD	323.8	8.86	73.8
34	6XXS	168.3	20.4	79.1
35	10SCH80	273.0	11.8	81.5
36	12XS	323.8	11.8	97.4
37	8XXS	219.1	20.7	108

For all the domes, the members that connect top joint to the ones on the first ring are categorized into the first group of variables. The members on the first ring are the second group and the members located between first and second rings are considered as the third group. This procedure is applied to the grouping of all members. In addition,  $\delta_{all} = 28$  mm.

### 6.1 Results of topology optimization

Topology optimization results for different spans are reported in Tables 3 to 5. The optimal weights are compared in Figs. 4 and 5 for  $Q=250$  and  $375$  kg/m<sup>2</sup>, respectively.

Table 3: Results of topology optimization for span = 20 m

Design variables	$Q = 250$ kg/m <sup>2</sup>			$Q = 375$ kg/m <sup>2</sup>		
	Lamella	Network	Geodesic	Lamella	Network	Geodesic
$nr$	5	5	5	5	5	5
$h$	4.0	5.0	5.0	5.0	5.0	5.0
$I_1$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH80	1/2SCH80	1/2SCH40
$I_2$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	3/4SCH80	1/2SCH40
$I_3$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40
$I_4$	1/2SCH40	1SCH40	1/2SCH40	1/2SCH40	1-1/2SCH40	1/2SCH40
$I_5$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40
$I_6$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH80
$I_7$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	3/4SCH80
$I_8$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	3/4SCH80
$I_9$	1/2SCH40	1/2SCH40	1/2SCH80	3/4SCH40	1/2SCH80	1/2SCH80
$I_{10}$	1/2SCH80	1/2SCH40	1/2SCH40	3/4SCH40	1/2SCH80	1/2SCH80
<b>Weight (kg)</b>	592.1	772.3	588.7	643.5	947.4	732.7

Table 4: Results of topology optimization for span = 30 m

Design variables	$Q = 250$ kg/m <sup>2</sup>			$Q = 375$ kg/m <sup>2</sup>		
	Lamella	Network	Geodesic	Lamella	Network	Geodesic
$nr$	6	6	6	6	6	6
$h$	7.5	7.5	7.5	7.5	7.5	7.5
$I_1$	3/4SCH80	3/4SCH80	1/2SCH40	1SCH80	1SCH80	1SCH40
$I_2$	1/2SCH80	1SCH80	1/2SCH40	3/4SCH80	1-1/2SCH40	3/4SCH80
$I_3$	1/2SCH40	1/2SCH80	1/2SCH40	1/2SCH80	3/4SCH80	3/4SCH40
$I_4$	1/2SCH40	1-1/2SCH40	1/2SCH40	1/2SCH40	3/4SCH40	1/2SCH80
$I_5$	1/2SCH40	1/2SCH40	1/2SCH40	1SCH80	1/2SCH40	1/2SCH40
$I_6$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40
$I_7$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	3/4SCH80
$I_8$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1SCH80
$I_9$	1/2SCH40	1/2SCH40	1/2SCH80	1/2SCH80	1/2SCH40	1SCH80
$I_{10}$	1/2SCH80	1/2SCH40	1/2SCH40	3/4SCH80	1SCH80	1SCH80
$I_{11}$	3/4SCH80	3/4SCH40	1/2SCH40	1SCH40	1-1/4SCH40	1SCH80
$I_{12}$	3/4SCH80	1/2SCH40	1/2SCH40	1SCH80	3/4SCH80	3/4SCH80
<b>Weight(kg)</b>	1387.3	1692.6	1592.4	1851.1	2379.1	2195.8

Table 5: Results of topology optimization for span = 40 m

Design variables	$Q = 250 \text{ kg/m}^2$			$Q = 375 \text{ kg/m}^2$		
	Lamella	Network	Geodesic	Lamella	Network	Geodesic
$nr$	7	7	7	7	7	7
$h$	10.0	10.0	8.0	10.0	10.0	10.0
$I_1$	1-1/2SCH40	1SCH80	1SCH80	1-1/2SCH80	1-1/2SCH80	1-1/2SCH40
$I_2$	1SCH80	1-1/4SCH40	1SCH80	1-1/2SCH40	2SCH40	1-1/4SCH40
$I_3$	1SCH40	1SCH40	1SCH80	1SCH80	1-1/4SCH80	1SCH80
$I_4$	1/2SCH80	2SCH40	1SCH40	3/4SCH80	2SCH80	1SCH80
$I_5$	1/2SCH40	1/2SCH40	3/4SCH80	3/4SCH80	3/4SCH40	3/4SCH80
$I_6$	3/4SCH80	2SCH80	1/2SCH80	1-1/4SCH40	2SCH40	3/4SCH40
$I_7$	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40	1/2SCH40
$I_8$	1/2SCH40	1/2SCH40	3/4SCH80	1/2SCH40	1/2SCH40	1SCH40
$I_9$	1/2SCH40	1/2SCH40	1SCH80	1/2SCH40	1/2SCH40	1-1/2SCH40
$I_{10}$	1/2SCH40	1/2SCH40	1SCH80	3/4SCH40	1/2SCH40	1-1/4SCH80
$I_{11}$	1/2SCH80	1/2SCH40	1SCH80	1SCH40	1/2SCH80	1-1/4SCH80
$I_{12}$	3/4SCH80	1/2SCH40	1SCH80	1-1/4SCH40	3/4SCH40	1-1/4SCH80
$I_{13}$	1SCH40	1/2SCH80	1-1/4SCH40	1-1/4SCH40	1SCH80	1-1/2SCH40
$I_{14}$	1SCH40	1/2SCH80	3/4SCH80	1-1/2SCH40	1-1/4SCH80	1SCH80
<b>Weight (kg)</b>	<b>2836.3</b>	<b>3672.3</b>	<b>4012.2</b>	<b>3983.9</b>	<b>5281.5</b>	<b>5364.1</b>

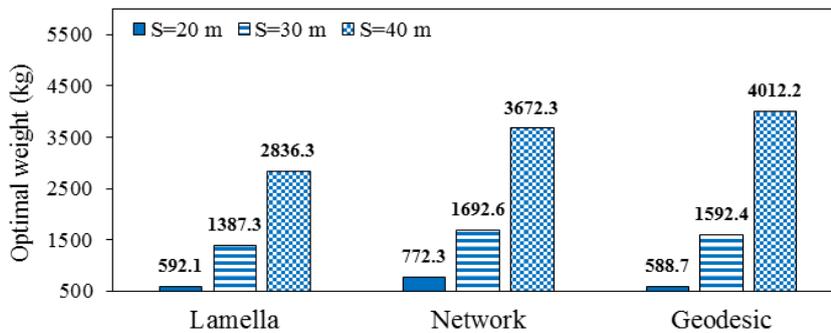


Figure 4. Optimal weight of domes for  $Q = 250 \text{ kg/cm}^2$

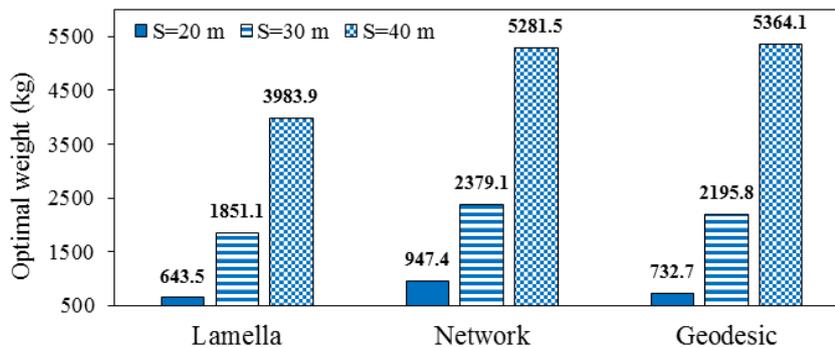


Figure 5. Optimal weight of domes for  $Q = 375 \text{ kg/cm}^2$

The results indicate that, in the case of  $Q = 250 \text{ kg/cm}^2$ , the best weight for  $S = 30$  and  $40$  m corresponds to lamella dome. For  $S = 20$  m, the optimal weights of lamella and geodesic domes are almost the same. In the case of  $Q = 375 \text{ kg/cm}^2$ , the best weights for all the spans belong to lamella dome. Furthermore, the network dome with  $20$  and  $30$  m spans and geodesic dome with span of  $40$  m have the maximum weight for both the  $250$  and  $375 \text{ kg/cm}^2$  load cases. Therefore, it can be concluded that the best form of single layer domes, in terms of optimal weight, with  $20$ ,  $30$ , and  $40$  m spans for both the  $250$  and  $375 \text{ kg/cm}^2$  loading cases is lamella dome.

### 6. 2 Results of reliability assessment

Reliability assessment is carried out for all the optimal domes by MCS method based on  $N=10^4$  samples. The values of  $RI$  obtained for all the domes are compared in Figs. (6) and (7) for  $Q=250$  and  $375 \text{ kg/m}^2$ , respectively.

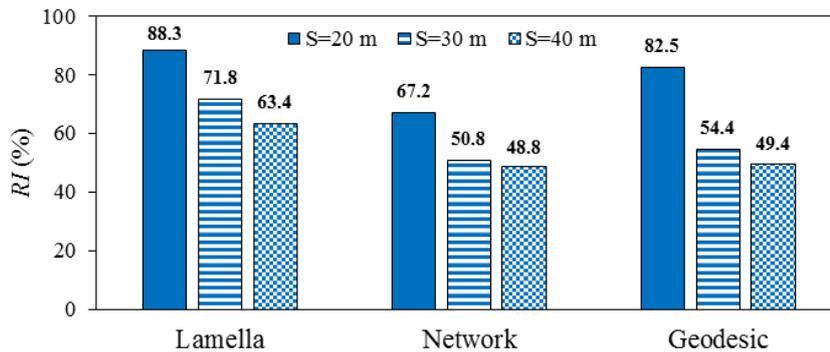


Figure 6. Reliability index of optimal domes for  $Q = 250 \text{ kg/cm}^2$

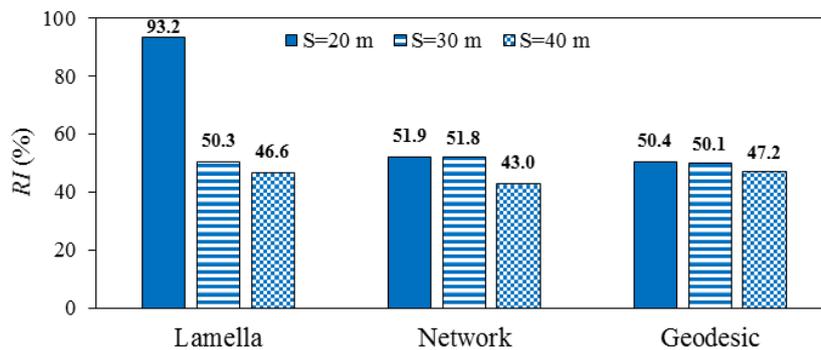


Figure 7. Reliability index of optimal domes for  $Q = 375 \text{ kg/cm}^2$

The results reveal that in the case of  $Q = 250 \text{ kg/cm}^2$  loading, the highest  $RI$  for all  $20$ ,  $30$ , and  $40$  m spans belongs to lamella dome. The second best form in terms of  $RI$  is geodesic dome. In the case of  $Q = 375 \text{ kg/cm}^2$  loading, the highest  $RI$  is obtained for lamella dome with span of  $20$  m and for the remaining ones  $RI$  is about  $50\%$ . These results indicate that lamella dome is the most reliable form compared to network and geodesic domes.

## 7. CONCLUSIONS

This study is devoted to reliability assessment of optimally designed nonlinear lamella, network, and geodesic lattice domes against overall collapse considering the uncertainties in strength and action parameters. To this end, three spans of 20, 30, and 40 m and two uniform external loading of 250 and 375 kg/m<sup>2</sup> are considered for lamella, network, and geodesic domes. Topology optimization is performed by using ECBO algorithm and reliability analysis is conducted by MCS method.

The numerical results demonstrate that, lamella dome is the best form of single layer domes, in terms of optimal weight, for all spans and both the loading cases. Moreover, it is concluded that lamella dome is the most reliable form compared to network and geodesic domes. This means that among lamella, network and geodesic domes, the best one in terms of optimal weight and collapse safety is lamella dome.

The most important finding of the present study is that the reliability index of most of the optimally designed nonlinear lamella, network, and geodesic lattice domes is low and therefore it can be concluded that these optimally designed structures are not safe against overall collapse. The computational burden of considering a constraint on overall collapse of lattice domes would be prohibitively high and is out of scope of this study. However, reliability-based topology optimization of lattice domes must be achieved to establish a fine balance between cost and safety of these structures considering the uncertainties in strength and action.

## REFERENCES

1. Makowshi ZS. Analysis, Design and Construction of Braced Domes, Granada Publishing Ltd, London, UK, 1984.
2. Kaveh A, Mahdavi Dahoei VR. Colliding bodies optimization for size and topology optimization of truss structures, *Struct Eng Mech* 2015; **53**: 847-65.
3. Carbas S, Saka MP. Optimum topology design of various geometrically nonlinear latticed domes using improved harmony search method, *Struct Multidisc Optim* 2012; **45**: 377-99.
4. Gholizadeh S, Barati H. Topology optimization of nonlinear single layer domes by a new metaheuristic, *Steel Compos Struct* 2014; **16**: 681-701.
5. Kaveh A, Talatahari S. Geometry and topology optimization of geodesic domes using charged system search, *Struct Multidisc Optim* 2011; **43**: 215-29.
6. Kaveh A, Rezaei M. Optimum topology design of geometrically nonlinear suspended domes using ECBO, *Struct Eng Mech* 2015; **56**: 667-94.
7. Kaveh A, Zolghadr A. Optimal analysis and design of large-scale domes with frequency constraints, *Smart Struct Syst* 2016; **18**: 733-54.
8. Kaveh A, Rezaei M, Shiravand MR. Optimal design of nonlinear large-scale suspendome using cascade optimization, *Int J Space Struct* 2017; **33**: 3-18.
9. Bigham A, Gholizadeh S. Topology optimization of nonlinear single-layer domes by an improved electro-search algorithm and its performance analysis using statistical tests,

- Struct Multidisc Optim* 2020; **62**: 1821-48.
10. Kaveh A, Ilchi Ghazaan M. Enhanced colliding bodies optimization for design problems with continuous and discrete variables, *Adv Eng Softw* 2014; **77**: 66-75.
  11. Shooman ML. Probabilistic Reliability: An Engineering Approach, McGraw-Hill, New York, 1968.
  12. Park GJ, Lee TH, Lee KH, Hwang KH. Robust design: An overview, *AIAA J* 2006; **44**: 181-91.
  13. Cardoso JB, de Almeida JR, Dias JM, Coelho PG. Structural reliability analysis using Monte Carlo simulation and neural networks, *Adv Eng Softw* 2008; **39**: 505-13.
  14. AISC 360-16. Specification for Structural Steel Buildings. American Institute of Steel Construction, Chicago, 2016.
  15. Kaveh A, Mahdavi VR. Colliding bodies optimization: A novel meta-heuristic method. *Comput Struct* 2014; **139**: 18-27.
  16. Lagaros ND, Garavelas AT, Papadrakakis M. Innovative seismic design optimization with reliability constraints, *Comput Meth Appl Mech Eng* 2008; **198**: 28-41.