ABSTRACT

One of the most important problems discussed recently in structural engineering is the structural reliability analysis considering uncertainties. To have an efficient optimization process for designing a safe structure, firstly it is required to study the effects of uncertainties on the seismic performance of structure and then incorporate these effects on the optimization process. In this study, a new procedure developed for incorporating two important sources of uncertainties in design optimization process of steel moment resisting frames, is proposed. The first source is related to the connection parameter uncertainties and the second one to seismic demand uncertainty. Additionally Mont Carlo (MC) simulation and a variance reduction technique (VRT) are utilized to deal with uncertainties and to reduce the corresponding computational cost. In the proposed procedure two design objectives are considered, which are structural weight and collapse prevention reliability index for a moment resisting frame in such a way that leads to a set of optimum designs with minimum weight and less possible amounts of sensitivity to connection parameters uncertainties and spectral acceleration uncertainty as seismic demand variation. Additionally, in this procedure the reliability index is computed considering all FEMA-356 performance acceptance criteria, the approach that has never been investigated in other studies. The efficiency of this approach is illustrated by exhibiting a set of optimum designs, in the form of both objective values and investigating nonlinear behavior of optimum designs compared with non-optimum designs. This procedure is introduced in this paper with emphasize on the collapse limit state and applying pushover analysis for studying the nonlinear behavior of structural elements.

Keywords: reliability analysis; connection parameters; uncertainties; monte carlo simulation; variance reduction; seismic demand variation; collapse limit state.

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1. INTRODUCTION

In conventional structural design optimization, usually formulated under deterministic design parameters, stochastic nature and uncertainty of governing design parameters are considered through several safety factors incorporated into the optimum design problem as constraints. In other words, safety concerns are mitigated by compliance to the design standards (e.g. ASCE7 [1]) and the economic part of the design problem is assumed to be only related to the initial construction cost. Consequently, because of complexity of the mentioned design optimization problem and unavoidable limitations on the design budgets and the designers’ effort, the final selected design is likely not to be the most optimal in its cost and safety. In order to find more reliable and safe designs, in recent decades non-deterministic performance measures are increasingly being taken into consideration in many engineering design problems while involve various reliability requirements, (e.g. FEMA 350 [2]). Mainly, this concept has been introduced as Performance-Based Design (PBD) in order to increase the safety against natural hazards, especially seismic hazards, to make them having a predictable and reliable performance.

In other words, in this approach the structures should be able to resist earthquakes in a quantifiable manner and to preset levels of desired possible damage. Therefore, according to PBD approach, seismic structural design is that a structure should meet performance-based objectives for a pre-defined hazard levels. The problem of incorporating different of kind of uncertainties in the PBD design process has attracted the attention of many researchers in recent years. Many of parameters that affect design process including aleatory uncertainties related to their uncertain nature. Moreover, epistemic uncertainties related to lack of sufficient knowledge, engineering and manufacturing errors are continuously involved in theoretical and executive problems [3, 4]. In order to precisely deal with uncertainties, it is necessary to know different types of uncertainties, how they affect seismic performance of structure and the amount of these effectiveness. It should be taken an efficient approach to incorporate important sources of uncertainties in design process, especially when a set of optimum designs is desired. For this purpose, suitable quantitative criteria should be proposed from the sensitivity of designs to different sources of uncertainties. By applying this measure, it will be possible to design a building with less sensitivity to discussed uncertainties. In this context, several studies have been proposed about studying structures based on reliability analysis. The common point among all of these reliability studies is the necessity of probabilistic reliability analysis with simulating nonlinear behavior of structures under the effect of uncertain parameters. In this way, sensitivity analysis is considered as one of the applicable strategies for studying the effectiveness rate of uncertain parameters on seismic response of structure. According to this approach, the effectiveness rate of each parameter on the seismic response of structure will be evaluated for its various amounts while others are taken to be constantly. Several researchers like Esteva and Ruiz [5], Ibarra and Krawinkler [6] and Aslani [7] have used sensitivity analyses in different reliability studies. They have recognized and used most effective parameters for nonlinear analysis of structure through this approach. Among the several reliability methods proposed in literature, First order second moment (FOSM) and MC are usually known as the most applicable methods. Other methods such as First order reliability method (FORM) and Second order reliability method (SORM) are utilized more restrictively.
use linear and quadratic approximations for limit state function $g(x)$ respectively. In SORM, the limit state function $g(x)$ achieved from second order-Taylor expansion about the $\mu_g = g(E[x])$, will be used for investigating the structure under the effect of uncertain parameters $x_i$. There is an important problem in application of FOSM/FORM when $g(x)$ tends to have nonlinear behavior around $\mu_g$. In this condition, the obtained approximations do not have exact precision and other reliability methods such as SORM and MC can be substituted. It is recommended to refer to studies [6,8] as an example of applying FOSM for reliability analysis. Haselton [9] combines sensitivity analysis and FOSM for evaluating the effects of uncertain variables on seismic response and calculating collapse capacity of a 4-story reinforced concrete frame. The other practical method for reliability analyses is MC simulation. In general, MC provides a good statistical simulation for probabilistic analysis using a sample set of stochastic random variables. These stochastic uniform random variables will be produced based on inverse transformation method (ITM) for uncertain parameters as for their cumulative distributions [8]. This procedure, if only implemented without applying numerical techniques for decreasing variance of stochastic generated values, is called as crude Monte Carlo (CMC) which may need much computational effort in structural reliability analyses [10]. For solving this problem, one can employ some techniques known as VRTs which lead to smaller dispersion in random generated variables. In this way, MC simulation can be implemented with a smaller sample set through applying VRTs compared with CMC [11, 12]. Among the last studies in literature, we can find an example for applying MC in the study of Porter [13] in which this approach is used for studying structural damage for non-ductile reinforced frames incorporating some uncertainties related to modeling parameters. In another study, Abbie and Liel [14] demonstrated that MC computational efforts could be reduced in combination with response surface and applying LHS sampling method for generating sample set. Moreover these prevalent methods, there are some novel methods for reliability studies in literature like the algorithm that introduced by Kaveh and Massoudi [15]. The proposed approach is capable of finding a design point by achieving minimum reliability index under the limit state function. Additionally, Danesh and Gholizade [16] performed a PBD optimization for three steel moment frames by employing four meta heuristic algorithms and they have investigated the efficiency of these algorithms by investigating the seismic collapse safety of optimal obtained designs. Kaveh and Zolghadr [17] performed a comparative study on meta-heuristic algorithms for optimal design of truss structures with frequency constraints. Gholizade and Fattahi [18] have proposed a novel approach for PBD optimization of steel frames using sequential enhanced colliding bodies algorithm. The optimization problem has been solved for achieving the minimum total cost defined as summation of initial construction cost and seismic damage cost. Ganjavi and Hajirasouliha [19] presented an optimum PBD methodology for concentrically steel braced frames based on the concept of uniform distribution for shear story drift along the height of building. Mahallati and Ghoohani [20] have presented an improved multi-objective evolutionary algorithm for optimization of planar steel moment frames. Gholizade and Kamyab [21] performed a PBD optimization for steel moment resisting frames by applying four different metaheuristic algorithms to minimize the structural weight subjected to inter-story drift ratio constraints. Kaveh and Talatahari [22] presented a performance-based optimal seismic design of steel frames applying the Ant colony optimization. Kaveh and Laknejadi [23] presented a new
framework for multi-objective optimization of large steel structures using genetic algorithm, NSGA-II. Kaveh and Fahimi [24,25] presented novel efficient approaches for multi-objective PBD optimization of steel moment frames applying wavelet analyses procedure for reducing the computational burden of time history analysis. Liu and Atamturkter [26] proposed a performance based robust design optimization for a steel moment resisting frame incorporating ground motion and connection parameters uncertainties. The problem has been solved for three objectives of initial cost, average of maximum inter story drift ratios as seismic demand measure and standard deviation of maximum inter story drift ratios as safety measure. Modal pushover analysis is applied for studying seismic response of structure under the effect of uncertainties. Algorithm NSGA-II is applied as optimizer and the results are determined as a set of optimum designs called as Pareto front. In another study, Liu and Atamturkter [27] solved the problem of optimization for a steel moment frame by incorporating spatial variability of connection parameters. The problem has been solved for two design objectives of initial cost and reliability index for collapse prevention and both of IDA and pushover analyses are employed for studying the effect of spatial variability of connection parameters on seismic response of a steel moment resisting frame. Firstly, they have used sensitivity analysis to evaluate the effectiveness rates of IK model parameters and the results show that among these parameters, $F_y$, $\frac{M_y}{M_{yp}}$, $\theta_{pc}$ and $\frac{M_c}{M_y}$ have more significant effects on seismic response of the structure. Then, they have solved the optimization problem by incorporating uncertainties related to four effective recommended parameters and spatial variability related to these parameters.

In this paper, we propose a new multi stage procedure for optimal Performance-Based Design of steel moment frames considering two sources of uncertainties. This procedure consists of as a bi-objective optimization problem, solved by well-known NSGA-II [28], while structural weight and reliability index are two contradicting objectives of this problem. Nonlinear static analysis, MC simulation, VRT and performance acceptance check according to FEMA-356 are four employed sub-steps with the aim of evaluation of structures subjected to uncertainties.

This paper is organized as follows. In Sections 2 and 3, the existing uncertainty in beam-column connections and seismic load are explained in detail, respectively. The effective uncertainties on collapse capacity of steel moment frames are introduced in Section 4 and the procedure for incorporating uncertainties in the main optimal design optimization process is described with contributing requirements in Section 5. The employed VRT method is introduced in Section 6 and a case study and the structure of the proposed PBD optimal design procedure are presented in Sections 7 and 8. Finally, in Section 9 a detailed discussion on the obtained results and the accuracy and efficiency of the proposed procedure is presented. Section 10 is the closure of this paper.

2. CONNECTION PARAMETERS UNCERTAINTY

In order to investigate the CP (collapse prevention) performance level of structures due to the seismic action, the nonlinear nature of structural response should be modeled correctly and the important sources of uncertainties related to modeling parameters shall be
considered in analysis and design process. Uncertainties related to damping, structural mass and strength of materials have relatively low impact on dispersion of structural responses for CP performance level of buildings [29]. Ibarra and Krawinkler [6] have described that the connection parameters uncertainties can have a significant effect on evaluating the collapse performance of buildings. These parameters determined usually based on experiments and suggested statistical values, are used for defining the relation of force (moment) – deformation (rotation) in modeling nonlinear behavior of structural elements. In present study, the Ibarra-Krawinkler (I-K) model [30, 31] is utilized for studying the behavior of connection parameters. This model forms the basis of modeling deterioration of structural elements. I-K model is described based on three resistance parameters, three deformation parameters and one cyclic deterioration parameter. Resistance parameters are effective yield strength \((M_y, F_y)\), maximum plastic strength \((M_c, F_c)\) and residual strength \((M_r, F_r)\) while deformation parameters includes of yield deformations \((\theta_y, \delta_y)\), ultimate deformation \((\theta_u, \delta_u)\) and capping deformation \((\theta_c, \delta_c)\). Amounts of deformation in the range of yielding and capping points is called as pre-capping plastic deformation \((\theta_p, \delta_p)\) and post-capping plastic deformation \((\theta_{pc}, \delta_{pc})\) is defined as deformation in the range of capping point and ultimate deformation. Consequently, this model determines the resistance boundaries of structural elements using backbone curve represented in Fig. 1 and describes hysteretic behavior of elements applying a set of rules among these deformation boundaries [6]. Among all existing hysteretic deterioration models, bilinear, peak oriented and pinching models are the most widely used models. Backbone curve that describes monotonic response of structural components is the same part among all of these hysteretic models. As shown in Fig. 1, this curve consists of four parts of elastic, hardening, post capping and residual strength branches. The boundaries among these branches is determined by yielding deformation \((\delta_y, \theta_y)\), maximum plastic deformation \((\delta_c, \theta_c)\) and ultimate deformation \((\delta_u, \theta_u)\), respectively.

Figure 1. Uniform backbone curve according to modified I-K model

One challenge from modeling connection parameters is related to uncertainty of their values. Lignos and Krawinkler have described this uncertainty in [32]. Variations of these
parameters are presented in their study after investigating several factors like beam depth ($d$) and ratios of shear span to beam depth ($\frac{l}{d}$), beam length to section gyration radius ($\frac{I}{r}$), width to thickness of the beam’s flange ($\frac{b_f}{2t_f}$) and depth to thickness of the beam’s web ($\frac{h}{t_w}$). They have presented the probability distribution of connection parameters and have determined values of mean and standard deviation for each of corresponding cumulative distribution functions after experimental study on 300 steel W sections. These distributions, which are applied for modeling connection parameters in this study, are visible in Table 1. In addition to the principal connection parameters introduced in this section, Ibarra & Krawinkler [6] have employed other parameters including pre-capping stiffness deterioration ($\alpha_s$), post-capping stiffness deterioration ($\alpha_c$) and cyclic deterioration parameters ($y_{s,c,k,a}$) for evaluating the collapse capacity of frame structures under seismic excitations. Parameters $\alpha_s$ and $\alpha_c$ are defined as the slope of backbone curve in strain-hardening and softening branches respectively.

### Table 1: Distribution properties for uncertain parameters of I-K model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Dispersion coefficient</th>
<th>Type of distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_p$</td>
<td>0.022</td>
<td>0.270</td>
<td>Log normal</td>
</tr>
<tr>
<td>$\theta_{pc}$</td>
<td>0.170</td>
<td>0.350</td>
<td>Log normal</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1.100</td>
<td>0.440</td>
<td>Log normal</td>
</tr>
<tr>
<td>$M_y$</td>
<td>1.170</td>
<td>0.210</td>
<td>Normal</td>
</tr>
<tr>
<td>$M_{y,p}$</td>
<td>1.110</td>
<td>0.050</td>
<td>Normal</td>
</tr>
<tr>
<td>$M_c$</td>
<td>0.400</td>
<td>0.100</td>
<td>Normal</td>
</tr>
<tr>
<td>$F_y$</td>
<td>55.100</td>
<td>0.120</td>
<td>Normal</td>
</tr>
</tbody>
</table>

### 3. SEISMIC DEMAND UNCERTAINTY

In order to design a safe building for a determined performance level without applying a probabilistic approach, it is expected to design the building for the worst-case earthquake excitation. For this purpose, designer may be looking for the worst-case earthquake that may result from closest faults. For example suppose that the aim is designing a building at a site located at point O by distance of $d_A$ and $d_B$ from predicted rupture locations on faults A and B according to Fig. 2 ($d_A > d_B$). By assuming that maximum magnitude earthquakes from faults A and B are estimated respectively $M_A$ and $M_B$ ($M_A < M_B$), the median response spectrums predicted from these two earthquakes can be demonstrated as Fig. 3. It can be found from comparison between these two median spectrums that maximum spectral acceleration can be resulted from fault A for short periods ($T < 1.3$ sec) and it can be resulted from fault B for long periods ($T > 1.3$ sec). In fact, on either side of $T=1.3$ sec, each of these two faults may cause the worst earthquake. This simple example explains obviously that it is not possible to determine the worst case earthquake resulting from known faults around a site, because of the uncertain seismic properties, especially uncertainty related to
surface wave frequency produced by ground motion shakes. Therefore, it can be concluded that uncertainty related to frequency content of earthquakes can lead to a variety in the amount of resulted spectral accelerations as a measure of ground motion intensity. Another problem associated to seismic demand uncertainties is related to variability in values of spectral accelerations recorded from one earthquake. For this reason, median spectrum is usually employed for analysis and design of structures, but we are facing with dispersion in spectral accelerations of recorded ground motions around the mean spectrum in reality. Fig. 4 illustrates variation in the records of 1999 earthquake in Chi-Chi Taiwan. Campbell and Bozorgnia [33], Abrahamson [34] and Wang [35] have proposed normal distribution for Ln ($S_a$) in different studies, for modeling this variation related to recorded spectral accelerations.

Figure 2. Map view of a structure adjacent to faults A and B on a circular domain

Figure 3. Two predicted response spectrums from two probable earthquakes resulted from faults A and B
Figure 4. Spectral accelerations recorded from the 1999 Chi-Chi, Taiwan earthquake [36]

4. EFFECTIVE UNCERTAINTIES ON COLLAPSE CAPACITY OF MOMENT RESISTING FRAME

Ibarra and Krawinkler have investigated all parameters affecting the collapse capacity of structures in a comprehensive study and they have studied the effects of these uncertainties on collapse capacity of structures [6]. According to their study, the most important factors are subdivided into record-to-record variability, connection and deterioration parameters uncertainties, and P-∆ effects. They have demonstrated the contribution rates of first two categories for a single degree of freedom system in their study according to Fig. 5. This result is for the case of no correlation included among deterioration parameters. According to [6], post capping stiffness deterioration ($\alpha_c$) has the most influence on collapse capacity of structures among the deterioration parameters, especially for medium fundamental periods (0.5 sec $\leq T \leq$ 2 sec). The influence of this uncertain parameter in creating dispersion is more related to when it has small values in range of $0.1 < \alpha_c < 0.3$. In this condition, for multi degree of freedom (MDOF) systems, collapse capacity decreases even to 30% with respect to the case of $\alpha_c = 0.1$, and for $\alpha_c \geq 0.3$ differences will be smaller [6]. After this parameter, ductility capacity has more effect in scattering on collapse capacity especially for medium periods. For example it can be more than 30% decrease in collapse capacity from value of 6 to 2 for ratio of $\frac{\delta_c}{\delta_y}$ as ductility capacity. Cyclic deterioration (CD) is the last factor related to I-K model that can affect the collapse capacity. The collapse capacity generally decreases for rapid rates of CD and its effect is usually smaller for long period structures. Nevertheless, generally, the effect of CD is less than the effects of post capping stiffness and ductility capacity for all of period values.
5. Procedure of incorporating uncertainties

One of the two design objectives considered for optimization process in this study, is defined based on reliability index for collapse prevention performance level. Therefore, it is necessary to consider variations related to effective sources of uncertainties mentioned in previous sections to have a set of robust designs with less possible amounts of dependency to these variations. For this purpose, an appropriate approach should be chosen for modeling nonlinear behavior of beam-column elements in such a way that describes nonlinear behavior of elements based on I-K model. In this study, pushover analysis is performed in OpenSees [37] for nonlinear structural analysis during the optimization procedure of a 2-D moment resisting frame for collapse prevention performance level. Variations related to connection parameters of $\theta_p$, $\theta_{pc}$ and $\frac{M_e}{M_y}$ and three deterioration parameters of $\alpha_c$, $\frac{\delta_c}{\delta_y}$ and CD ($y_{s,c,k,a}$) from I-K model are considered for incorporating variations of uncertain parameters which are needed for defining features of hypothetical springs assigned to beam-column elements for modeling concentrated plasticity. Therefore, the cumulative distribution functions (CDFs) and statistical properties of aforementioned uncertain parameters are needed to perform MC simulation. These properties for connection parameters $\theta_p$, $\theta_{pc}$ and $\frac{M_e}{M_y}$ are in access from results of Liu’s study[27] according to Table 1. For deterioration parameters $\alpha_c$, $\frac{\delta_c}{\delta_y}$ and CD, the assumption of lognormal distribution can be used by $\text{Ln}(\sigma) = 0.6$ and the mean values for these parameters are taken equal to values of $\mu_{\alpha_c} = 0.1$, $\mu_{\delta_c} = 2$, $y_{s,c,k,a} = 100$ according to Ref. [6].

For incorporating ground motion uncertainty in optimization process, the procedure proposed by Abrahamson [34] is applied for determining standard deviation of $\text{Ln}(S_a)$. Then, $\sigma_{\text{Ln}(S_a)}$ is obtained equal to 0.8$g$. According to this procedure, the mean value for $S_a$ is taken equal to 1.614$g$ based on FEMA-356 instructions [38] for estimating spectral acceleration of case study frame, illustrated in Fig. 6. Additionally, for this case study 2% critical damping and site class A are other assumptions. MC simulation is applied for reliability analysis of structure and stochastic values are produced using inverse
transformation method (ITM) by having CDFs of uncertain parameters. In this study, variance reduction techniques (Antithetic variables and Control variables techniques [11,12]) is used in procedure of producing sample set for parameters $S_a$ and $CD (\gamma_{S,C,k,a})$, because these uncertain parameters have a larger range of variations compared with others. Consequently, applying VRTs in ITM procedure helps to produce stochastic values in more realistic range of variations and improving performance of MC simulation. A summary description about these numerical techniques is presented in the following sections.

6. VARIANCE REDUCTION TECHNIQUES

Initial approach in MC simulation is estimating the expectation of variables based on corresponding probability density functions (PDFs) by producing a sample set of stochastic values for uncertain parameters. The process consists of three initial steps including: 1) Determining the type of CDF for input variables, 2) Producing a sample set of stochastic numbers for each of variables based on corresponding CDFs and 3) Estimating the expectation of variables using stochastic generated sample sets. To have more authentic results from MC simulation, it is needed to take a large sample set for each of stochastic variables in second step of simulation. VRTs are numerical techniques that could be applied for solving this problem and decreasing computational costs of simulation. In this study, we use combination of Control variables (CV) and Antithetic variables (AV) [11,12] for improving performance of MC simulation.

6.1 Antithetic variables

Assuming that $X=\{x_1, x_2, ..., x_n\}$ is a vector of stochastic values produced by ITM for uncertain parameter $x$, while the aim of simulation is estimating $\hat{x} = E(H(X))$. If $q = \frac{x_i + x_j}{2}$ (i≠j) is taken as an unbiased estimator of $\hat{x}$, following expression can be written for $\text{var}(q)$:

$$\text{Var}(q) = \frac{1}{4}(\text{var}(x_i) + \text{var}(x_j) + 2\text{cov}(x_i, x_j))$$  \hspace{1cm} (1)

According to equation 1, when $\text{cov}(x_i, x_j)$ is a negative value, variance of $q$ is smaller than variance of each parameters $x_i$ and $x_j$. This variance reduction among input variables of simulation leads to more realistic outputs.

6.2 Control variables

This method improves performance of MC simulation by employing one or more than one auxiliary variable called as control variable in which expectation of this parameter is a known value. Assuming that $x$ is a vector of stochastic values produced by ITM for uncertain parameter $x$ and the aim of simulation is estimating $\hat{x} = E(H(X))$. Considering $y$ as control variable and $E(y)=e$, $q$ is an unbiased estimator according to following expression for the state of single control variable:

$$q = \hat{x} - \alpha(x-e)$$  \hspace{1cm} (2)
\[ \alpha = -\frac{\text{cov}(\hat{x}, y)}{\text{var}(y)} \]  

(3)

Minimum variance for \( q \) is achieved equal to equation 4 from applying \( \alpha \) according to equation 3.

\[ \text{Var}(q) = (1 - \rho^2) \text{var}(\hat{x}) \]  

(4)

where \( \rho \) is correlation coefficient between \( \hat{x} \) and \( y \). Applying this approach improves performance of MC simulation by using control variable \( y \) without any intervention in producing stochastic values in ITM procedure.

7. CASE STUDY

The case study considered for implementing optimization process is a 4 story – 4 bay steel moment resisting frame according to Fig. 6. The gravity load imposed on all 4 stories includes of live load \( q_l = 50.25 \text{ lb/in} \) and dead load \( q_d = 60 \text{ lb/in} \). According to Fig. 6, the elements of this frame are subdivided into eight categories. Therefore, eight section numbers should be assigned to beam-column elements in each cycle of optimization process. Section numbers are taken from an initial list of W Standard AISC sections presented in Table 2.

<table>
<thead>
<tr>
<th>No</th>
<th>Section</th>
<th>No</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>W36 × 800</td>
<td>11</td>
<td>W40×503</td>
</tr>
<tr>
<td>2</td>
<td>2 × W36 × 441</td>
<td>12</td>
<td>2×W44×230</td>
</tr>
<tr>
<td>3</td>
<td>2×W36×395</td>
<td>13</td>
<td>W36×529</td>
</tr>
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<td>4</td>
<td>2×W44×290</td>
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<td>5</td>
<td>2×W36×361</td>
<td>15</td>
<td>W40×431</td>
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<td>6</td>
<td>2×W40×324</td>
<td>16</td>
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<td>7</td>
<td>W36×652</td>
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<td>2×W36×330</td>
<td>19</td>
<td>W40×392</td>
</tr>
<tr>
<td>10</td>
<td>2×W40×277</td>
<td>20</td>
<td>W40×372</td>
</tr>
</tbody>
</table>
8. OPTIMIZATION PROCESS

The optimization process is designed for solving problem with two objectives of weight and reliability index ($\beta$) which is demonstrated by the flowchart as in Fig. 7. This optimization process is set in form of a reference program in MATLAB [39]. As shown in this flowchart, the optimization process designed for this problem consists of seven steps. After introducing a list of input W section numbers to program, one set of section numbers beside together create an initial design in first step. Static analysis is done for this initial design by OpenSees in second step and control of designs to fulfill the requirements of AISC 341-16 [40]. In this step an auxiliary constraint $C$ is applied according to equation 5 for controlling elements based on instruction codes:

$$C = \sum \max\left(\frac{\text{demand}}{\text{capacity}} - 1, 0\right)$$  \hspace{1cm} (5)

If the value of $C$ becomes anything except zero, this investigated design will not be used in the next steps and the process returns to first step with new set of section numbers. If the value of $C$ becomes zero the third step will be done by producing stochastic values for uncertain parameters based on their cumulative distribution functions (CDFs) using ITM in MC simulation. After assigning these values to uncertain parameters, several designs with the same set of section numbers and different inelastic behavior are formed. In the next step, OpenSees is used for second time in each cycle of process and nonlinear behavior of structure for each of designs is investigated by pushover analysis. Then, designs will be controlled by comparing results of analysis with allowable values based on FEMA-356 in the fifth step and the sixth step includes calculating the reliability index for each of acceptable designs based on the ratio of seismic capacity to demands resulted from pushover analysis in each cycle. In the seventh step, values for both objective designs are available. Eventually the values of weights and reliability indexes arrives to NSGA-II and the optimization will be performed based on values of weight and ($\beta$) and responses goes to the optimum designs using parent–offspring technique after several cycles.

8.1 Modeling nonlinear behavior of elements

As previously mentioned, pushover analysis is performed in this study for investigating nonlinear behavior of structural elements. With this aim, the element type of “beamwithhinge” is employed as one of the prevalent approaches for modeling nonlinear behavior of elements in OpenSees. This approach is applicable for modeling beam-column elements with concentrated plasticity distribution in both ends of elements and the middle part is modeled by elastic behavior. The values of connection parameters and deterioration parameters uncertainties should be assigned to hypothetical rotational springs defined by modified I-K model specified at the distance of plasticity concentrated region in OpenSees. These input variables are assigned to elements after producing by ITM in MATLAB.
8.2 Reliability index

The process explained in previous section for modeling nonlinear behavior of structure is set in such a way that 50 stochastic values for uncertain parameters are produced in each cycle of program. Thus, 50 offspring designs are produced from each of initial parent designs which are formed as a set of W sections chosen by program in each simulation. Thus, if the population size of sample set is taken equal to n for each cycle of optimization, (50×n) pushover analyses will be performed in this cycle. Given that the range of variations related to the connection and deterioration parameters are expected to be different with the range of
spectral acceleration variations, three sets of reliability indexes are computed by limit state function defined as equation 6. In this function, \( R \) & \( S \) denote seismic capacity and demands respectively. The reliability index \( (\beta) \) is defined as Cornell reliability index based on equation 7, for each of initial designs in each cycle of the optimization process according to flowchart of Fig. 8.

\[
g(R,S) = \ln\left(\frac{R}{S}\right) \quad (6)
\]

\[
\beta = \frac{\mu_{\beta}}{\sigma_{\beta}} \quad (7)
\]

According to above relations, the limit state function \( (g) \) is defined by ratio of \( R/S \). \( R \) is determined as seismic capacity of elements according to provisions of FEMA-356 and \( S \) is determined based on pushover analysis for structural elements. Also, \( \mu \) and \( \sigma \) are defined as mean and standard deviation of limit state function respectively. The first and second reliability indexes \( (\beta_1, \beta_2) \) are determined based on the performance of force-control and deformation-control actions respectively, in the case of incorporating connection and deterioration parameters uncertainties. The third reliability index \( (\beta_3) \) is considered for
deformation-control actions in the case of incorporating variations of $S_a$ as seismic demand uncertainty. These variations are considered according to lognormal distribution which is explained in section 5 and affect directly the target displacement of pushover analysis according to following equations [38].

\[
\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_r^2}{4\pi^2} g
\]

\[
T_e = T_1 \sqrt{\frac{k_i}{k_e}}
\]

where $C_0$, $C_1$, $C_2$, $C_3$ are modification factors determined 1.25, 1, 1, 1 respectively for case study frame according to FEMA-356[38] instructions. Also, effective fundamental period ($T_e$) could be determined from equation 9 in which $T_1$ denotes elastic fundamental period of structure and $k_i$, $k_e$ denote elastic and effective lateral stiffness of building respectively. Therefore, variations of $S_a$ affect directly values of target displacement and the analysis’ step in which force-control and deformation-control elements should be controlled based on provisions [38]. Regarding the point that the vast majority of elements are categorized as deformation control elements, the reliability index of $\beta_3$ is utilized for evaluating reliability of generated designs by MC against seismic demand uncertainty.

It should be mentioned that in this study, the optimization process is set up in such a way that $\beta_1$ has just a controller role for controlling the performance of force-control elements in each structure with this description that if $\beta_1 \leq 1$, the investigated design will be considered unacceptable for recommending in the optimization process but, for acceptable designs ($\beta_1 > 1$), the process will be continued by determining a new reliability index ($\beta$) from integrating $\beta_2$ and $\beta_3$ using SRSS. This final reliability index is used as one of the two objectives alongside the weight of structure for optimizing solutions.

9. RESULTS

9.1 Design objective changes

By implementing the optimization process for the case study displayed in Fig. 7, optimum design solutions resulted from parent-offspring technique are obtained for each iteration of optimization process according to steps outlined in Fig. 8 and these solutions are used as input designs for next iteration of process. It is obvious, as we have more iterations, the results are more exact. However, we have similar solutions after several iterations of the algorithm that means differences between design objectives are very small and desired convergence is achieved. In this study, after implementing 100 iterations of process for the case study, this convergence is achieved and the performance of optimization process is as shown in Figs. 9 and 10.
Figure 9. The obtained mean values of the design objectives in different steps of the optimization process.

Figure 10. The obtained variance of reliability indexes in different steps of the optimization process.

It can be seen that the changing process of mean values of design objectives ($\mu_w, \mu_\beta$) relative to each other in Fig. 9. Moreover, the changing process of variance of reliability index ($\sigma_\beta$) during the optimization process is demonstrated in Fig. 10. These changes are indicating that the mean values of weights (initial cost) and reliability indexes against collapse occurrence is improved in direction of optimum values. In other words, the proposed process in this study shows a proper performance for increasing the reliability indexes and decreasing weights (initial costs) of initial designs after 100 iterations. According to Table 3 the mean value of reliability indexes has maximum increment of 41.4% in the iteration 75 relative to iteration 1 with $\mu_\beta=3.06$. Additionally, the mean value of weights (initial costs) has maximum decrement of 19.4% in iteration 100 relative to iteration 1 with $\mu_w=477.95$ ton. In other iterations, as it is obvious from Fig. 10, the procedure of increasing $\beta$ and decreasing weight is apparently visible from right to left.
Table 3: Mean and standard deviation values of design objectives in various iterations

<table>
<thead>
<tr>
<th>No. Iteration</th>
<th>%decrease of $\mu_w$</th>
<th>%increase of $\beta$</th>
<th>Var($\beta$)</th>
<th>Mean value of $\beta(\mu_\beta)$</th>
<th>Mean value of weight($\mu_w$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1.42</td>
<td>3.06</td>
<td>477.95</td>
</tr>
<tr>
<td>70</td>
<td>17.0</td>
<td>39.1</td>
<td>0.77</td>
<td>4.26</td>
<td>394.90</td>
</tr>
<tr>
<td>75</td>
<td>18.0</td>
<td>41.4</td>
<td>0.78</td>
<td>4.33</td>
<td>392.04</td>
</tr>
<tr>
<td>80</td>
<td>19.0</td>
<td>37.0</td>
<td>0.83</td>
<td>4.21</td>
<td>387.10</td>
</tr>
<tr>
<td>85</td>
<td>19.3</td>
<td>36.0</td>
<td>0.86</td>
<td>4.16</td>
<td>385.64</td>
</tr>
<tr>
<td>90</td>
<td>19.4</td>
<td>35.0</td>
<td>0.81</td>
<td>4.14</td>
<td>385.19</td>
</tr>
<tr>
<td>95</td>
<td>19.4</td>
<td>35.0</td>
<td>0.87</td>
<td>4.13</td>
<td>385.05</td>
</tr>
<tr>
<td>100</td>
<td>19.4</td>
<td>36.2</td>
<td>0.85</td>
<td>4.17</td>
<td>385.00</td>
</tr>
</tbody>
</table>

To have a better understanding from performance of optimization process for improving design objectives, the values of these parameters are shown for initial and obtained designs of iterations 1, 75, 85 and 100 in Figs. 11, 12, 13 and 14 respectively. In addition, some of the optimum designs has been introduced with $\beta$>4, from 100$^{th}$ iteration of optimization process in Table 4.
Table 4: Properties of some optimum designs with $\beta > 4$

<table>
<thead>
<tr>
<th>No.</th>
<th>Design</th>
<th>$\beta$</th>
<th>Weight (ton)</th>
<th>$S_B$</th>
<th>$S_7$</th>
<th>$S_6$</th>
<th>$S_5$</th>
<th>$S_4$</th>
<th>$S_3$</th>
<th>$S_2$</th>
<th>$S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.56</td>
<td>400.67</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>10</td>
<td>4</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>5.19</td>
<td>396.91</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>390.46</td>
<td>20</td>
<td>18</td>
<td>20</td>
<td>18</td>
<td>18</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>5.05</td>
<td>393.62</td>
<td>18</td>
<td>20</td>
<td>18</td>
<td>18</td>
<td>17</td>
<td>10</td>
<td>4</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>5.01</td>
<td>387.18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>17</td>
<td>10</td>
<td>8</td>
<td>10</td>
<td>8</td>
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<tr>
<td>F</td>
<td>4.87</td>
<td>391.07</td>
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<td>18</td>
<td>20</td>
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<td>18</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>G</td>
<td>4.74</td>
<td>388.60</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>4.46</td>
<td>392.27</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>I</td>
<td>4.34</td>
<td>384.39</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>7</td>
<td>10</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>J</td>
<td>4.21</td>
<td>392.00</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>20</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

9.2 Seismic responses of optimum designs

Since we have set the reliability index based on the capacity to demand deformation ratio of elements, it is expected to have a sensible improvement in performance of structural elements belong to the optimum designs. In this regard, Table 5 presents a comparative evaluation among three optimum designs (A, B, J) related to the iteration 100 and two non-optimum designs related to the iteration 1 (K, L). Moreover, the force-displacement curves for designs A, B, K, L for six series of different values of connection parameters uncertainties are presented in Figs. 15, 16, 17, 18, respectively. Values of uncertain parameters are determined for each case according to Table 6. These figures are well representing the sensitivity of seismic responses of optimum designs (A, B) and non-optimum designs (K, L) deal with variations of uncertain variables.

Table 5: Features of three optimum designs related to iteration 100 and two non-optimum designs related to iteration 1

<table>
<thead>
<tr>
<th>Design</th>
<th>$\beta$</th>
<th>Weight (ton)</th>
<th>$\mu_{capacity/demand}$</th>
<th>Var(\theta)</th>
<th>$\mu_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.56</td>
<td>400.67</td>
<td>1.865</td>
<td>1.13 $\times 10^{-5}$</td>
<td>0.0153</td>
</tr>
<tr>
<td>B</td>
<td>5.19</td>
<td>396.91</td>
<td>1.335</td>
<td>1.03 $\times 10^{-4}$</td>
<td>0.0267</td>
</tr>
<tr>
<td>J</td>
<td>5.05</td>
<td>390.46</td>
<td>2.134</td>
<td>1.00 $\times 10^{-5}$</td>
<td>0.0139</td>
</tr>
<tr>
<td>K</td>
<td>3.23</td>
<td>433.17</td>
<td>2.217</td>
<td>1.59 $\times 10^{-5}$</td>
<td>0.0151</td>
</tr>
<tr>
<td>L</td>
<td>2.38</td>
<td>476.60</td>
<td>5.054</td>
<td>3.03 $\times 10^{-6}$</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

Table 6: Six different series of connection parameters values

<table>
<thead>
<tr>
<th>Series</th>
<th>$\frac{M_c}{M_y}$</th>
<th>$\theta_{pc}$</th>
<th>$\theta_p$</th>
<th>$\alpha_c$</th>
<th>$\gamma_{s,c,k,a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.110</td>
<td>0.170</td>
<td>0.022</td>
<td>-0.100</td>
<td>100.000</td>
<td></td>
</tr>
<tr>
<td>0.965</td>
<td>0.074</td>
<td>0.013</td>
<td>-0.088</td>
<td>82.855</td>
<td></td>
</tr>
<tr>
<td>1.041</td>
<td>0.120</td>
<td>0.017</td>
<td>-0.094</td>
<td>91.457</td>
<td></td>
</tr>
<tr>
<td>1.160</td>
<td>0.203</td>
<td>0.028</td>
<td>-0.116</td>
<td>110.346</td>
<td></td>
</tr>
<tr>
<td>1.208</td>
<td>0.257</td>
<td>0.032</td>
<td>-0.123</td>
<td>121.669</td>
<td></td>
</tr>
<tr>
<td>1.258</td>
<td>0.291</td>
<td>0.037</td>
<td>-0.136</td>
<td>131.587</td>
<td></td>
</tr>
</tbody>
</table>
9.3 Evaluating results

In order to evaluate the results of this study, we can compare these results with those of paper [27]. PBD optimization has been performed in [27] by incorporating spatial variability of connection parameters. The optimization problem has been solved by using algorithm NSGA-II for a steel moment-resisting frame with the same geometrical dimensions, the same modulus of elasticity and the same mechanical strength of steel in this paper, with a difference that gravity loads imposed on floors of frame in [27] is limited to the weight of beam-columns sections. Researchers utilized inter story drift ratio as constraints for reliability analysis and then reliability index is calculated by equation 8. In this relation, \( SC \) is seismic demand for CP performance level that is taken 5% inter story drift ratio in [27]. \( \mu \) and \( \delta \) are average and standard deviation of logarithm of seismic responses respectively and
\( \delta_g \) relates to ground motion uncertainty that is taken equal to constant value of 0.3 in [27].

\[
\beta = \frac{\ln(SC) - \ln(\mu)}{\sqrt{\delta^2 + \delta^2_g}}
\]

(8)

By comparing the obtained results of present study and the results of [27] in the state of perfectly uncorrelated inter-member relationship, it can be found that weight loss values is approximately same and equal to 20\% in both studies. But, reliability indexes of optimum designs resulted from equation 4 in [27] is distributed in the range of 1 to 2.7 whereas the average of reliability indexes in present study in 100\textsuperscript{th} iteration is obtained equal to 4.17. Regarding to same essence of reliability indexes and same optimizer utilized in both studies, it can be concluded that fulfilling all performance acceptance criteria (for CP performance level) mentioned in FEMA-356 instead of considering only acceptance criterion for inter story drift ratios and also incorporating ground motion variability more exactly in present study, provides accessibility to more safe designs.

10. SUMMARY AND CONCLUSIONS

In this paper, a reliability based PBD optimization approach is proposed for designing steel moment resisting frames with two design objectives of weight and reliability index against collapse performance. The reliability index is introduced in such a way that it could incorporate the effect of two important sources of uncertainties including connection parameters and seismic demand uncertainties. The optimization problem is solved for a moment resisting frame with four stories and four bays. After surveying the obtained results from implementing the optimization process described in previous parts, the results of this study can be expressed according to following conclusions:

- It can be deduced from Fig. 9 that despite the existence of few fluctuations related to trade off relationship between design objectives, there is an incremental process for mean values of reliability indexes and a suitable decreasing trend for mean values of initial costs (weights) regarding the first iteration of optimization process. Therefore, it can be found from these changing processes that the optimization is successful from the aspect of improving mean values of design objectives.

- In addition to the ascending trend in mean values of reliability indexes, the optimization process resulted 41\% decrease in variance of reliability indexes at iteration 100 compared to the first iteration of optimization process. These procedures are representing a remarkable reduction in noise effect of uncertainties during the optimization process.

- Output designs obtained from the proposed optimization process provide a wide range of suitable designs, whereas all of them satisfy design requirements (according to AISC 341-11). Consequently, there is a wide domain of optimal designs to choose with certain values of design objectives regarding to various factors like accessibility to sections, ease of implementation and the possibility of implementing sectional patterns of each design.
• Connection parameters uncertainties cause dispersion in seismic response of structure especially in high plastic deformations. This dispersion is visible for optimum and non-optimum designs in Figs. 15, 16, 17 and 18. It is obvious from these figures, by increasing the values of I-K model parameters given the increasing ductility of beam-column elements, force-displacement curves tend to have zero and even positive slope in inelastic region. This variation in seismic response is accompanied by variation of structural element’s deformations.

• There is a variety in sensitivity of different designs to variation of uncertainties. Even a larger dispersion may be found in the force-displacement curve of an optimum design compared to a non-optimum design with smaller reliability index, but the optimizer recognizes designs with less vulnerability to dispersions. This superiority is achieved by employing the reliability index for collapse prevention defined by capacity to demand ratio of elements.

• Unlike the last studies performing PBD optimization based on drift ratios as the limit state for evaluating seismic performance of structure, the proposed approach which is based on deformations of all beam-column elements, provides a more exact evaluation from responses of structure in high plastic deformations under seismic excitations.

According to the results of this study, the capacity to demand ratio has a descending process for optimum designs compared to the non-optimum designs with smaller reliability indexes, however the optimum designs tolerate larger lateral displacements. Therefore, there is a significant superiority in performance of optimum designs from the aspect of tolerating non-linear deformations before collapse occurrence.

REFERENCES


12. Drik P. Kroese. Monte Carlo Methods, School of Mathematics and Physics, University of Queensland, 2011.


36. Baker JW. An Introduction to Probabilistic Seismic Hazard Analyses (PSHA), Stanford University, 2008, pp. 5-12.
37. OpenSees. Version 2.4.0 [Computer software], PEER, Berkeley.