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OPTIMAL DESIGN OF TRUSS BRIDGES USING TEACHING-LEARNING-BASED OPTIMIZATION ALGORITHM

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ABSTRACT

In this study, teaching-learning-based optimization (TLBO) algorithm is employed for the first time for optimization of real world truss bridges. The objective function considered is the weight of the structure subjected to design constraints including internal stress within bar elements and serviceability (deflection). Two examples demonstrate the effectiveness of TLBO algorithm in optimization of such structures. Various design groups have been considered for each problem and the results are compared. Both tensile and compressive stresses are taken into account. The results show that TLBO has a great intrinsic capability in problems involving nonlinear design criteria.

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KEY WORDS: Bridge, truss, sizing optimization, teaching-learning-based optimization

1. INTRODUCTION

Bridges are amazing structures usually regarded as landmarks. They play an important role in transportation and development of countries. Design, fabrication and installation of bridges are usually costly. Optimization methods can be used in order to reduce these expenses and hence such methods are of paramount importance. Unfortunately, despite of their effectiveness in economically design of real life structures, optimization techniques are not practically employed by engineers, especially in the area of bridge design.

Optimal design can be performed based on sizing, shape or topology of the structure. A

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combination of these optimization approaches is also possible. In sizing optimization of truss bridges, which is the main concern of this article, the cross sectional areas of members are considered as design variables and they should be optimized such that the weight of structure is minimized. Moreover, some design constraints should be satisfied at the same time. Generally, internal stress within bar elements (strength) and serviceability of the structure (deflection) are regarded as design constraints.

The design procedure of structures usually involves preliminary design, analysis of structure, controlling design constraints, re-analysis and re-design. On the other hand, procedure of finding optimum structure is usually carried out by evolutionary algorithms because of their robustness, effectiveness and ease of application. In population-based optimization algorithms several numbers of structures are generated randomly in the beginning of the procedure and the design is improved after evolutions. Some well-known and efficient population based algorithms such as Genetic Algorithm (GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Firefly Algorithm (FA) and so on have been developed so far.

In the last decade, several valuable studies have been carried out on the optimization of truss structures using evolutionary and metaheuristic algorithms [1-14]. However, a few studies have been published concerning optimization of real life bridge structures. Hong Goun et al. [15] utilized principal stress based evolutionary structural optimization method for optimization of arch, tied arch, cable-stayed and suspension bridges with stress, displacement and frequency constraints. By using genetic algorithm, Cheng [16] investigated size optimization of steel arch truss bridges. Chen [17] studied the shape optimization of bridge structures using hybrid genetic algorithm. Hasancebi [18] investigated the application of evolutionary strategies in size, shape and topology optimization of truss bridges. Baldimor et al. [19] studied optimization problem of cable cross section of a cable stayed bridge considering cable stress and deck displacement as design constraints.

In this paper, the effectiveness of a recently developed population-based optimization algorithms, i.e. teaching-learning-based optimization (TLBO) in sizing optimization of real life bridge structures is investigated. The optimization problem is first formulated for a general two-dimensional steel truss arch bridge structure and then a teaching- learning-base optimization algorithm is developed for the optimum design of steel truss arch bridges. Finally, two numerical examples involving detailed computational models of long span steel truss arch bridges with main spans of 680 ft and 778.0208 ft are presented to demonstrate the applicability and merits of the aforementioned optimization method. Both tensile and compressive stresses are considered; several design groups are tested and the results are compared.

2. FORMULATION OF THE PROBLEM

The problem of sizing optimization of truss bridge structures involves optimizing cross sections A_i of the bars such that the weight of the structure W is minimized and some constraints with respect to design criteria are satisfied. The mathematical formulation of the problem can be stated as follows:

$$\text{Minimize} \qquad W(A) = \sum_{k=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i \tag{1}$$

$$\text{Subject to:} \qquad \sigma_{low} \le \sigma_i \le \sigma_{up}, \qquad i = 1, 2, ..., nm \tag{2}$$

$$\sigma_i^b \le \sigma_i \le 0, \qquad i = 1, 2, ..., ncm \tag{3}$$

$$\delta_{low} \le \delta_i \le \delta_{up}, \qquad i = 1, 2, ..., nn \tag{4}$$

$$A_{low} \le A_i \le A_{up}, \qquad i = 1, 2, ..., n_g \tag{5}$$

in which A is the vector containing the design variables (i.e. cross sections $A = \{A_1, A_2, ..., A_{ng}\}$), $W(A)$ is the weight of the truss structure, ρ_i is the density of member i, L_i is the length of member i, nm is the number of members in the structure , ncm is the number of compression members, nn is the number of nodes, ng is the total number of member groups (i.e. design variables), A_k is the cross sectional area of the members belonging to group k, mk is the total number of members in group k , σ_i is the stress of the ith member, σ_i^b is the allowable buckling stress for the ith member, δ_i is the displacement of the ith node, and *low* and *up* are the lower and upper bounds for stress, displacement and cross-sectional area.

3. TEACHING-LEARNING-BASED- OPTIMIZATION (TLBO) ALGORITHM

In 2011 Rao et al. [20] presented a new metaheuristics called teaching-learning-based-optimization (TLBO). TLBO is a population-based algorithm which tries to simulate the process of teaching and learning in a classroom. The optimization process involves two stages including teacher phase and learner Phase. In teacher phase, learners first get information from a teacher and then from other classmates in learner phase. The best solution is regarded as the teacher ($X_{teacher}$) in the population. In the teacher phase, learners learn from the teacher and the teacher tries to enhance the results of other individuals (X_i) by increasing the mean result of the classroom (X_{mean}) towards his/her position $X_{teacher}$. Two randomly-generated parameters r in the range of 0 and 1 and T_F are applied in update formula for the solution X_i for stochastic purposes as follows:

$$X_{new} = X_i + r.(X_{teacher} - T_F . X_{mean}) \tag{6}$$

where X_{new} and X_i are the new and existing solution of i, and T_F is a teaching factor which can be either 1 or 2 [21,22].

In second phase, i.e. the learner phase, the learners increase their knowledge by communicating with other students in the classroom. Therefore, an individual will learn new knowledge if the other individuals have more knowledge than him/her. During this stage, the student X_i interacts randomly with another student X_j ($i \neq j$) in order to develop his/her knowledge. In the case that X_j is better than X_i (i.e. $f(X_j) < f(X_i)$ for minimization problems), X_i is moved toward X_j. Otherwise it is moved away from X_j:

$$X_{new} = X_i + r.(X_j - X_i) \quad if \quad f(X_i) > f(X_j) \tag{7}$$

$$X_{new} = X_i + r.(X_i - X_j) \quad if \quad f(X_i) < f(X_j) \tag{8}$$

If the new solution X_{new} is better, it is accepted in the population. The algorithm will continue until the termination condition is met. For more details about the algorithm, the interested reader is referred to relevant references [21,22].

4. DESIGN EXAMPLES

In order to investigate the effectiveness of TLBO algorithm in sizing optimization of truss bridge structures, two real life truss bridges are optimized. These bridges were selected because fairly complete information about the geometry, loading and design criteria of these structures are available. Since there are no published articles in the literature regarding optimization of these bridge structures, the results are compared with the actual weight of structures and other results obtained by re-grouping of the design variables. Therefore, the current study can be regarded as a benchmark problem for further investigations and comparison with our results in the future. A finite element code in MATLAB is used for analysis of structures combined with a code for the process of optimization based on TLBO.

To explore usefulness of the optimization technique in solving problems involving nonlinear design criteria, both tensile and compressive stresses are taken into account. For both examples, allowable tensile and compressive stresses are considered according to AISC ASD (1989) [23] code as follows:

$$\begin{cases} \sigma_{up} = 0.6F_y & for \ \sigma_i > 0 \\ \sigma_i^b & for \ \sigma_i < 0 \end{cases} \tag{9}$$

$$\sigma_i^b = \begin{cases} \left[\left(1 - \dfrac{\lambda_i^2}{2C_c^2}\right)F_y\right] \Big/ \left(\dfrac{5}{3} + \dfrac{3\lambda_i}{8C_c} - \dfrac{\lambda_i^3}{8C_c^3}\right) & for \ \lambda_i < C_c \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & for \ \lambda_i \geq C_c \end{cases} \tag{10}$$

where F_y is the yield stress of steel; E is Young's modulus of elasticity of steel; λ_i is slenderness ratio $\left(\lambda_i = kL_i/r_i\right)$; k is the effective length factor, L_i is the length of each member i; r_i is the radius of gyration of member i; and C_c is defined as:

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \tag{11}$$

For these structures, the Young's modulus of elasticity was 4.2×10^9 *lb/ft*^{2}; the material density was 495 *lb/ft*^{3}. The radius of gyration was expressed in terms of cross-sectional areas as $r_i = aA_i^b$ [24]. Here, *a* and *b* are the constants depending on the types of sections adopted for the members such as pipes, angles, and tees. For pipe sections considered in this study *a*=0.4993 and *b*=0.6777 were adopted.

Allowable displacement is determined based on recommendations of the Australian Bridge Code [25] where the deflection allowance under the service load should not exceed 1/800 of the main span of the bridge.

4.1 Burro Creek Bridge

Burro Creek Bridge is located in Arizona U.S. Highway 93 runs north to south through central Arizona and is the primary transportation corridor between Phoenix and Las Vegas. The Burro Creek Bridge, which carried two-way auto traffic, is a truss arch structure with spandrel columns supporting the roadway deck and plate girder approach spans. Two views of this bridge are shown in Figure 1.

Figure 1. Burro Creek Bridge

The main span of the bridge is 680 ft which consists of 34 panels of 20 ft in length. Both upper and lower chords shapes are quadratic parabola. The elevation view of the bridge is shown in Figure 2. The averaged dead loads for various parts of the structure are summarized in Table 1 [26-27]. Equivalent live load plus impact loading on each arch for fully loading structure is considered as 1420 lb/ft. According to Australian Bridge Code [25], allowable displacement is $0.85 ft$. Moreover, the minimum cross-sectional area was considered to be 0.2 ft^2 and F_y as 72.0×10^5 lb/ft^2.

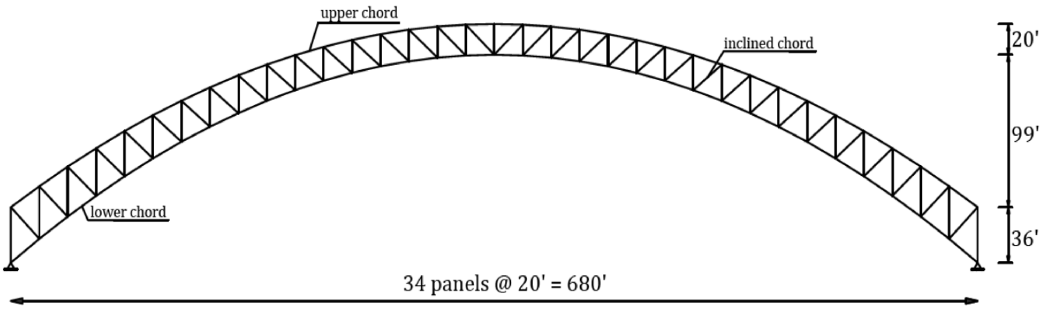

Figure 2. Elevation view of Burro Creek Bridge

Table 1: Average dead load on Burro Creek Bridge

Average dead load	*lb per ft*
Deck slab and surfacing for roadway	3140
Slabs for sidewalks	704
Railings and parapets	470
Floor steel for roadway	800
Floor bracing	203
Arch trusses	2082
Arch bracing	580
Arch posts and bracing	608
Total	8587

For simplification, a total uniform load of 5713.5 *lb/ft* for both dead and live loads is considered on the deck. Because of symmetry, half of the structure is considered in the analysis which is shown in Figure 3 including numbering of all bars.

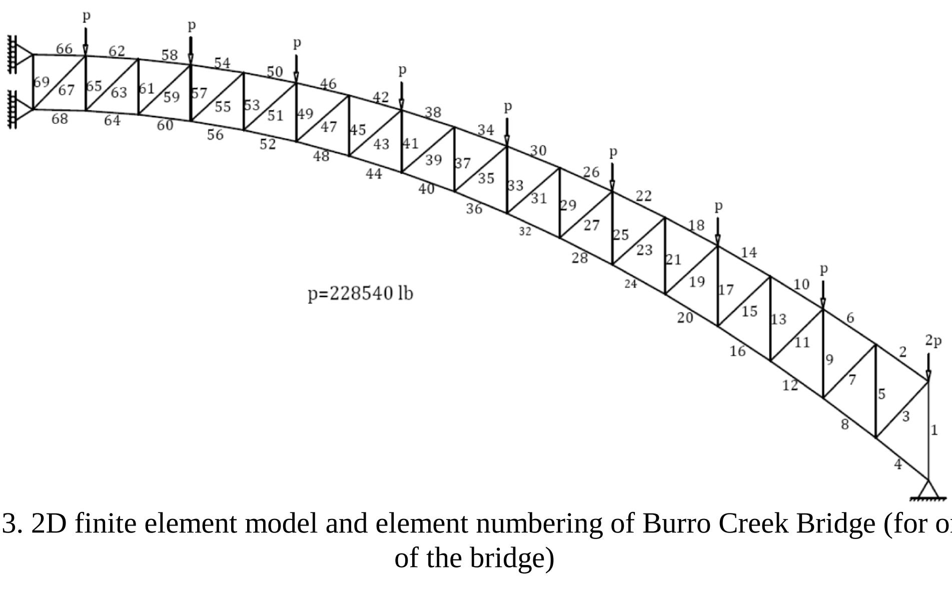

Figure 3. 2D finite element model and element numbering of Burro Creek Bridge (for one half of the bridge)

Optimization of the structure is accomplished considering three different groups of variables including 4, 8 and 12 variables in the design. Table 2 demonstrates the cross sections considered for these three different cases. Table 3 reports the results found after optimization of the structure for three aforementioned cases.

Table 2: Design variables for Burro Creek Bridge for three different cases

Design variables	Member number		
	Case I (4 variables)	Case II (8 variables)	Case III (12 variables)
1	67, 63, 59, 55, 51,47, 43, 39, 35, 31, 27, 23, 19, 15, 11, 7, 3	67, 63, 59, 55, 51,47, 43, 39, 35, 31, 27	67, 63, 59, 55, 51,47, 43
2	66, 62, 58, 54, 50, 46, 42, 38, 34, 30, 26, 22, 18, 14, 10, 6, 2	66, 62, 58, 54, 50, 46, 42, 38, 34, 30, 26	66, 62, 58, 54, 50, 46, 42
3	69, 65, 61, 57, 53, 49, 45, 41, 37, 33, 29, 25, 21, 17, 13, 9, 5, 1	69, 65, 61, 57, 53, 49, 45, 41, 37, 33, 29	69, 65, 61, 57, 53, 49, 45
4	68, 64, 60, 56, 52, 48, 44, 40, 36, 32, 28, 24, 20, 16, 12, 8, 4	68, 64, 60, 56, 52, 48, 44, 40, 36, 32, 28	68, 64, 60, 56, 52, 48, 44
5		23, 19, 15, 11, 7, 3	39, 35, 31, 27, 23, 19
6		22, 18, 14, 10, 6, 2	38, 34, 30, 26, 22, 18
7		25, 21, 17, 13, 9, 5, 1	41, 37, 33, 29, 25, 21
8		24, 20, 16, 12, 8, 4	40, 36, 32, 28, 24, 20
9			19, 15, 11, 7, 3
10			14, 10, 6, 2
11			17, 13, 9, 5, 1
12			16, 12, 8, 4

Table 3: Comparison of optimal design for Burro Creek Bridge for three different cases

Design variables	Optimal cross-sectional areas (ft^2)		
	Case I (4 variable)	Case II (8 variable)	Case III (12 variable)
A_1	0.20000	0.20000	0.20000
A_2	0.39202	0.46247	0.49843
A_3	0.41654	0.22233	0.20000
A_4	0.85487	0.57067	0.39476
A_5		0.20012	0.20000
A_6		0.31227	0.42170
A_7		0.42791	0.25346
A_8		0.84160	0.63739
A_9			0.20000
A_{10}			0.27992
A_{11}			0.43354
A_{12}			0.83483
Weight *(lb)*	368598.1371	315885.7516	298699.9356

The optimum weight of 368598.1371 lb is found when 4 group of variables is considered and the optimum weight of 298699.9356 lb if found in the case of 12 variables. As expected, including more design variables results in flexibility in the optimization procedure and finding lighter structures. It is worth pointing out that the actual weight of the structure is approximately 353940 lb. In Figure 4 a comparison among convergence rates in TLBO for three cases is presented.

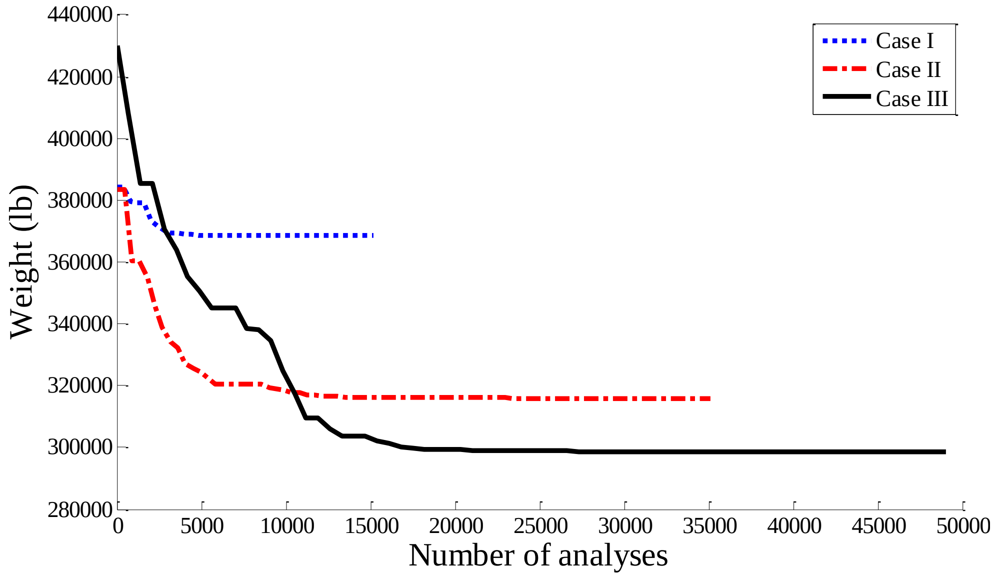

Figure 4. Comparison of the convergence rates for Burro Creek Bridge for three different cases

4.2 West End-North Side Bridge

The West End-North Side Bridge is a steel bowstring arch bridge over the Ohio River in Pittsburgh, Pennsylvania, approximately one mile below the confluence of the Allegheny and Monongahela Rivers. A view of the bridge is depicted in Figure 5.

Figure 5. The West End-North Side Bridge

The main span of the bridge is 778.0208 ft which consists of 28 panels of 27.786 ft in length. A elevation view of the bridge and its geometry is shown in Figure 6 with more details.

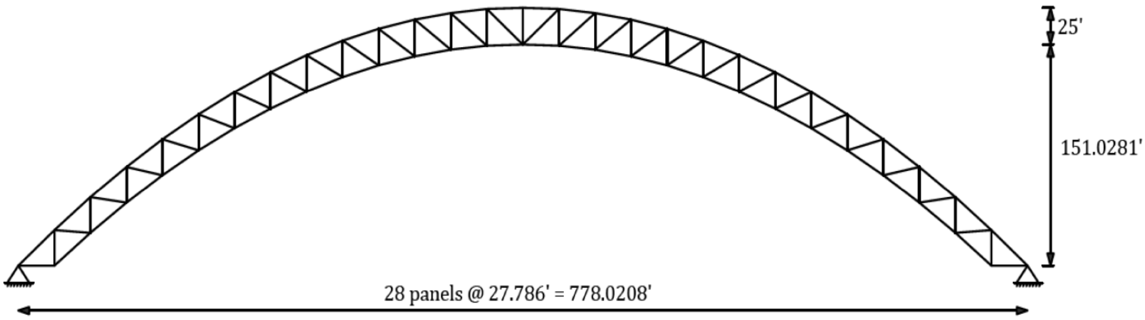

Figure 6. Elevation view of West End-North Side Bridge

The averaged dead loads for various parts of the structure are reported in Table 4 [26-28]. Equivalent live load plus impact loading on each arch for fully loading structure is considered as 1790 lb/ft. For this structure, $F_y$was considered as 57.6×10^5 *lb/ft*^{2} and the minimum cross-sectional area was 0.15 *ft*^{2}. According to Australian Bridge Code [25], allowable displacement is considered to be 0.97 ft.

Table 4: Average dead load on West End-North Side Bridge

Average dead load	*lb per ft*
Roadway, sidewalks, and railings	4870
Floor steel and Floor bracing	2360
Arch trusses	4300
Arch ties	2100
Arch bracing	550
Hangers	360
Utilities and excess	600
Total	15140

A total uniform load of 9360 lb/ft for both deal and live loads is considered on the deck. Similar to previous problem, half of the structure is considered in the finite element analysis which is depicted in Figure 7, including the bars numbering.

Design variables for this problem,, i.e. cross sectional areas, are categorized in four and eight groups for Case I and Case II, respectively. A list of members considered in each case is tabulated in Table 5.

Table 6 shows the optimum cross sectional areas found by TLBO and the optimum weight of structure in each case. The optimum weight of 551239.6752 lb is found for Case I in which 4 groups of variables are considered, and 506029.4052 lb for Case II when the variables are categorized in 8 groups. Same as the previous example, lighter structures can be found with increasing the number of involved variables. In Figure 8 a comparison between convergence rates in TLBO for two cases is depicted.

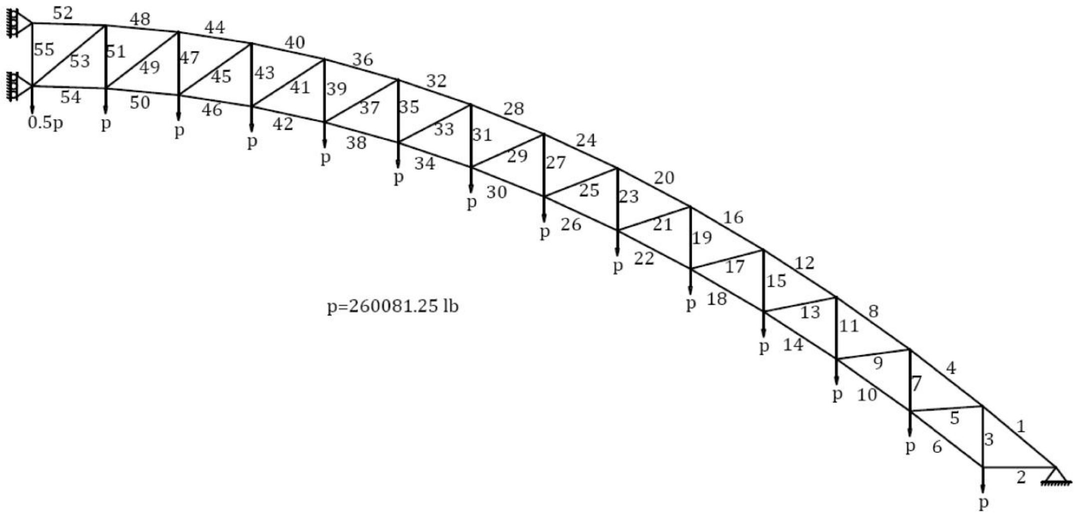

Figure 7. 2D finite element model and element numbering of West End-North Side Bridge (for one half of the bridge).

Table 5: Design variables for West End-North Side Bridge for two different cases

Design variables	Member number	
	Case Ⅰ (4 variables)	Case Ⅱ (8 variables)
1	53, 49, 45, 41, 37, 33, 29, 25, 21, 17, 13, 9, 5, 2	53,49,45,41,37,33,29, 25
2	52, 48, 44, 40, 36, 32, 28, 24, 20, 26, 12, 8, 4, 1	52, 48, 44, 40, 36, 32, 28, 24
3	55, 51, 47, 43, 39, 35, 31, 27, 23, 19, 15, 11, 7, 3	55, 51, 47, 43, 39, 35, 31, 27
4	54, 50, 46, 42, 38, 34, 30, 26, 22, 18, 14, 10, 6	54, 50, 46, 42, 38, 34, 30, 26
5		21, 17, 13, 9, 5, 2
6		20, 26, 12, 8, 4, 1
7		23, 19, 15, 11, 7, 3
8		22, 18, 14, 10, 6

Table 6: Comparison of optimal design for West End-North Side Bridge for two different cases

Design variables	Optimal cross-sectional areas (ft^2)	
	Case Ⅰ (4 variable)	Case Ⅱ (8variable)
A_1	0.19944	0.18775
A_2	1.56711	1.22934
A_3	0.15000	0.15006
A_4	0.46767	0.59447
A_5		0.22284
A_6		1.83206
A_7		0.15000
A_8		0.50057
Weight *(lb)*	551239.6752	506029.4052

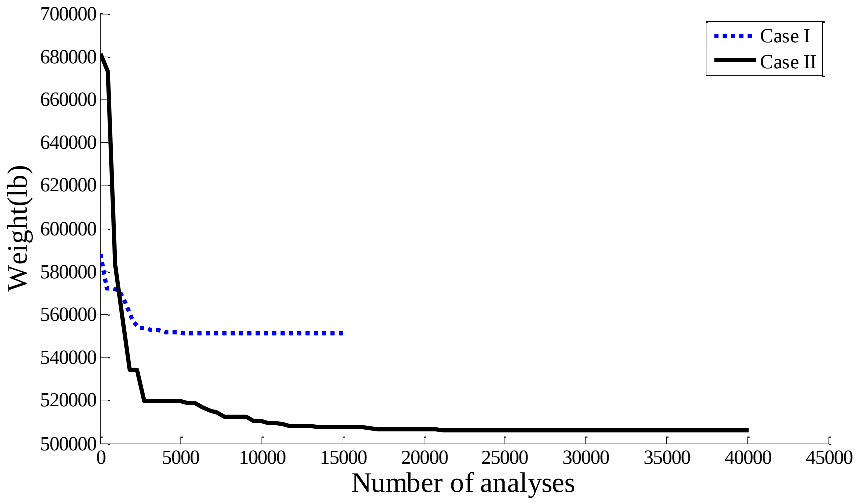

Figure 8. Comparison of the convergence rates for West End-North Side Bridge for two different cases

5. CONCLUSIONS

Application and effectiveness of one of the most recently developed optimization algorithms called teaching-learning-based optimization (TLBO) in design optimization of real world steel truss arch bridges is investigated in this paper. Two bridges are optimized via TLBO, taking both tensile and compressive stresses into account. Various groups of variables are considered and the results show that TLBO is very effective in sizing optimization of this kind of structures with nonlinear design criteria.

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