



OPTIMUM DESIGN OF REINFORCED CONCRETE FRAMES USING BAT META-HEURISTIC ALGORITHM

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ABSTRACT

The main aim of the present study is to achieve optimum design of reinforced concrete (RC) plane moment frames using bat algorithm (BA) which is a newly developed meta-heuristic optimization algorithm based on the echolocation behaviour of bats. The objective function is the total cost of the frame and the design constraints are checked during the optimization process based on ACI 318-08 code. Design variables are the cross-sectional assignments of the structural members and are selected from a data set containing a finite number of sectional properties of beams and columns in a practical range. Three design examples including four, eight and twelve story RC frames are presented and the results are compared with those of other algorithms. The numerical results demonstrate the superiority of the BA to the other meta-heuristic algorithms in terms of the frame optimal cost and the convergence rate.

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1. INTRODUCTION

Optimum design of reinforced concrete (RC) structures is more complicated compared with steel structures. In the case of steel structures, the cost is directly proportional to the structural weight while for RC structures the cost of concrete, steel and formwork influence the total cost. Also the design space of RC structures optimization problem can be extremely large in comparison to steel structures.

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In the past, many studies have been performed in the field of cost optimization of RC structures and a comprehensive literature review may be found in [1]. This literature review reveals that among the meta-heuristic algorithms, genetic algorithm (GA) has been widely used to perform RC frames optimization. However, in the work of Kaveh and Sabzi [1] big bang-big crunch (HBB-BC) and heuristic particle swarm ant colony optimization (HPSACO) algorithms have been used to tackle RC frame optimization problems. In other studies, Kaveh and Sabzi [2-3] utilized other meta-heuristic algorithms for optimization of RC frames for strength constraints. In the most recent work of Kaveh and Behnam [4], 3D RC frames were optimized for natural frequency constraints by charged system search (CSS) meta-heuristic algorithm.

Employing stochastic search techniques allows exploration of a larger fraction of the design space in comparison with gradient-based optimization methods. In order to approach the region containing the global optimum by spending low computational cost in terms of the number of required structural analyses, a variety of meta-heuristic optimization methods inspired by nature were developed. The meta-heuristics demonstrate their efficiency in many of the structural optimization problems and this is why these methods have been extensively employed in the field of structural engineering [5].

A very promising recent development in the field of meta-heuristic algorithms is bat algorithm (BA) proposed by Yang [6]. BA is a new search algorithm based on the echolocation behavior of microbats. The capability of echolocation of microbats is fascinating, as these bats can find their prey and discriminate different types of insects even in complete darkness. Preliminary studies indicate that BA is superior to GA and particle swarm optimization (PSO) [7] for solving engineering optimization problems [8-9]. Recently, Carbas and Hasancebi [10] employed BA for optimum design of steel frames.

In the present paper, BA is employed to implement design optimization of RC frames subject to gravity and lateral loads. Three design examples are presented and the numerical simulations demonstrate the efficiency of BA compared with other algorithms.

2. FORMULATION OF RC FRAMES OPTIMIZATION PROBLEM

The main aim of the size optimization of RC structures is to minimize the total cost of the frame. Therefore, in this paper total cost of frame is chosen to be the objective function of the optimization problem. The total cost of a frame includes the cost of concrete, steel reinforcement and framework of all beams and columns. In this case the objective function can be stated as follows:

$$F = \sum_{i=1}^{nb} (C_C b_{b,i} h_{b,i} + C_S A_{S,b,i} + C_F (b_{b,i} + 2h_{b,i})) L_i + \sum_{j=1}^{nc} (C_C b_{c,j} h_{c,j} + C_S A_{S,c,j} + 2C_F (b_{c,j} + h_{c,j})) H_j \quad (1)$$

where F is objective function; nb is the number of beams; $b_{b,i}$, $h_{b,i}$, L_i and $A_{S,b,i}$ are the i th beam width, depth, length and area of the steel reinforcement, respectively; nc is the number of columns; $b_{c,j}$, $h_{c,j}$, H_j and $A_{S,c,j}$ are the j th column width, depth, length and area of the steel reinforcement, respectively; C_C , C_S and C_F are the unit cost of concrete, steel and

framework, respectively. As mentioned in [1], in the present work the following unit costs are also considered: $C_C=105 \text{ \$/m}^3$, $C_S=7065 \text{ \$/m}^3$, $C_F=92 \text{ \$/m}^2$.

It is clear that a semi-infinite set of member width, depth and steel reinforcement arrangements can be considered for RC structure elements. In this case, as the dimensions of the design space are very large, the computational burden of the optimization process increases. In order to reduce the dimensions of design space and consequently the computational cost, a countable number of cross-sections can be employed by constructing data sets in a practical range. This idea has been successfully utilized in the past researches [1, 11-12] by using the tables of the reinforced concrete column and beam sections. In the present study, the section databases constructed for beams and columns in [1] are employed. these databases of beams and columns are shown in Figures 1 and 2, respectively. Further information about the databases can be found in [1].

Beam number	Width (mm)	Depth (mm)	Area ($\times 10^3 \text{mm}^2$)	Moment of inertia ($\times 10^6 \text{mm}^4$)	Number of bars		Factored moment resistance (kN.m)		Cost per unit length (\$)
					Center (D19)	End (D22)	Center	End	
1	300	450	1350	2278.1	2	2	74.41	97.67	133.95
2	300	450	1350	2278.1	3	2	108.84	97.67	135.96
.....
1013	450	900	4050	27338	10	12	761.3	1152.3	301.78
1014	450	900	4050	27338	12	12	893.43	1152.3	305.79

Figure 1. Database for the beams [1]

Column number	Width (mm)	Depth (mm)	Number of bars (D25)	P_0 (kN)	P_1 (kN)	P_3 (kN)	P_5 (kN)	M_2 (kN.m)	M_3 (kN.m)	M_4 (kN.m)	Cost per unit length (\$)
1	300	300	4	1643.3	1314.7	429	692.7	22.19	81.98	70.52	133.72
2	300	300	6	1880.7	1504.6	405.7	1039.1	25.97	101.82	100.61	140.66
.....
54	900	900	22	13128	10503	4961.3	3810	717.17	2230.7	1504.5	492.55
55	900	900	24	13366	10693	4954.9	4156.3	739.31	2326.2	1638.1	499.48

Figure 2. Database for the columns [1]

During the optimization process, axial force and bending moments for each column and only bending moments for each beam are computed via finite element analysis. In this paper, frames are analyzed for the following load cases according to ACI 318-08 code [13]:

$$\begin{cases} \text{Load Case 1: } 1.2DL+1.6LL \\ \text{Load Case 2,3: } 1.2DL+1.0LL\pm 1.4EL \\ \text{Load Case 4,5: } 0.9DL\pm 1.4EL \end{cases} \quad (2)$$

where DL , LL and EL are dead, live and earthquake loads, respectively.

In order to design a beam the externally applied moment at mid-span (M_u^+), left (M_{uL}^-) and right (M_{uR}^-) joints of beams should be respectively less than the factored moment capacities at the middle (ϕM_n^+), and near the ends (ϕM_n^-). The factored moment capacity for beams is computed as follows:

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) \quad (3)$$

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad (4)$$

where $\phi=0.9$ is the strength reduction factor; A_s is the area of the tensile bars; f_y is specified yield strength of the reinforcing bars; d is the effective depth of the section which is measured as the distance from extreme compression fiber to centroid of the longitudinal tensile reinforcing bars of the section; a is the depth of the equivalent rectangular compressive stress block; f'_c is compressive strength of the concrete and b is the width of the cross-section.

The strength of a column subject to bending moment and axial force is evaluated using a simplified linear P-M interaction diagram [1] shown in Figure 3.

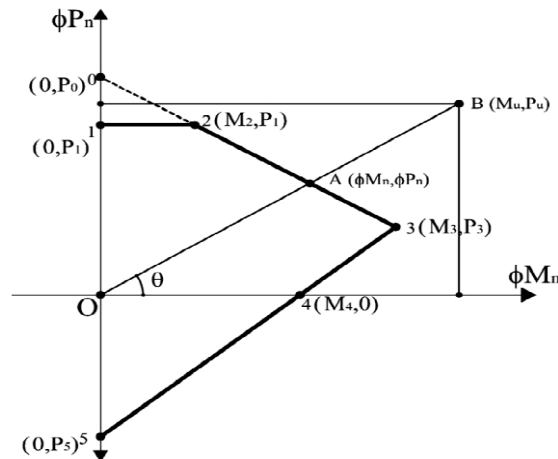


Figure 3. A simplified linear P-M interaction diagram [1]

In a designed column the corresponding pair (M_u, P_u) under the applied loads does not fall outside the interaction diagram. In Figure 3, if point B shows the position of the pair (M_u, P_u) and A is the crossing point of the line connecting B to the O and the interaction diagram, then the distance of the points A and B from O can be calculated. The ratio of the mentioned distances can be used as the constraint of the columns resistance. The angle between line OB and the horizontal axis (θ) is required to specify the point A. The lengths of OA (L_n) and OB (L_u) lines, can be computed as follows:

$$L_n = \sqrt{(\phi M_n)^2 + (\phi P_n)^2} \quad , \quad L_u = \sqrt{(M_u)^2 + (P_u)^2} \quad (5)$$

Therefore, if for a column section $L_u \leq L_n$ it can be concluded that the section is suitable and safe enough. Besides the strength requirements, for columns of a frame, the dimensions of the top column (including width and height of the cross section *i.e.*, b_T, h_T) should not be larger than those of the bottom one (b_B, h_B), and also the number of reinforcing bars in the top

column (n_T) should not be greater than that of the bottom column (n_B).

In a sizing structural optimization problem, the aim is usually to minimize the weight of the structure under some behavioural constraints. For a RC frame structure, a discrete sizing optimization problem can be formulated as follows:

$$\text{Minimize: } F \quad \text{Subject to:} \quad (6)$$

$$g_1 = \frac{M_u^+}{\phi M_n^+} - 1 \leq 0 \quad (7)$$

$$g_2 = \frac{|M_{uL}^-|}{\phi M_n^-} - 1 \leq 0 \quad (8)$$

$$g_3 = \frac{|M_{uR}^-|}{\phi M_n^-} - 1 \leq 0 \quad (9)$$

$$g_4 = \frac{L_u}{L_n} - 1 \leq 0 \quad (10)$$

$$g_5 = \frac{b_T}{b_B} - 1 \leq 0 \quad (11)$$

$$g_6 = \frac{h_T}{h_B} - 1 \leq 0 \quad (12)$$

$$g_7 = \frac{n_T}{n_B} - 1 \leq 0 \quad (13)$$

In this study, the constraints of the optimization problem are handled using the concept of penalty functions method (PFM) [14]. In this case, the pseudo unconstrained objective function is expressed as follows:

$$\Phi = F(1 + P_{beam} + P_{column}) \quad (14)$$

$$P_{beam} = r_p \sum_{i=1}^{nb} \left((\max\{0, g_1\})^2 + (\max\{0, g_2\})^2 + (\max\{0, g_3\})^2 \right)_i \quad (15)$$

$$P_{column} = r_p \sum_{j=1}^{nc} \left((\max\{0, g_4\})^2 + (\max\{0, g_5\})^2 + (\max\{0, g_6\})^2 + (\max\{0, g_7\})^2 \right)_j \quad (16)$$

where Φ and r_p are the pseudo objective function and positive penalty parameter, respectively; P_{beam} , and P_{column} are the penalty functions of beams and columns of the frame,

respectively.

In this paper, the newly developed bat algorithm (BA) [6] is employed to tackle RC frames optimization problem. The BA is explained in the subsequent section.

3. BAT META-HEURISTIC ALGORITHM

The BA meta-heuristic optimization algorithm is inspired from the echolocation behavior of microbats. Echolocation is an advanced hearing based navigation system used by bats to detect objects in their surroundings by emitting a sound to the environment. While they are hunting for preys or navigating, these animals produce a sound wave that travels across the canyon and eventually hits an object or a surface and return to them as an echo. The sound waves travel at a constant speed in zones where atmospheric air pressure is identical. By following the time delay of the returning sound, these animals can determine the precise distance to circumjacent objects. Further, the relative amplitudes of the sound waves received at each individual ear are used to identify shape and direction of the objects. The information collected this way of hearing is synthesized and processed in the brain to depict a mental image of their surroundings [10].

The echolocation characteristics of microbats in BA are idealized as the following rules [8]:

1. All bats use echolocation to sense distance, and they also “know” the difference between food/prey and background barriers in some magical way;
2. Bats randomly fly with velocity V_i at position X_i with a fixed frequency f_{\min} , varying wavelength λ and loudness A^0 to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission $r \in [0,1]$, depending on the proximity of their target;
3. Although the loudness can vary in many ways, it is assumed that the loudness varies from a large (positive) A^0 to a minimum constant value A_{\min} ;

The position and velocity of each bat should be updated in the design space. As optimization of RC frames using the section databases constructed for beams and columns is a discrete optimization problem, the following equations can be employed for updating position and velocity of i th bat:

$$f_i = f_{\min} + (f_{\max} - f_{\min})u_i \quad (17)$$

$$V_i^{k+1} = \text{round} \left(V_i^k + (X_i^k - X^*)f_i \right) \quad (18)$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (19)$$

where f_{\min} and f_{\max} are the lower and upper bounds imposed for the frequency range of bats. In this study, $f_{\min}=0.0$ and $f_{\max}=1.0$ are used; $u_i \in [0,1]$ is a vector containing uniformly distribution random numbers; X^* is the current global best solution;

A local search is implemented on a randomly selected bat from the current population using the following equation:

$$X^{k+1} = X^k + \text{round}(\varepsilon_j A^{k+1}) \quad (20)$$

where ε_j is a uniform random number in the range of $[-1, 1]$ selected for each design variable of the selected bat. A^{k+1} is the average loudness of all the bats at the current iteration.

The loudness A_i and the rate r_i of pulse emission have to be updated accordingly as the iterations proceed. As the loudness usually decreases once a bat has found its prey, while the rate of pulse emission increases, the loudness can be chosen as any value of convenience. In this work, $A^0 = 1$ and $A_{\min} = 0$ also, $r^0 = 0$ and $r_{\max} = 1$.

$$A_i^{k+1} = \alpha A_i^k \quad (21)$$

$$r_i^{k+1} = r_i^0 (1 - e^{-\gamma \cdot k}) \quad (22)$$

where α and γ are constants. In this study, $\alpha = 0.9$, and $\gamma = 0.01$.

The pseudocode of standard BA containing its basic steps is summarized as shown in Figure 4.

Bat Algorithm

Pseudo objective function $\Phi(X)$
 Initialized the bat population X_i ($i=1, \dots, n$) and V_i
 Define pulse frequency f_i at X_i
 Initialize loudness A_i and pulse emission rate r_i
while ($k < \text{maximum number of iterations}$)
 Generate a new population by updating positions and velocities of the previous population
 if $\text{rand} > r_i$
 Select the i th solution from the current population
 Generate a local solution around the selected solution by a local random walk
 endif
 if $\text{rand} < A_i$ & $\Phi(X_i) < \Phi(X^*)$
 Accept the new solution
 Update A_i and r_i
 endif
 Update the best solution X^*
end while
 Present the final solution

Figure 4. Pseudocode of standard bat algorithm (BA)

BA is a new meta-heuristic algorithm which has two main computational merits in comparison with existing meta-heuristics:

- 1- *Frequency tuning*: the frequency-based tuning mechanism of BA leads to better convergence and simpler implementation compared with other algorithms.
- 2- *Dynamic control of exploration and exploitation*: the balance of exploration and exploitation plays a very important role in convergence behaviour of an optimization algorithm. BA uses a dynamic strategy for exploration and exploitation. In fact,

automatic switch from exploration to more extensive exploitation can be achieved when the optimality is approaching; thus, the algorithm can be very efficient in practice [8].

In the next section the results of RC frames optimization employing BA are presented and compared with other meta-heuristic algorithms.

4. NUMERICAL RESULTS

In this work, three RC plane frames are selected from [1]. In these examples, lateral equivalent static earthquake loads (EL) are applied as joint loads, and uniform gravity loads are assumed for a dead load $DL = 22.3$ kN/m and a live load $LL = 10.7$ kN/m. For all examples, five loading cases of Eq. (2) are considered for strength design. The assumed specified compressive strength of concrete and yield strength of reinforcement bars are $f'_c = 23.5$ and $f_y = 392$ MPa, respectively.

In this paper, all of the required computer programs are coded in MATLAB [15]. Also for computer implementation a personal Pentium IV 3.0 GHz has been used.

In all the presented design examples, the number of bats in the population is 50 and the total number of generations is limited to 1000. A termination criterion is considered for BA in all the presented examples. If the best solution is repeated in 40 consecutive iterations the algorithm will be terminated.

The demand/capacity ratio (DCR) in the members of the optimum solutions, which is defined in the following equations, are given in all examples.

$$\text{For beams: } DCR = \max \left\{ \frac{M_u^+}{\phi M_n^+}, \frac{|M_{uL}^-|}{\phi M_n^-}, \frac{|M_{uR}^-|}{\phi M_n^-} \right\} \quad (23)$$

$$\text{For columns: } DCR = \frac{L_u}{L_n} \quad (24)$$

4.1 Example 1: Three bay, four-story RC frame

For the three bay, four-story RC frame, the geometry, lateral equivalent static earthquake loads and grouping details are shown in Figure 5.

Table 1 compares the results of optimization reported in [1] with those of obtained by BA in the present study. DCR in the members of the optimum solution obtained by BA for the three bay, four-story RC frame is given in Figure 6. It can be observed that the optimum solution is feasible. Figure 7 shows the convergence rate of BA for optimization of three bay, four-story frame.

The results indicate that the optimum cost obtained by BA is slightly less than those of reported in [1] however, BA requires 4550 structural analyses while the algorithm employed in [1] requires 8500 ones. These results demonstrate that BA not only found the best design overall but required also much less structural analyses than the other algorithm.

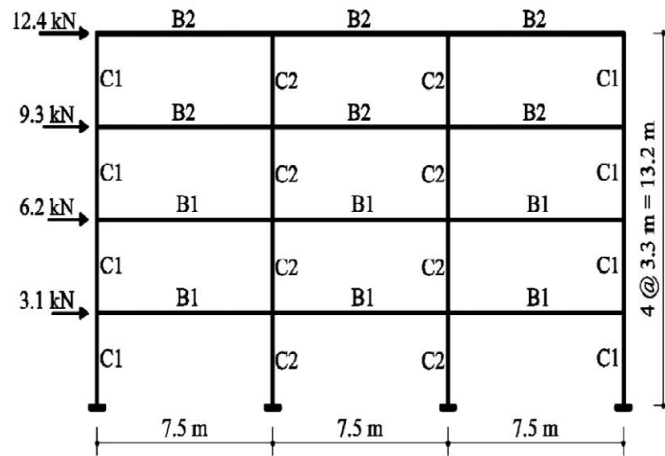


Figure 5. Three bay, four-story RC frame

Table 1: Optimum designs of three bay, four-story RC frame

Element		BA				Kaveh and Sabzi [1]			
		Sectional dimensions		Reinforcements		Sectional dimensions		Reinforcements	
Type	Group	Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	B1	300	450	3-D19	5-D22	300	500	3-D19	5-D22
	B2	300	450	4-D19	5-D22	300	500	4-D19	5-D22
Column	C1	350	350	8-D25		350	350	8-D25	
	C2	350	350	4-D25		300	300	6-D25	
Frame cost (\$)				21630		22207			
Number of structural analyses				4550		8500			

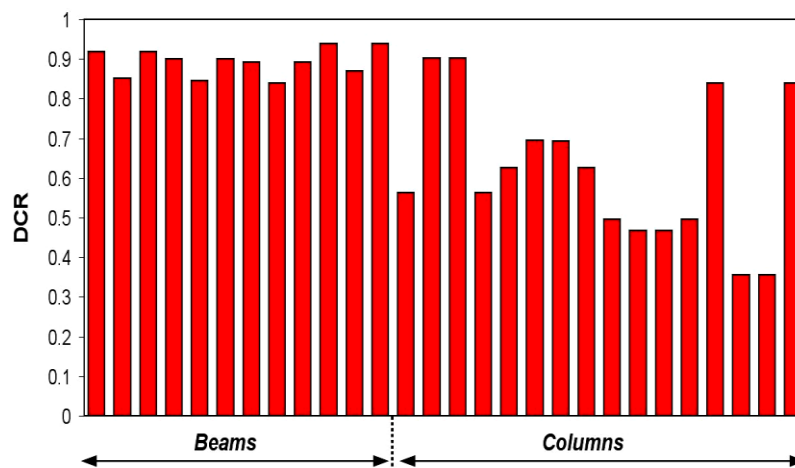


Figure 6. DCR in the members of the optimum three bay, four-story RC frame

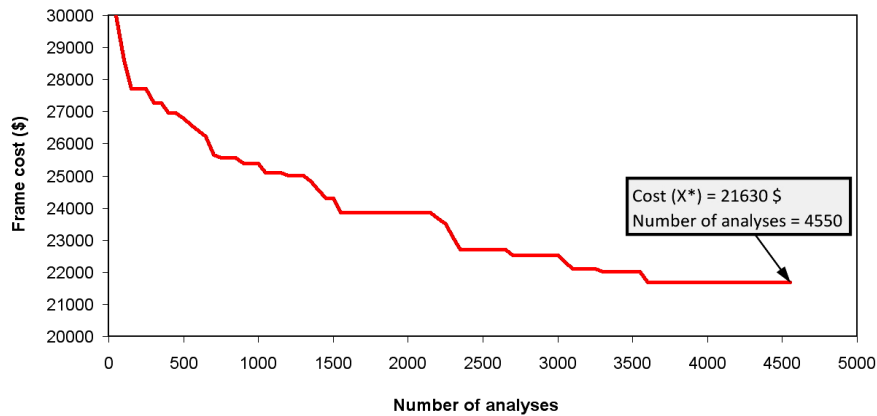


Figure 7. Convergence history of BA for three bay, four-story RC frame

4.2 Example 2: Three bay, eight-story RC frame

The geometry of the three bay, eight-story RC frame, its element groups and lateral equivalent static earthquake loads are shown in Figure 8.

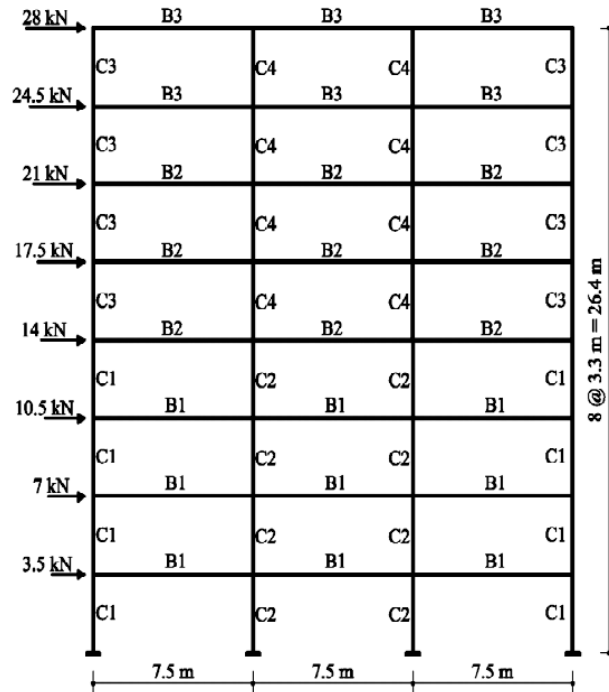


Figure 8. Three bay, eight-story RC frame

The results of three bay, eight-story RC frame optimization obtained by BA are compared with those of the reported in [1] in Table 2. DCR in the members of the optimum solution obtained by BA for the three bay, eight-story RC frame is given in Figure 9. The results depicted in this figure imply that the optimum solution is feasible.

The convergence history of BA for optimization of three bay, eight-story frame is shown

in Figure 10. As well as the first example the optimum cost obtained by BA in this example is also less than those of the reported in [1]. The results indicate that BA requires 18400 structural analyses which is considerably less than that of the reported in [1].

Table 2: Optimum designs of three bay, eight-story RC frame

Element		BA				Kaveh and Sabzi [1]			
		Sectional dimensions		Reinforcements		Sectional dimensions		Reinforcements	
Type	Group	Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	B1	300	500	3-D19	5-D22	300	500	3-D19	6-D22
	B2	300	500	3-D19	5-D22	300	500	3-D19	6-D22
	B3	300	500	3-D19	4-D22	300	500	3-D19	5-D22
Column	C1	400	400		8-D25	400	400		8-D25
	C2	450	450		8-D25	450	450		12-D25
	C3	350	350		8-D25	350	350		8-D25
	C4	350	350		4-D25	350	350		8-D25
Frame cost (\$)				47047		48263			
Number of structural analyses				18400		39500			

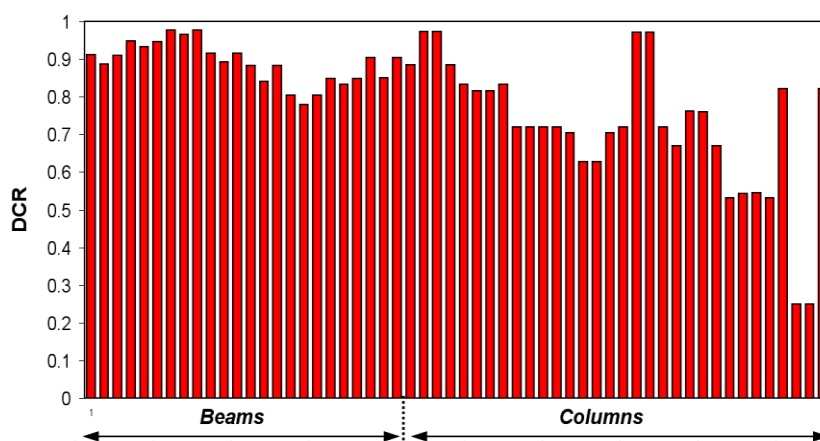


Figure 9. DCR in the members of the optimum three bay, eight-story RC frame

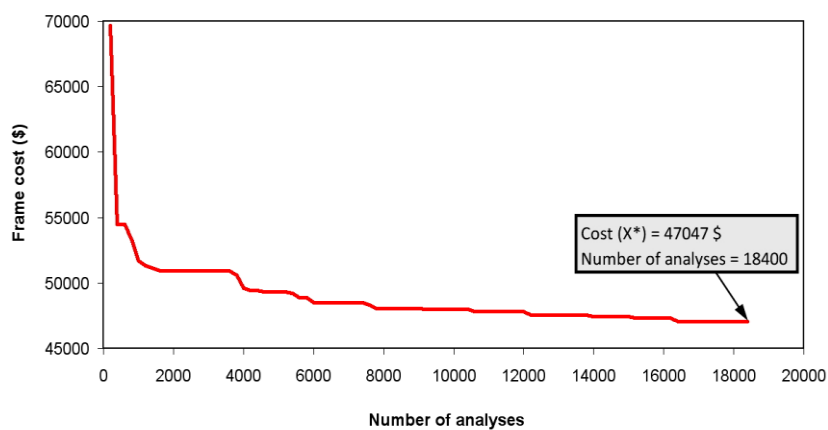


Figure 10. Convergence history of BA for three bay, eight-story RC frame

4.3 Example 3: Three bay, twelve-story RC frame

The geometry, element groups and lateral loading of the three bay, twelve-story RC frame, are presented in Figure 11.

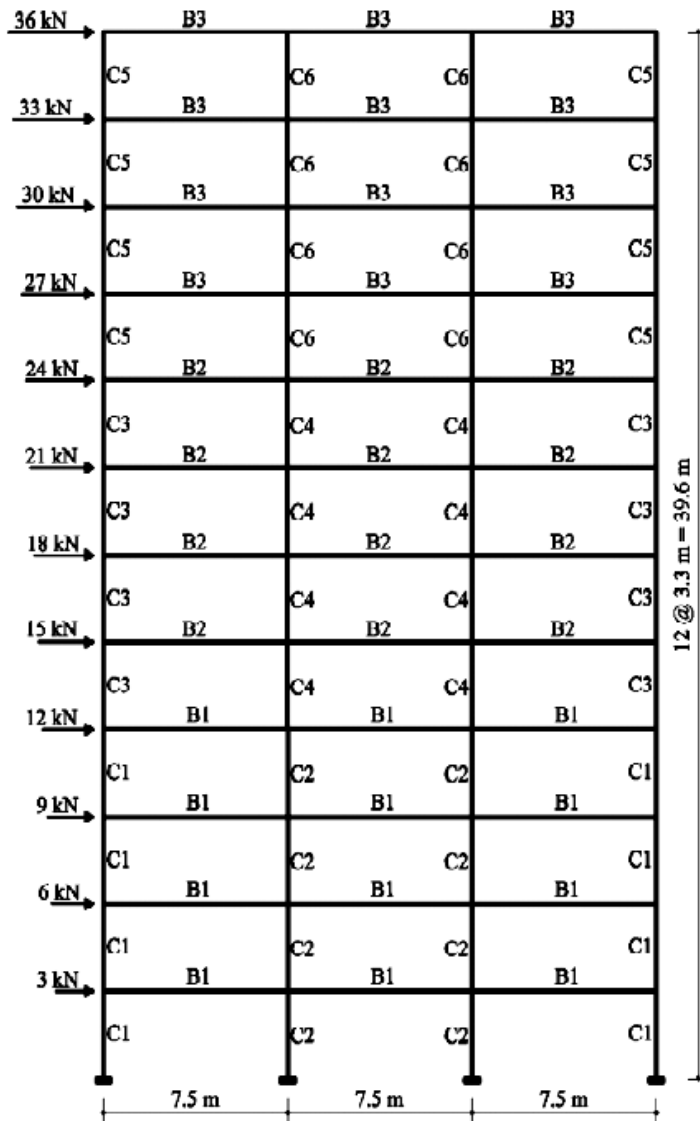


Figure 11. Three bay, twelve-story RC frame

Comparison of the results of the RC frame optimization obtained by BA with those of the reported in [1] is given in Table 3. For the optimum solution obtained by BA the DCRs in the members of the the RC frame are depicted in Figure 12 indicating that the optimum solution is feasible. The convergence history of BA for optimization of three bay, twelve-story frame is shown in Figure 13.

Table 3: Optimum designs of three bay, twelve-story RC frame

Element		BA				Kaveh and Sabzi [1]					
		Sectional dimensions		Reinforcements		Sectional dimensions		Reinforcements			
Type	Group	Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment		
Beam	B1	350	550	3-D19	6-D22	350	600	3-D19	6-D22		
	B2	350	550	3-D19	6-D22	300	550	5-D19	6-D22		
	B3	350	550	3-D19	5-D22	300	500	3-D19	6-D22		
Column	C1	450	450	8-D25		450	450	10-D25			
	C2	600	600	10-D25		600	600	12-D25			
	C3	400	400	8-D25		400	400	10-D25			
	C4	450	450	10-D25		500	500	10-D25			
	C5	350	350	8-D25		350	350	8-D25			
	C6	350	350	8-D25		400	400	4-D25			
Frame cost (\$)				80470				81138			
Number of structural analyses				25800				54600			

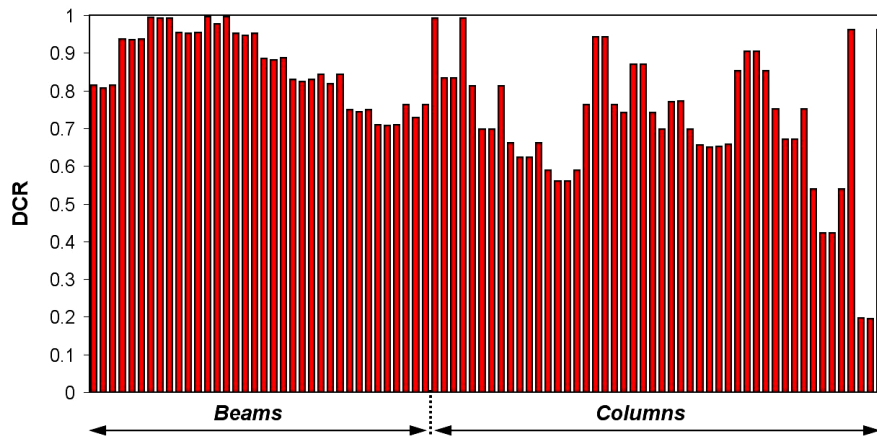


Figure 12. DCR in the members of the optimum three bay, twelve-story RC frame

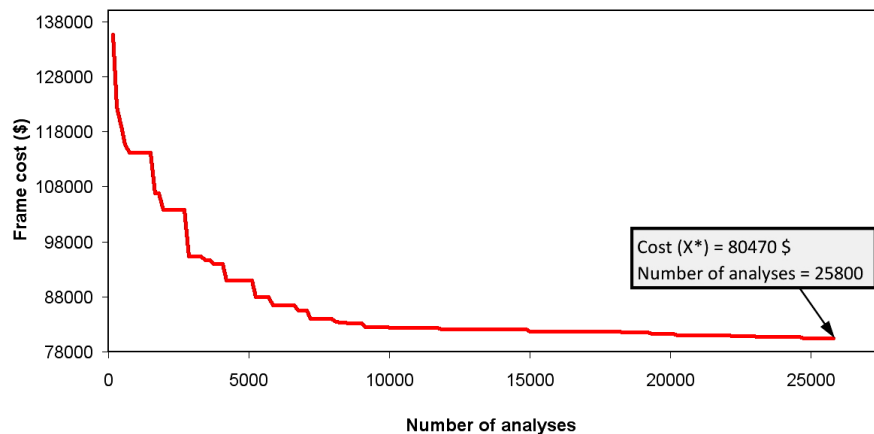


Figure 13. Convergence history of BA for three bay, twelve-story RC frame

As well as the first two examples, in the present example also the optimum cost obtained by BA is less than those of the reported by Kaveh and Sabzi [1]. As shown in Figure 13 the BA converges after 25800 structural analyses which is considerably less than the number of structural analyses required by the algorithm presented in [1].

5. CONCLUSIONS

The present paper deals with design optimization of reinforced concrete (RC) frames using bat algorithm (BA). The BA meta-heuristic optimization method is inspired from the echolocation behavior of bats. Recently achieved researches have demonstrated the high efficiency of BA in comparison with existing meta-heuristic algorithms. The robustness of BA lies in its interesting ability in making a satisfactory balance between exploration and exploitation characteristics. Automatic switch from exploration to more extensive exploitation is achieved in BA when the optimality is approaching. The efficiency of the BA meta-heuristic is numerically examined using four, eight and twelve story RC frames and the results are compared with those of reported in literature. In the presented design example, the optimum cost found by BA is slightly better compared with those of other algorithms. However, the computational demands of BA is considerably better in comparison with other algorithms.

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