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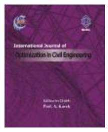

OPTIMUM WEIGHTED MODE COMBINATION FOR NONLINEAR STATIC ANALYSIS OF STRUCTURES

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ABSTRACT

In recent years some multi-mode pushover procedures taking into account higher mode effects, have been proposed. The responses of considered modes are combined by the quadratic combination rules, while using the elastic modal combination rules in the inelastic phases is not valid. Here, an optimum weighted mode combination method for nonlinear static analysis is presented. Genetic algorithm is used for optimization of the modal weight. The proposed procedure is applied for a sample building. The results show that the resulted response from the proposed method has minimal error in comparison with the response of the nonlinear time history analysis.

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KEY WORDS: pushover; modal combination; optimization; genetic algorithm

1. INTRODUCTION

Referring to the philosophy of the seismic design and inelastic behavior of structures at low performance levels such as life safety and collapse prevention, it is clear that the damageability of structure under earthquake is controlled by inelastic deformation capacity of the structural elements. Therefore revising design codes force control to displacement control has been widely recognized by researchers [1]. This revision can be achieved only by introducing nonlinear analysis into the seismic design methodology. Regarding the nature of seismic loads in the form of base acceleration, nonlinear time history analysis (NTHA) is the most rigorous procedure to compute the seismic demand. Because of complexity of this method, during the last decade nonlinear static procedure (NSP), so-

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called pushover analysis, has been developed as a practical tool to estimate the inelastic response of sytructures [2-4].

In the pushover analysis, a structural model that directly incorporates nonlinear material characteristics is subjected to monotonically increasing invariant lateral force pattern until a predetermined target displacement is reached. Define of the load pattern and target displacement are based on the assumption that the response of a multi-degree-of-freedom (MDOF) structure is directly related to the response of an equivalent single-degree-of-freedom (SDOF) system with a fundamental mode shape. While the invariant load pattern according to fundamental mode may be adequate for regular and low-rise structures whose response is effectively dominated by fundamental mode [5], it can misleading for irregular and high-rise building with significant higher mode contribution.

The major drawback of the conventional pushover analysis [2-4] lies in the fact that it is basically restricted with a single-mode response, and it cannot account for the contributions of higher modes and changing of modes shape because of structural yielding. Recently to eliminate this drawback several multi-mode pushover procedure based on the modal combination concept have been proposed which could consider the effect of higher modes [6-8]. Also in order to consider the effect of the progressive changes in the structural properties during the nonlinear response, some researchers have proposed the adaptive form of the modal procedures [9-14] where, in each step, the load patterns are updated with respect to the progressive changes in the structural modal properties.

In the multi-mode pushover procedures whether in adaptive or non-adaptive form the responses resulting from different considered modes are combined by quadratic combination rules (e.g. square-root-of-sum-of-squares SRSS). While using these elastic modal combination rules in the inelastic phases is not valid and may using an effective alternative modal combination rule could improve the results. Therefore, in this paper, an optimum weighted mode (OWM) combination method for using in multi-mode pushover procedures is presented.

2. MULTI-MODE PROCEDURES

The first multi-mode pushover procedure was proposed by Paret et al. [6], which help to identify the effect of higher modes, without providing any solution to estimate the seismic response in the higher modes. In order to quantify the effect of higher modes, Moghadam has proposed PRC (Pushover Results Combination) method [7]. In which the total seismic response of the structure is estimated by combining the responses due to multiple pushover analyses. Each of the pushover analysis is conducted by using a mode shape as its load pattern. The resulted responses from different modal pushover analysis are combined using Equation (1).

$$R = \sum_{i=1}^{N} B_i R_i \tag{1}$$

where, R: total estimation of response, B_i: mass participating factor for mode i, R_i: value of response resulted from pushover analysis using mode shape i as load pattern.

Furthermore modal pushover analysis (MPA) was proposed by Chopra and Goel [8]. This method is an efficient extension of the single-mode traditional pushover to multi-mode procedure while retaining the simplicity of it. The seismic responses are estimated by combining the response of some first independent modal pushover analysis which, use the elastic mode shape as load pattern. In the MPA procedure in spite of the PRC, the target displacement for every mode of pushover analysis is not the same and determined from NTHA of equivalent SDOF system. The responses of the pushover analysis are combined using SRSS combination rule (Equation 2).

$$R = \sqrt{\sum_{i=1}^{n} R_i^2} \tag{2}$$

where, R and R_i are the same terms as defined for Equation (1).

Also an adaptive modal combination (AMC) procedure was proposed by Kalkan and Kunnath [12] which try to consider the variation of modal shapes during the MPA procedure and incorporate the adaptive method in the MPA procedure. In the MPA and AMC procedures the structural model is pushed and pulled simultaneously in each mode according to the modal forces and the effect of the reversal force storey, observed in dynamic analysis is considered in each mode. However in these procedures, because of using the quadratic combination rules (e.g. SRSS) to compute the total response, the effect of the sign reversal is finally removed.

The main issue in this paper is to answer this question: can be estimated the seismic responses exactly by using another combination rules in the MPA procedure and how can be found this optimum combination rule? In this regard, a new combination rules is tentatively proposed in which, the weighted result of the considered modes are combined through a direct algebraic summation without removing any sign. The optimum weight (contribution coefficient) of each mode is obtained by using genetic algorithm (GA) optimization procedure. So that the inter-story drift profile resulted from the proposed combination rule has minimal error in comparison with drift profile resulted from the NTHA. A twelve story moment resistant steel frame as a case study is investigated in this research program.

3. PROPOSED PROCEDURE

Since the goal of this procedure is to define an optimal combination rule for using in MPA procedure, all the established steps of MPA procedure except the SRSS combination step are included in the proposed method. The error between responses of NTHA and sum of factored responses resulted from each pushover analysis is defined as an error function that must be minimized and the quantity of each factors be determined. Every factor shows the contribution of each mode and the GA method is used for minimization of error (objective) function.

The procedure is implemented in a sequence of steps as fallow:

1) Compute the natural frequencies, ω_n and mode shapes, $\O_n$, for linearly elastic

vibration of the structure.

2) For a selected number of first modes, develop the base shear-roof displacement ($V_{b\,n}$–$U_{r\,n}$) pushover curve. The lateral force distribution for the n^{th} mode is defined using Equation (3),

$$S_n = m\,Ø_n \tag{3}$$

where, m is the mass matrix of the structure and $Ø_n$ is mode shape for the n^{th} mode.

3) Idealize each pushover curve as a bilinear curve.

4) Obtain the force–displacement ($F_{s\,n}/L_n$ – D_n) relation, for the n^{th} mode inelastic SDOF system from the idealized pushover curve using Equation (4),

$$\frac{F_{s\,n\,y}}{L_n} = \frac{V_{b\,n\,y}}{M_n^*} \qquad D_{n\,y} = \frac{U_{r\,n\,y}}{\Gamma_n \varnothing_{r\,n}} \tag{4}$$

where, $M_n^* = L_n\,\Gamma_n$ is the effective modal mass and $Ø_{r\,n}$ is the value of $Ø_n$ at the roof level. Γ_n and L_n are calculated by Equation (5)

$$\Gamma_n = \frac{\sum_{k=1}^{N} m_k \varnothing_{k\,n}}{\sum_{k=1}^{N} m_k \varnothing_{k\,n}^2} \qquad L_n = \sum_{k=1}^{N} m_k \varnothing_{k\,n} \tag{5}$$

5) Compute the peak deformation of the n^{th} inelastic SDOF system through NTHA. The elastic period of vibration of the n^{th} system is calculated using Equation (6).

$$T_n = 2\pi \sqrt{\frac{L_n D_{n\,y}}{F_{s\,n\,y}}} \tag{6}$$

6) Calculate the peak roof displacement associated with the n^{th} mode using Eq. 7.

$$u_{r\,n} = \Gamma_n \varnothing_{r\,n} D_n \tag{7}$$

7) From the pushover database values at roof displacement $u_{r\,n}$, extract values of desired responses R_n (floor displacements, story drifts, etc).

8) Perform NTHA of the structure under the desired acceleration record (in this study El Centro 1940) and determine the envelop of inter-story drift profiles.

9) Define the combination rule as:

$$R = aR_1 + bR_2 + cR_3 + dR_4 \tag{8}$$

where, R is final (total) response, R_1, R_2, R_3 and R_4 is obtained at step 7 and a, b, c and d are contribution factors of each mode that obtained from minimized objective function at step 12.

10) Since inter-story drift is important factor in damage of structure, so it is desired that the differences between drift resulted from NTHA and proposed combination rule is minimized and the error vector is defined as:

$$\vec{\Delta}_{error} = \vec{\Delta}_{NTHA} - \left(a\vec{\Delta}_1 + b\vec{\Delta}_2 + c\vec{\Delta}_3 + d\vec{\Delta}_4\right) \tag{9}$$

where, $\vec{\Delta}_{NTHA}$ is vector of story drift profile (at every story) resulted from NTHA and $\vec{\Delta}_1, \vec{\Delta}_2, \vec{\Delta}_3$ and $\vec{\Delta}_4$ is vector of story drift profile (at every story) extracted from each pushover analysis according with every mode.

11) To minimize the error vector, all its components and sum of them must be minimized, so the objective function is defined as:

$$norm(\vec{\Delta}_{error}) = \sqrt{d_{error1}^2 + d_{error2}^2 + d_{error3}^2 + ... + d_{error12}^2} \tag{10}$$

where, $d_{error\ i}$ is i^{th} component of the error drift story vector, $\vec{\Delta}_{error}$.

12) Use GA optimization method to minimize the objective function defined at step 11 and obtain the optimum quantities of a, b, c and d.

4. GENETIC ALGORITHMS (GAS)

In the traditional procedures to optimize an objective function, gradient of the objective function is used to search the domain of the function. Main drawback of these procedures arises when the objective functions and constraints are not continuous and it is not possible to calculate the gradient of the functions. In the latest years of the twentieth century, along with development of the computers, Genetic algorithm (GA) procedure as a numerical optimization method has been developed, in which the objective functions is not required to be continuous [15-19]. In recent years, GA, as a practical method, is used extensively for solving optimization problems in different fields of science, including civil engineering [20-22]. In the application of GA for solving optimization problems, a design vector can be considered as a chromosome, its design components as the genes, and its value of the objective function as a measure of the fitness. GA starts with a discrete set of design vectors (chromosomes) and changes the current set towards generating a fitter generation of design points, through three genetic algorithm operators including selection, cross over and mutation [16-17]. In each generation, a set of chromosomes is selected for mating based on their relative fitness. The fitter chromosomes are given more chance of passing their genes into the next generation. This process is operated by selection.

In this paper the stochastic universal sampling method [18] has been used for selecting a number of chromosomes for mating, based on their fitness values in the current population.

The selected chromosomes are then chosen randomly through cross over to produce offspring. In the present study discrete recombination has been used for cross over operator. In order to maintain the variability of the population, mutation at a specified low rate should be performed in certain chromosomes. The mutation helps GA to provide a guarantee that the probability of searching any given chromosomes will never be zero and helps the GA escape local minima. At the final generation the chromosome which has the best fitness is chosen as the optimum point. Though in the early stages of string coding development, design variables were represented in their binary format but they have some drawbacks in taking continuous problems and it has been shown that for real-valued numerical optimization problems, real-valued coding representations offer certain advantages such as simple programming, less memory required and greater freedom to use different genetic operators over binary versions [19]. Hence in this paper the real-valued coding has been used to represent the chromosomes. Also in this paper the elitist strategy has been used which allows some of the best chromosomes in the current population to go to the next generation without modification.

5. CASE STUDY

Investigated structure is a 12 story structure presented in Reference [23]. It is a 12 story moment resisting frame that conforms to the requirements of the UBC [24] provisions. The building designs are based on a configuration presented in the SEAOC seismic Design Manual [25]. The building's lateral force resisting system is composed of steel perimeter moment resisting frames (MRF). The floor-plan and elevation view of the investigated frame are illustrated in Figure 1. Masses assigned for this frame at the story floors and roof level are 550 and 510 tons, respectively. The yield strength of steel is assumed to be F_y=345MPa (50 Ksi) for all structural members.

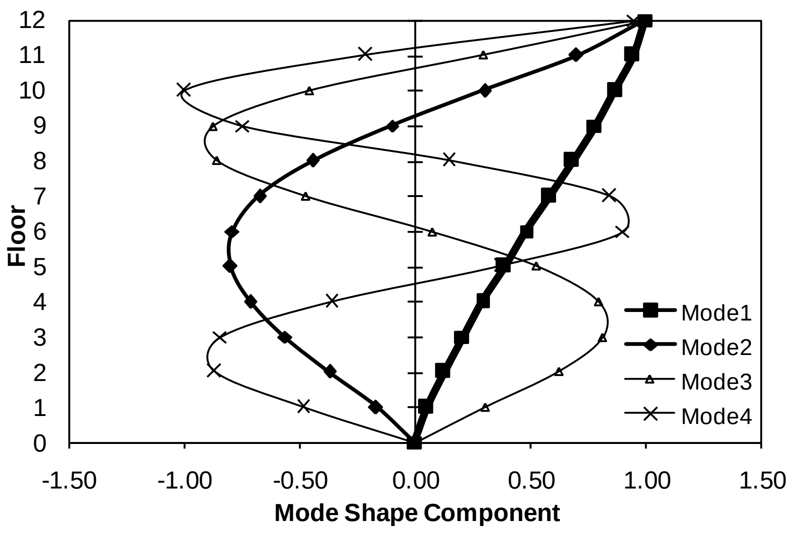

Figure 1. The first four mode shapes

5.1. Comparison and Analysis of the Results

The proposed procedure was implemented for the 12 story steel frame subjected to El Centro 1940 ground acceleration record (PGA= 0.348g, at T=2.12 sec) and the optimum weight of each mode was obtained. The nonlinear analyses were carried out using DRAIN-2DX computer program [26]. The properties and shape of four first modes is illustrated in Table 1 and Figure 1, respectively.

Table 1. Properties of the first four modes

	Mode 1	Mode 2	Mode 3	Mode 4
ω_n	1.89	5.25	9.10	13.28
L_n	355595.11	−151523.41	95089.52	−75634.70
Γ_n	1.40	−0.61	0.34	−0.23
M_n^*	498055.64	92374.26	32255.65	17140.85
B_n	0.74	0.14	0.05	0.03

The base shear-roof displacement relations, $V_{b\,n} - U_{r\,n}$ and $F_{s\,n}/L_n - D_n$ resulted from each pushover analysis using S_n=m $Ø_n$ as load pattern are presented in Figures 2, 3 and Table 2. The maximum displacement of the equivalent SDOF system subjected to El Centro ground motion ($D_{n\text{-NTHA}}$) and its associated maximum roof displacement ($U_{r\,n\text{-NTHA}}$) are also shown in Table 2. In the second mode the value of $D_{n\text{-NTHA}}$ is less than $D_{n\,y}$, which means the equivalent SDOF system associated with these modes is not yielded.

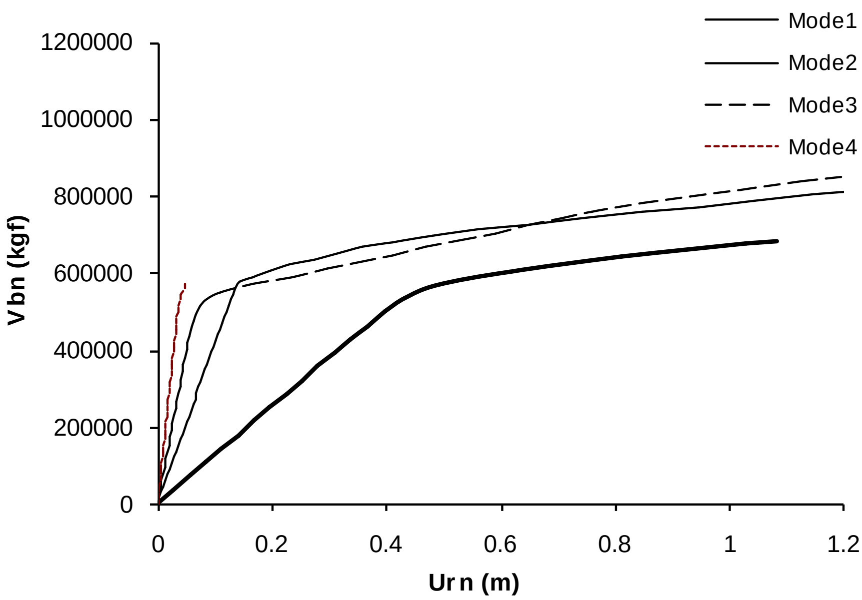

Figure 2. Modal pushover curves

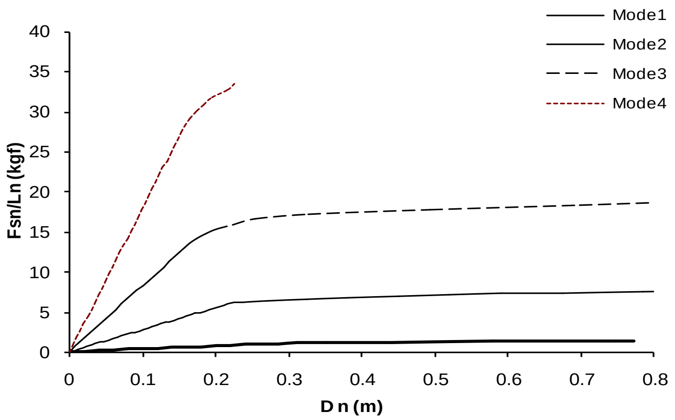

Figure 3. Equivalent SDOF pushover curves

To determine the factor (contribution) of each mode in the optimum weighted mode (OWM) combination procedure, the GA procedure is employed. The factor of each mode (a, b, c and d) are chosen as variable parameters while their upper and lower bound values are [−1,1]. The parameters of GA to be used in this study are taken as follows: population size = 50; number of generation = 500 and mutation rate = 0.05. After performing the GA it is found that the optimum value of factors are a =0.6651, b=0.3267, c=0.580 and d=0.4023. The evolving best fitness generation can be seen in Figure 4.

Table 2. Properties of equivalent nonlinear SDOF systems

Mode	$V_{b\,n\,y}$	$U_{r\,n\,y}$	$F_{s\,n\,y}/L_n$	$D_{n\,y}$	$D_{n\text{-NTHA}}$	$U_{r\,n\text{-NTHA}}$
1	578192.80	0.46	1.16	0.33	0.28	0.39
2	629604.46	0.15	6.82	0.25	0.27	0.16
3	539795.10	0.07	16.73	0.20	0.13	0.04
4	527634.69	0.04	30.78	0.17	0.14	0.03

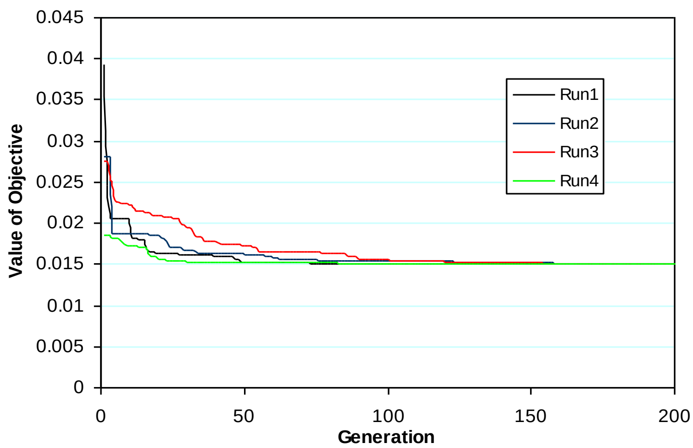

Figure 4. Evolving best fitness generation

The Total inter-story drifts profiles resulting from the different combination rules used in the multi-mode pushover procedure (OWM, PCR and MPA) and also the peak inter-story drifts profiles resulting from the NTHA and the conventional pushover analysis (mode 1) for the sample building are shown in Figure 5.

(Note: in PRC method in spite of MPA method, all models are pushed until the same displacement, $U_{r\text{-NTHA}}$ is achieved while in this study, because of instability of the model in mode 4 when pushed until Ur-NTHA = 0.41 meter, only first three modes are included in the combination process).

Furthermore the inter-story drift error of the different pushover methods with respect to the NTHA in each story are calculated by Eq. 11 and shown in Figure 6.

$$Error(\%) = 100 \times \left(\frac{d_{i-P} - d_{i-NTHA}}{d_{i-NTHA}} \right) \tag{11}$$

where di-NTHA is the peak response of drift at a given level i from the NTHA, di-P is the corresponding drift from the pushover analysis.

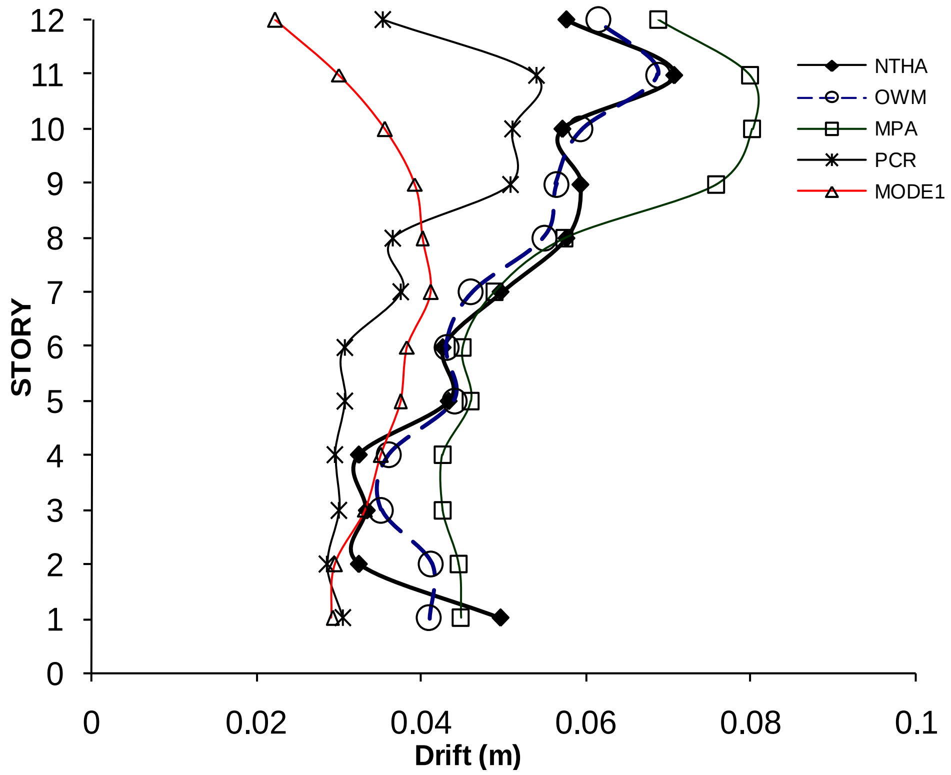

Figure 5. Peak inter-story drift profiles resulting from the NTHA and different pushover procedure.

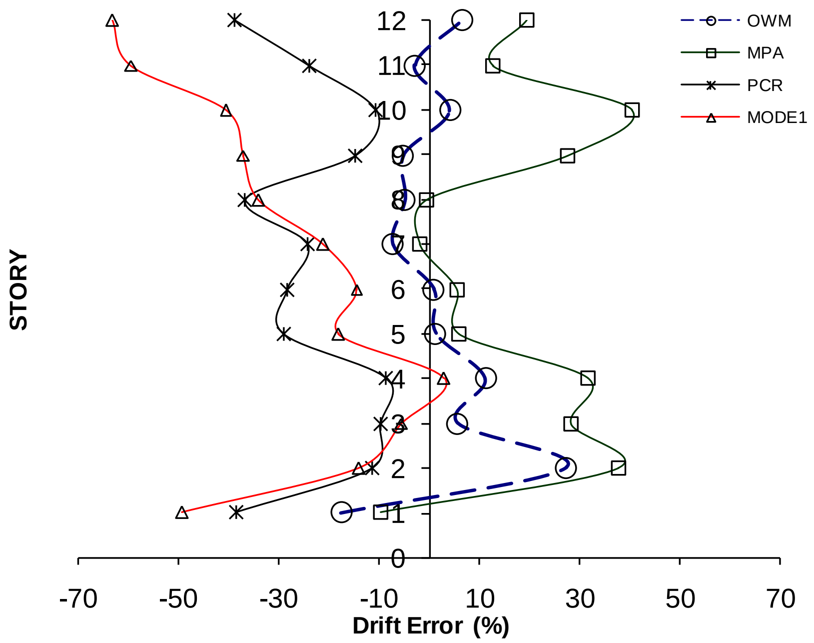

Figure 6. Inter-story drift profiles of different pushover procedures with respect to NTHA

To compare the accuracy of different methods, an error index is defined using Eq. 11 [27].

$$Error(\%) = 100 \times \frac{1}{n} \sqrt{\sum_{i=1}^{n} \left(\frac{d_{i-P} - d_{i-NTHA}}{d_{i-NTHA}} \right)^2} \tag{12}$$

where $d_{i\text{-}NTHA}$ is the peak response of drift at a given level i from the NTHA, $d_{i\text{-}P}$ is the corresponding drift from the pushover analysis and n is the number of stories. Whenever the error index is close to zero, the pushover response approaches the NTHA response. The error index is calculated for different methods as presented in Table 3.

Table 3. Value of Error Index (%) for different pushover procedures

Methods	OCM	MPA	PRC	Mode 1
Error Index (%)	3.115	6.650	7.343	9.519

6. CONCLUSION

In this study it is shown that an optimum combination rule for using in modal pushover analysis could be found while it estimates the seismic responses satisfactorily with minimum error index. The optimal weight of the each mode is defined so that the resulted response of the proposed combination rule has a minimum error to the nonlinear time history analysis (NTHA). Genetic algorithm is used to find the optimum weight of each mode. The responses resulted from the direct algebraic combination of modes factored by optimum weight are very close to responses resulted from the NTHA. Therefore the modal response vector can be interpreted as the ordered basis of the vector space of the NTHA responses. It is expected that by using this procedure for different groups of structures, the participation of each mode for every group of structures can be defined statistically.

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