

Benders Decomposition Algorithm for a Build-to-Order Supply Chain Problem Under Uncertainty

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ABSTRACT

Since customization increases, build-to-order systems have received greater attention from researchers and practitioners. This paper presents a new build-to-order supply chain model with multiple objectives that minimize the total cost and lead time and, also, maximize the quality level. The model is first formulated in a deterministic condition and, then, the uncertainty of the cost and quality by the scenario-based approach to solving a robust optimization was investigated. The return policy and outsourcing are the new issues in a build-to-order supply chain considering the cost and inventory. A Benders decomposition algorithm is used to solve and validate the model. Finally, the related results are analyzed and compared with the results obtained by CPLEX for deterministic and scenario-based models.

KEYWORDS *Build-to-order, Multi-objective supply chain, Benders decomposition.*

1. Introduction

A supply chain consists of integrated suppliers, manufacturers, and all external actors (e.g., retailers and distributors), all of whom affect organizational performance [1]. Therefore, quantity and pricing decisions are tactical decisions [2]. Increasing competition makes individual firms members of the supply chain. Planning in a supply chain has an important role in the success of it. A build-to-order (BTO) system is a proper production strategy that faces changes in customer's interests [3]. The BTO like make-to-order (MTO) leads to the variety and flexibility, as well as higher customer satisfaction [4]. The raw materials inventory and inventory of modules are based on short-term anticipation; however, production or assembling of final products is done after receiving customer's orders in a BTO supply chain [5]. Lead time has an important role in attracting customers and growing demand. The highest quality with the lowest cost in the lowest lead time satisfies customers. Satisfying customers represent an important aim in customized systems. Because

the BTO is a customized system, three objective functions are considered; minimizing the total cost, maximizing the quality of products, and minimizing the lead time.

Che and Chiang [6] presented three objectives, namely cost, delivery, and quality; however, their model is different from our model. In addition, in the supply chain, a few firms produce all components of products. They outsourced some parts of products due to cost advantage. However, outsourcing may require more lead times rather than in-house producing [7]. In addition, higher quality of some suppliers is considerable. However, one of the major gaps in the BTO problems is that they did not consider outsourcing. To the best of our knowledge, three objective functions presented in this paper can make a trade-off between advantages and disadvantages of outsourcing and, then, outsource products with higher quality, lower cost, and lower lead time rather than in-house manufacturing. Makhopadhyay and Setoputro [8] proposed a cost function-related return policy in a BTO system. Additionally, Konstantaras et al. [9] suggested a cost function related to return policy. However, they did not consider a supply chain and the return products in the inventory system. The other contribution of our paper is to consider a return policy in a stock system and cost function.

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2. Literature Review

The studies in the field of BTO systems are related to production planning, etc. Li and Chen [10] modeled a cost function in a BTO environment based on customers' views. Li and Chen [11] presented two segments of customers with one segment priority in a BTO system and made a model based on a queuing theory with price and capacity as decision variables.

Due to existing uncertainty in real BTO systems, fuzzy modeling was used in this system as mentioned by Deenadayaian and Malliga [12], who studied computer assembling in a BTO environment. Yibin et al. [13] compared the performance of two systems: build-to-stock (BTS) and combined BTO & BTS. A few studies such as Xiao and Choi used game theory in BTO systems [14]. They used a game theory to model the competition between two manufacturers under a BTO environment. Yohanes [15] used Stackelberg a game competition between market and production. In the field of inventory control, Moses et al. [16] presented a real-time promising method for estimating due date correctly and tested performance measure of this approach. Shimoda et al. [17] proposed a recommendation method that balances component's inventory and substitutes a customer specification by alternative specifications.

Xu and Ma [18] investigated bottleneck in BTO systems and proposed a production control model using the input/output control and constraint's theory. Volling and Spengler [19] studied BTO systems and proposed order-driven planning in this field considering master production, scheduling, and order promising. Matzke et al. [20] presented auctions in BTO systems to match capacity and demand.

Flexibility is the issue considered in BTO [21-23], and knowledge management and information technology are the other conceptual issues in the BTO literature [24-25].

One of the solution methods for optimizing supply chains is a Benders decomposition algorithm that has received greater attention in

recent years. Khatami et al. [26] used a Bender's decomposition algorithm for optimizing a supply chain problem. Makui et al. [27] considered an aggregate production planning problem solved by benders decomposition. Shaw et al. [28] applied a Benders decomposition algorithm for optimizing stochastic programming in supply chain systems. This algorithm was used in supply chain problems in the stochastic model by Keyvanshokoo et al. [29]. Jeihoonian et al. [30] formulated durable products in a supply chain solved by a Benders decomposition algorithm.

According to the literature, BTO models do not address outsourcing. BTO systems must be flexible and have a high variety of products so that outsourcing can be useful in these systems, especially when cheaper components and higher quality can be gained by outsourcing. Due to the benefit of outsourcing, outsourcing is a way for manufacturers to obtain components in this model. The manufacturer can fabricate them in the firm too. The return policy is another issue that is considered in this model. The return policy is a new issue in BTO systems that has not been found in supply chains or inventory systems, and a few literature reviews related to a return policy studied it in demand and cost only. This paper presents a BTO supply chain. There are multi-suppliers, multi-manufacturers, and multi-retailers. Three objective functions are considered here to minimize the total cost, maximize the quality, and minimize the lead time.

The rest of this paper is as follows. Notations and formulation of the problem are presented in Section 2. Section 3 gives a solution algorithm involving Benders decomposition. Some numerical examples, results, and some analyses are provided in Sections 4 and 5. Section 6 presents the conclusion and suggestions for future research.

3. Problem Formulation

This section illustrates mathematical notations, objective function, and constraints.

3-1. Mathematical notations

Indices

w	Index of suppliers
s	Index of component' supplier
m	Index of manufacturer
k	Index of retailer
r	Index of raw materials
j	Index of final products
n	Index of components

t Index of periods

Parameters

- $C_{j,m}$ Cost of assembling product j for manufacturer m
- $BC_{j,k}$ Shortage cost of product j for retailer k
- $JCT_{j,i}$ Transportation cost of product j from manufacture m to retailer k
- NCM Cost of component n for manufacturer m
- NCO_s Outsourcing cost of component n for manufacturer m from outsourcer s
- HCN_i Inventory cost of component n for manufacturer m
- HCR_r Inventory cost of raw material r for manufacturer m
- $RC_{r,n}$ Cost of raw material r for manufacturer m from supplier w
- $UJ_{j,m}$ Quality of product j for manufacturer m
- $UO_{n,s}$ Quality of component n for outsourcer s
- UM_n Quality of component n for manufacturer m
- $UR_{r,v}$ Quality of row material r in supplier w
- TMN Time to fabricate component n for manufacturer m
- TON_s Time to fabricate and transport component n in outsourcer s for manufacturer m
- $TJ_{j,m}$ Time to assemble product j for manufacturer m
- $TT_{m,i}$ Time to transport from manufacturer m to retailer k
- $DA_{j,k}$ Real demand of product j for retailer k in period t
- $SL_{j,k}$ Customer service level needed in % demand
- DJ_j Demand of product j without the influence of price, return policy, and time
- β Price-sensitive coefficient in demand
- α Sensitivity of return policy in demand
- η Sensitivity of time in demand
- σ Price coefficient of upper bound for r
- δ Return policy coefficient to use them
- φ Rate of return related to r
- mp Upper bound of price
- $\gamma_{n,r}$ Amount of raw material r required in component n
- $\mu_{n,j}$ Amount of component n required in product j

Decision variables

- $Q_{j,m,i}$ Quantity of product j transformed from manufacturer m to retailer k in period t
- $QM_{j,i}$ Quantity of product j produced by manufacturer m in period t
- $Qk_{j,k}$ Quantity of product j transformed to retailer k in period t
- $DF_{j,m}$ Predicted demand of product j for manufacturer m in period t
- $DN_{n,i}$ The demand of component n for manufacturer m in period t
- $P_{j,m,t}$ Price of product j in manufacturer m in period t
- $r_{j,m,t}$ Refund of product j in manufacturer m in period t
- $B_{j,k,t}$ Shortage amount of product j in retailer k in period t
- MN_n Quantity of component n produced by manufacturer m in period t
- $ON_{n,i}$ Quantity of component n outsourced to outsourcer s by manufacturer m in period t
- MND Quantity of component n fabricated by manufacturer m in period t to use in period t (in the same period fabricated and used)
- OND Quantity of component n outsourced to outsourcer s by manufacturer m in period t to use in period t
- $IN_{n,m}$ Inventory of component n by manufacturer m in period t
- $IR_{r,m}$ Inventory of raw material r for manufacturer m in period t

Binary variable

- $\theta_{j,m,k}$ 1 if product j is transformed from manufacturer m to retailer k in period t ; 0, otherwise

$Y_{n,m,t}$ 1 if component n is fabricated by manufacturer m in period t ; 0, otherwise
 $X_{n,m,s}$ 1 if component n is outsourced to outsourcer s by manufacturer m in period t ; 0, otherwise
 $Z_{n,m,t}$ 1 if X and Y are 0; 0, otherwise

3-2. Main assumptions

As mentioned earlier, the presented BTO supply chain model is a multi-objective model that minimizes the cost, maximizes the quality, and minimizes the lead time. There are multi-components suppliers to outsource parts. The raw materials and components of each supplier have its quality and lead time that may differ from each other. The fabricating components by each manufacturer have their own quality and lead time, as well. Due to the BTO systems under which there is not any final product inventory, the lead time is crucial to prevent the loss of demand if the component inventory is not enough. One supplier with lower lead time and higher quality can help the manufacturer to compensate for the shortage of component inventory. In addition, if suppliers fabricate component with higher quality and lower cost, the manufacturers can outsource the components while having enough time or for the next period. The demand depends on price, time, and refund amount. The returned product is disassembled and used in component inventory. The return policy has its related cost.

3-3. Mathematical model

By employing the afore-mentioned notations and assumptions, the associated mathematical model can be formulated by:

$$\begin{aligned} \text{Min } Z_1 & \quad (1) \\ = & \sum_t \sum_j \sum_n \sum_r \sum_k \sum_m \sum_s \sum_w C_{j,m} \cdot QM_{j,m,t} \\ & + BC_{j,k} \cdot B_{j,k,t} + JCT_{j,m,k} \cdot Q_{j,m,k,t} \\ & + \phi \cdot 2 \cdot r_{j,m,t} + NCM_{n,m} \cdot (MN_{n,m,t} \\ & + MND_{n,m,t}) + NCO_{n,m,s} \cdot (ON_{n,m,s,t} \\ & + OND_{n,m,s,t}) + HCN_{n,m} \cdot IN_{n,m,t+1} \\ & + HCR_{r,m} \cdot IR_{r,m,t+1} \\ & + RC_{r,m,w} \cdot OR_{r,m,w,t} \end{aligned}$$

$$\begin{aligned} \text{Max } Z_2 & \quad (2) \\ = & \sum_t \sum_j \sum_n \sum_r \sum_w QM_{j,m,t} \cdot UJ_{j,m} \\ & + \sum_s (ON_{n,m,s,t} \\ & + OND_{n,m,s,t}) \cdot UO_{n,s} \\ & + (MN_{n,m,t} + MND_{n,m,t}) \cdot UM_{n,m} \\ & + OR_{r,m,w,t} \cdot UR_{r,w} \end{aligned}$$

$$\begin{aligned} \text{Min } Z_3 & \quad (3) \\ = & \sum_t \sum_j \sum_n \sum_m \sum_k TMN_{n,m} \cdot Y_{n,m,t} \\ & + \sum_s TON_{n,m,s} \cdot X_{n,m,s,t} \\ & + TJ_{j,m} \cdot \theta_{j,m,k,t} + TT_{m,k} \cdot \theta_{j,m,k,t} \end{aligned}$$

$$B_{j,k,t} \leq SL_{j,k,t} * DA_{j,k,t} \quad (4)$$

$$B_{j,k,t} = DA_{j,k,t} - Qk_{j,k,t} \quad (5)$$

$$Qk_{j,k,t} = \sum_m Q_{j,m,k,t} \cdot \theta_{j,m,k,t} \quad (6)$$

$$QM_{j,m,t} = \sum_k Q_{j,m,k,t} \cdot \theta_{j,m,k,t} \quad (7)$$

$$\begin{aligned} DF_{j,m,t} & \quad (8) \\ = & DJ_j - \beta \cdot P_{j,m,t} + \alpha \cdot r_{j,m,t} \\ & - \eta \cdot \left(\sum_n TMN_{n,m} \cdot Y_{n,m,t} \right. \\ & \left. + \sum_s TON_{n,m,s} \cdot X_{n,m,s,t} \right) \end{aligned}$$

$$\begin{aligned} P_{j,m,t} > & \sum_n NCM_{n,m} \cdot MN_{n,m,t} + \\ & NCO_{n,m,s} \cdot ON_{n,m,s,t} \\ & + HCN_{n,m} \cdot IN_{n,m,t+1} \\ & + C_{j,m} \end{aligned} \quad (9)$$

$$P_{j,m,t} < mp \quad (10)$$

$$DN_{n,m,t} = \max \left\{ \sum_j \mu_{n,j} \cdot DF_{j,m,t}, \sum_j \mu_{n,j} \cdot QM_{j,m,t} \right\} \quad (11)$$

$$DN_{n,m,t} > IN_{n,m,t} - M \cdot z_{n,m,t} \quad (12)$$

$$IN_{n,m,t+1} \leq MN_{n,m,t} + ON_{n,m,t} \quad (13)$$

$$+ \sum_j \delta \cdot (\varphi \cdot r_{j,m,t}) \cdot \mu_{n,j} + M_2 \cdot z_{n,m,t}$$

$$IN_{n,m,t+1} \geq MN_{n,m,t} + ON_{n,m,t} \quad (14)$$

$$+ \sum_j \delta \cdot (\varphi \cdot r_{j,m,t}) \cdot \mu_{n,j} - M'_2 \cdot z_{n,m,t}$$

$$DN_{n,m,t} \leq IN_{n,m,t} + MND_{n,m,t} \cdot Y_{n,m,t} \quad (15)$$

$$+ OND_{n,m,t} \cdot X_{n,m,t} + M_3 \cdot z_{n,m,t}$$

$$DN_{n,m,t} \geq IN_{n,m,t} + MND_{n,m,t} \cdot Y_{n,m,t} \quad (16)$$

$$+ OND_{n,m,t} \cdot X_{n,m,t} - M'_3 \cdot z_{n,m,t}$$

$$DN_{n,m,t} \leq IN_{n,m,t} + M'' \cdot (1 - z_{n,m,t}) \quad (17)$$

$$IN_{n,m,t+1} \leq IN_{n,m,t} + MN_{n,m,t} \quad (18)$$

$$+ ON_{n,m,t} + \sum_j \delta \cdot (\varphi \cdot r_{j,m,t}) \cdot \mu_{n,j}$$

$$- DN_{n,m,t} + M_4 \cdot (1 - z_{n,m,t})$$

$$IN_{n,m,t+1} \geq IN_{n,m,t} + MN_{n,m,t} + \quad (19)$$

$$ON_{n,m,t} + \sum_j \delta \cdot (\varphi \cdot r_{j,m,t}) \cdot \mu_{n,j}$$

$$- DN_{n,m,t} - M'_4 \cdot (1 - z_{n,m,t})$$

$$Y_{n,m,t} \leq (1 - z_{n,m,t}) \quad (20)$$

$$X_{n,m,t} \leq (1 - z_{n,m,t}) \quad (21)$$

$$IR_{r,m,t+1} \quad (22)$$

$$= IR_{r,m,t} + \sum_w OR_{r,m,w,t}$$

$$- \sum_n (MN_{n,m,t} + MND_{n,m,t}) \cdot \gamma_{n,r}$$

$$r_{j,m,t} < \sigma \cdot P_{j,m,t} \quad (23)$$

$$\sum_j \sum_k Q_{j,m,k,t} \leq J_m \quad (24)$$

$$\sum_n (MN_{n,m,t} + MND_{n,m,t}) \leq N_m \quad (25)$$

3-4. Description of the model statements

The first objective function minimizes the total cost including manufacturing, shortage, transportation, inventory, outsourcing, and refund. The second objective maximizes the quality of raw materials, component, and final product. The third objective minimizes lead time consisting of time to manufacture and delivery to retailers. Time to fabricate or outsource components will be considered if the component inventory is not enough. The shortage amount is shown in Equations (4) and (5). Equations (6) and (7) determine the amount of final product transformed to each retailer and assembled in each firm. The predicted demand is shown in Equation (8). The maximum and minimum of the price are shown in Equations (9) and (10). Equation (11) defines the requirement for components.

Equations (12)-(19) are associated with inventory balance. Therefore, if the demand of component in period t is more than an inventory of component in period t (Eq. 12), the manufacturer must decide on fabricating or outsourcing (Eqs. 15 and 16) to have enough component to assemble the demanded products in period t . In this condition, at least, one of X and Y binary variables should have one value. The inventory for the next period ($t+1$) is determined through Equations (13) and (14). If the inventory of component in period t is more than the demand of component in period t (Eq. 17), the manufacturer must only decide on the next inventory of component (Eqs. 18 and 19).

If x and y variables have one value (Eqs. 20 and 21), it means that the component demand is more than component inventory, and then variable z must be zero in Equation (12). Equation (22) is related to the inventory balance of raw materials. The maximum refund is stated in Equation (23). The capacity of manufacturing for the final product and fabricating of components are respectively shown in Equations (24) and (25).

3-5. Mathematical model under uncertainty

Some cost and quality parameters under specific scenario are listed below:

Parameter

- $C_{j,m,se}$ Cost of assembling product j in manufacturer m under scenario se
- $BC_{j,k,se}$ Shortage cost of product j for retailer k under scenario se
- $JCT_{j,m,i}$ Transportation cost of product j from manufacturer m to retailer k under scenario se
- $NCM_{n,i}$ Cost of component n in manufacturer m under scenario se
- $NCO_{n,n,se}$ Outsourcing cost of component n for manufacturer m from outsourcer s under scenario se
- $RC_{r,m,w}$ Cost of raw material r for manufacturer m from supplier w under scenario se
- $UJ_{j,m,se}$ Quality of product j in manufacturer m under scenario se
- $UO_{n,s,se}$ Quality of component n in outsourcer s under scenario se
- $UR_{r,w,se}$ Quality of raw material r in supplier w under scenario se
- P_{se} Probability of scenario se

Therefore, the model under scenario se changes as follows:

$$\begin{aligned} \text{Min } Z_1(se) = & \sum_t \sum_j \sum_n \sum_r \sum_k \sum_m \sum_s \sum_w C_{j,m,se} \cdot QM_{j,m,t} + BC_{j,k,se} \cdot B_{j,k,t} \\ & + JCT_{j,m,k,se} \cdot Q_{j,m,k,t} + \phi \cdot 2 \cdot r_{j,m,t} + NCM_{n,m,se} \cdot (MN_{n,m,t} + MND_{n,m,t}) \\ & + NCO_{n,m,s,se} \cdot (ON_{n,m,s,t} + OND_{n,m,s,t}) + HCN_{n,m} \cdot IN_{n,m,t+1} \\ & + HCR_{r,m} \cdot IR_{r,m,t+1} + RC_{r,m,w,se} \cdot OR_{r,m,w,t} \end{aligned} \tag{26}$$

$$\begin{aligned} \text{Max } Z_2(se) = & \sum_t \sum_j \sum_n \sum_r \sum_w QM_{j,m,t} \cdot UJ_{j,m,se} \\ & + \sum_s (ON_{n,m,s,t} + OND_{n,m,s,t}) \cdot UO_{n,s,se} + (MN_{n,m,t} + MND_{n,m,t}) \cdot UM_{n,m} \\ & + OR_{r,m,w,t} \cdot UR_{r,w,se} \end{aligned} \tag{27}$$

Equations (3)-(8)

$$P_{j,m,t} > \sum_n NCM_{n,m,se} \cdot MN_{n,m,t} + NCO_{n,m,s,se} \cdot ON_{n,m,s,t} + HCN_{n,m} \cdot IN_{n,m,t+1} + C_{j,m,se} \tag{28}$$

Equations (10) - (25).

4. Solution Method

As mentioned earlier, the supply chain BTO model is a multi-objective one. There are suppliers, manufacturers, retailers, components, and raw materials that constitute a large-sized problem. The computational time will be increased exponentially by increasing the model size. The mixed-integer non-linear problem can be divided into small-sized problems; therefore, a Benders decomposition algorithm (BDA) is appropriate for our model. This exact solution method splits the problem into integer programming (master problem) and

linear programming (sub-problem) to decrease the complexity of the original problem.

Before applying the BDA, firstly, the three-objective model is converted to the single-objective problem; therefore, the utility function method is used [31, 32]. Second, the terms such as

$MND_{n,m,t} \cdot Y_{n,m,t}$, $OND_{n,m,s,t} \cdot X_{n,m,s,t}$, $Q_{j,m,k,t} \cdot \theta_{j,m,k,t}$) make the problem non-linear; thus, the new variables and constraints are added to the model, making the problem linear.

4-1. Implementation of benders decomposition for the BTO model

To apply the BDA, the sub-problem (SP), dual subproblem (DSP), and the master problem (MP) must be formulated. Therefore, binary variables should be fixed ($Y_{n,m,t} = \overline{Y_{n,m,t}}$, $X_{n,m,s,t} =$

$\overline{X_{n,m,s,t}}$, $\theta_{j,m,k,t} = \overline{\theta_{j,m,k,t}}$, $z_{n,m,t} = \overline{z_{n,m,t}}$). Then, the sub-problem is determined. The upper bound for the objective function of the original problem in this model is a dual sub-problem, as illustrated below:

$$\text{Max DSP} = \sum_t \sum_k \sum_m \sum_n \sum_j -V1_{j,k,t} \cdot SL_{j,k,t} \cdot DA_{j,k,t} + DA_{j,k,t} \cdot V2_{j,k,t} - \overline{\theta_{j,m,k,t}} \cdot m \cdot V8_{j,m,k,t} - m \cdot (1 - \overline{\theta_{j,m,k,t}}) \cdot V9_{j,m,k,t} \tag{50}$$

$$+ (DJ_j - \eta) \cdot \left(\sum_n TMN_{n,m} \cdot \overline{Y_{n,m,t}} + \sum_s TON_{n,m,s} \cdot \overline{X_{n,m,s,t}} \right) \cdot V11_{j,m,t} + C_{j,m} \cdot V13_{j,m,t} - mp \cdot V14_{j,m,t} - M \cdot \overline{z_{n,m,t}} \cdot V17_{n,m,t} - M_2 \cdot \overline{z_{n,m,t}} \cdot V18_{n,m,t} - M'_2 \cdot \overline{z_{n,m,t}} \cdot V19_{n,m,t} - M_3 \cdot \overline{z_{n,m,t}} \cdot V20_{n,m,t} - M'_3 \cdot \overline{z_{n,m,t}} \cdot V21_{n,m,t} - \overline{Y_{n,m,t}} \cdot M \cdot V22_{n,m,t} - M \cdot (1 - \overline{Y_{n,m,t}}) \cdot V23_{n,m,t} - \overline{X_{n,m,s,t}} \cdot M \cdot V25_{n,m,s,t} - M \cdot (1 - \overline{X_{n,m,s,t}}) \cdot V26_{n,m,s,t} - M'' \cdot (1 - \overline{z_{n,m,t}}) \cdot V28_{n,m,t} - M_4 \cdot (1 - \overline{z_{n,m,t}}) \cdot V29_{n,m,t} - M'_4 \cdot (1 - \overline{z_{n,m,t}}) \cdot V30_{n,m,t} - mjj \cdot V36_{m,t} - mnn \cdot V37_{m,t} + \sum_n \sum_m IN_{n,m,1} \cdot V17_{n,m,1} - IN_{n,m,1} \cdot V20_{n,m,1} + IN_{n,m,1} \cdot V21_{n,m,1} - IN_{n,m,1} \cdot V28_{n,m,1} - IN_{n,m,1} \cdot V29_{n,m,1} + IN_{n,m,1} \cdot V30_{n,m,1} + \sum_r \sum_m IR_{r,m,1} \cdot V33_{r,m,1} - V1_{j,k,t} + V2_{j,k,t} \leq BC_{j,k} \cdot A_1 \tag{51}$$

$$V2_{j,k,t} + V4_{j,k,t} \leq 0 \tag{52}$$

$$-V4_{j,k,t} - V6_{j,m,t} - V8_{j,m,k,t} + V9_{j,m,k,t} - V10_{j,m,k,t} \leq 0 \tag{53}$$

$$V6_{j,m,t} - \sum_n \mu_{n,j} \cdot V16_{n,m,t} \leq A_1 \cdot C_{j,m} - A_2 \cdot UJ_{j,m} \tag{54}$$

$$-V9_{j,m,k,t} + V10_{j,m,k,t} - V36_{m,t} \leq A_1 \cdot JCT_{j,m,k} \tag{55}$$

$$V11_{j,m,t} - \sum_n \mu_{n,j} \cdot V15_{n,m,t} \leq 0 \tag{56}$$

$$+\beta \cdot V11_{j,m,t} + V13_{j,m,t} - V14_{j,m,t} + \sigma \cdot V35_{j,m,t} \leq 0 \tag{57}$$

$$-\alpha \cdot V11_{j,m,t} + \sum_n \delta \cdot (\varphi) \cdot \mu_{n,j} \cdot V18_{n,m,t} - \sum_n \delta \cdot (\varphi) \cdot \mu_{n,j} \cdot V19_{n,m,t} + \sum_n \delta \cdot (\varphi) \cdot \mu_{n,j} \cdot V29_{n,m,t} - \sum_n \delta \cdot (\varphi) \cdot \mu_{n,j} \cdot V30_{n,m,t} - V35_{j,m,t} \leq 2 \cdot \emptyset \cdot A_1 \tag{58}$$

$$-NCM_{n,m} \cdot \sum_j V13_{j,m,t} + V18_{n,m,t} - V19_{n,m,t} + V29_{n,m,t} - V30_{n,m,t} + \sum_r \gamma_{n,r} \cdot V33_{r,m,t} - V37_{m,t} \leq NCM_{n,m} \cdot A_1 - UM_{n,m} \cdot A_2 \tag{59}$$

$$- \sum_s NCO_{n,m,s} \cdot \sum_j V13_{j,m,t} + V18_{n,m,t} - V19_{n,m,t} + V29_{n,m,t} - V30_{n,m,t} \leq \sum_s NCO_{n,m,s} \cdot A_1 - UO_{n,s} \cdot A_2 \tag{60}$$

$$-HCN_{n,m} \cdot \sum_j V13_{j,m,t} - V18_{n,m,t} + V19_{n,m,t} - V29_{n,m,t} + V30_{n,m,t} - V17_{n,m,t+1} + V20_{n,m,t+1} - V21_{n,m,t+1} + V28_{n,m,t+1} + V29_{n,m,t+1} - V30_{n,m,t+1} \leq HCN_{n,m} \cdot A_1 \tag{61}$$

$$V15_{n,m,t} + V16_{n,m,t} + V17_{n,m,t} - V20_{n,m,t} + V21_{n,m,t} - V28_{n,m,t} - V29_{n,m,t} + V30_{n,m,t} \leq 0 \tag{62}$$

$$V20_{n,m,t} - V21_{n,m,t} - V22_{n,m,t} + V23_{n,m,t} - V24_{n,m,t} \leq 0 \tag{63}$$

$$V20_{n,m,t} - V21_{n,m,t} - V25_{n,m,s,t} + V26_{n,m,s,t} - V27_{n,m,s,t} \leq 0 \tag{64}$$

$$-V23_{n,m,t} + V24_{n,m,t} + \sum_r \gamma_{n,r} \cdot V33_{r,m,t} - V37_{m,t} \leq NCM_{n,m} \cdot A_1 - UM_{n,m} \cdot A_2 \tag{65}$$

$$-V26_{n,m,s,t} + V27_{n,m,s,t} \leq NCO_{n,m,s} \cdot A_1 - UO_{n,s} \cdot A_2 \tag{66}$$

$$V33_{r,m,t} - V33_{r,m,t+1} \leq HCR_{r,m} \cdot A_1 \tag{67}$$

$$-V33_{r,m,t} \leq \sum_w RC_{r,m,w} \cdot A_1 - UR_{r,w} \cdot A_2 \tag{68}$$

The lower bound for objective functions of original problem in this model is the master problem defined by:

$$\text{Min MP} = \sum_t \sum_j \sum_n \sum_m \sum_k TMN_{n,m} \cdot Y_{n,m,t} + \sum_s TON_{n,m,s} \cdot X_{n,m,s,t} + TJ_{j,m} \cdot \theta_{j,m,k,t} \tag{69}$$

$$\Gamma \geq \sum_t \sum_k \sum_m \sum_n \sum_j -V1_{j,k,t} \cdot SL_{j,k,t} \cdot DA_{j,k,t} + DA_{j,k,t} \cdot V2_{j,k,t} - \overline{\theta_{j,m,k,t}} \cdot m \cdot V8_{j,m,k,t} - m \cdot (1 - \overline{\theta_{j,m,k,t}}) \cdot V9_{j,m,k,t} + (DJ_j - \eta \cdot \left(\sum_n TMN_{n,m} \cdot \overline{Y_{n,m,t}} + \sum_s TON_{n,m,s} \cdot \overline{X_{n,m,s,t}} \right)) \cdot V11_{j,m,t} \tag{70}$$

$$+ C_{j,m} \cdot V13_{j,m,t} - mp \cdot V14_{j,m,t} - M \cdot \overline{z_{n,m,t}} \cdot V17_{n,m,t} - M_2 \cdot \overline{z_{n,m,t}} \cdot V18_{n,m,t} - M'_2 \cdot \overline{z_{n,m,t}} \cdot V19_{n,m,t} - M_3 \cdot \overline{z_{n,m,t}} \cdot V20_{n,m,t} - M'_3 \cdot \overline{z_{n,m,t}} \cdot V21_{n,m,t} - \overline{Y_{n,m,t}} \cdot M \cdot V22_{n,m,t} - M \cdot (1 - \overline{Y_{n,m,t}}) \cdot V23_{n,m,t} - \overline{X_{n,m,s,t}} \cdot M \cdot V25_{n,m,s,t} - M \cdot (1 - \overline{X_{n,m,s,t}}) \cdot V26_{n,m,s,t} - M'' \cdot (1 - \overline{z_{n,m,t}}) \cdot V28_{n,m,t} - M_4 \cdot (1 - \overline{z_{n,m,t}}) \cdot V29_{n,m,t} - M'_4 \cdot (1 - \overline{z_{n,m,t}}) \cdot V30_{n,m,t} - mjj \cdot V36_{m,t} - mnn \cdot V37_{m,t} + \sum_n \sum_m IN_{n,m,1} \cdot V17_{n,m,1} - IN_{n,m,1} \cdot V20_{n,m,1} + IN_{n,m,1} \cdot V21_{n,m,1} - IN_{n,m,1} \cdot V28_{n,m,1} - IN_{n,m,1} \cdot V29_{n,m,1} + IN_{n,m,1} \cdot V30_{n,m,1} + \sum_r \sum_m IR_{r,m,1} \cdot V33_{r,m,1} \tag{71}$$

$$\sum_t \sum_k \sum_m \sum_n \sum_j -V1_{j,k,t} \cdot SL_{j,k,t} \cdot DA_{j,k,t} + DA_{j,k,t} \cdot V2_{j,k,t} - \overline{\theta_{j,m,k,t}} \cdot m \cdot V8_{j,m,k,t} - m \cdot (1 - \overline{\theta_{j,m,k,t}}) \cdot V9_{j,m,k,t} + (DJ_j - \eta \cdot \left(\sum_n TMN_{n,m} \cdot \overline{Y_{n,m,t}} + \sum_s TON_{n,m,s} \cdot \overline{X_{n,m,s,t}} \right)) \cdot V11_{j,m,t} \tag{71}$$

$$+ C_{j,m} \cdot V13_{j,m,t} - mp \cdot V14_{j,m,t} - M \cdot \overline{z_{n,m,t}} \cdot V17_{n,m,t} - M_2 \cdot \overline{z_{n,m,t}} \cdot V18_{n,m,t} - M'_2 \cdot \overline{z_{n,m,t}} \cdot V19_{n,m,t} - M_3 \cdot \overline{z_{n,m,t}} \cdot V20_{n,m,t} - M'_3 \cdot \overline{z_{n,m,t}} \cdot V21_{n,m,t} - \overline{Y_{n,m,t}} \cdot M \cdot V22_{n,m,t} - M \cdot (1 - \overline{Y_{n,m,t}}) \cdot V23_{n,m,t} - \overline{X_{n,m,s,t}} \cdot M \cdot V25_{n,m,s,t} \tag{72}$$

$$- M \cdot (1 - \overline{X_{n,m,s,t}}) \cdot V26_{n,m,s,t} - M'' \cdot (1 - \overline{z_{n,m,t}}) \cdot V28_{n,m,t} - M_4 \cdot (1 - \overline{z_{n,m,t}}) \cdot V29_{n,m,t} - M'_4 \cdot (1 - \overline{z_{n,m,t}}) \cdot V30_{n,m,t} - mjj \cdot V36_{m,t} - mnn \cdot V37_{m,t} \tag{73}$$

$$+ \sum_n \sum_m IN_{n,m,1} \cdot V17_{n,m,1} - IN_{n,m,1} \cdot V20_{n,m,1} + IN_{n,m,1} \cdot V21_{n,m,1} - IN_{n,m,1} \cdot V28_{n,m,1} - IN_{n,m,1} \cdot V29_{n,m,1} + IN_{n,m,1} \cdot V30_{n,m,1} + \sum_r \sum_m IR_{r,m,1} \cdot V33_{r,m,1} \leq 0 \tag{72}$$

$$\overline{Y_{n,m,t}} \leq (1 - \overline{z_{n,m,t}}) \tag{72}$$

$$\overline{X_{n,m,t}} \leq (1 - \overline{z_{n,m,t}}) \tag{73}$$

5. Empirical Results

5-1. Empirical results of the BTO model

At this part, the results of solving the model by CPLEX and implementing the Benders decomposition algorithm for the numerical example are illustrated and analyzed. The GAMS software is used to solve the model, and practical experiments are performed by a Pentium® CPU 2117U @ 1.80 GHz computer. Table 1 illustrates the objectives functions(OF) under different importance weights by CPLEX solver. As can be seen from this table, according to the importance of cost, quality, and lead time in each company, the objects and following aggregated OF value are different. The interaction between the two

objects is shown in Fig. 1. It is reasonable that by increasing the quality target, the cost object increases too; in addition, a growth in the lead time object leads to a decrease in the cost object. Some test problems are created randomly to implement the Benders decomposition algorithm. Table 3 shows the detailed empirical results. As the empirical results demonstrate, the Benders decomposition algorithm can be useful for different sizes of problems. Fig. 3 shows the convergence progress of the Benders decomposition algorithm for one instance. The upper and lower bounds in each iteration are given in Table 4, too.

Tab. 1. Computational results under different importance weights of objectives (OFs).

Importance weight of OFs	Aggregated OF value	OF values		
		Cost	Quality	Lead time
(w_1, w_2, w_3)				
(1, 0, 0)	570208.264	570208.264	30625.098	224.000
(0.8, 0.1, 0.1)	453114.003	570226.794	31018.327	236.000
(0.6, 0.15, 0.25)	337554.870	570403.260	31820.574	254.000
(0.4, 0.2, 0.4)	221934.789	570403.260	31820.574	272.000
(0.37, 0.3, 0.33)	201616.554	570403.260	31820.574	263.600
(0.3, 0.3, 0.4)	161712.406	570403.260	31820.574	272.000
(0.17, 0.03, 0.8)	96283.205	570226.794	31018.327	320.000
(0.1, 0.6, 0.3)	38046.027	570420.862	31832.099	260.000

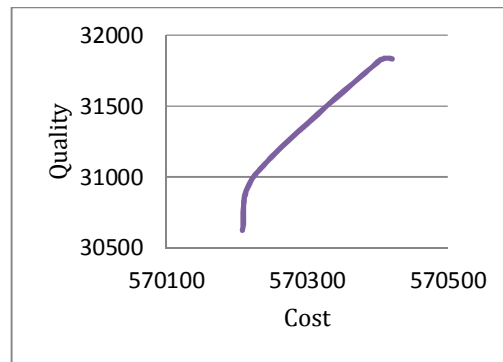


Fig. 1. Reciprocal performance of the quality and cost objectives

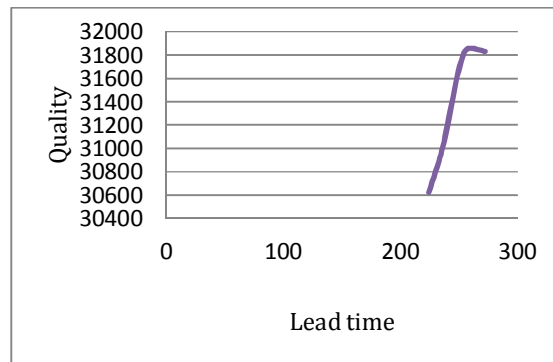


Fig. 2. Reciprocal performance of the quality and lead time objectives

Tab. 3. Implementing BDA

Problem size $ J \times N \times R $ $\times K \times M \times S $ $\times W \times T $	Importance weight of OFs	No. of iterations	CPU time (second)	BDA	
				Lower bound	Upper bound
$2 \times 2 \times 3 \times 2 \times 1 \times 1 \times 1 \times 1$	(0.8, 0.1, 0.1)	14	07.256	340430	340430
	(0.8, 0.05, 0.15)	14	10.694	341633.4	341633.4
$2 \times 3 \times 4 \times 2 \times 1 \times 2 \times 2 \times 1$	(0.8, 0.1, 0.1)	29	16.172	449853.8	449853.8
	(0.8, 0.05, 0.15)	30	15.013	451654.1	451654.1
$2 \times 2 \times 3 \times 2 \times 1 \times 1 \times 1 \times 2$	(0.8, 0.1, 0.1)	40	22.253	682859.6	682859.6
	(0.8, 0.05, 0.15)	40	23.039	685277.8	685277.8
$2 \times 2 \times 3 \times 2 \times 1 \times 2 \times 2 \times 2$	(0.8, 0.1, 0.1)	83	48.376	682859.6	682859.6
	(0.8, 0.05, 0.15)	76	47.616	685277.8	685277.8
$3 \times 4 \times 5 \times 2 \times 1 \times 1 \times 1 \times 2$	(0.8, 0.1, 0.1)	165	92.150	1674774.4	1674774.4
	(0.8, 0.05, 0.15)	164	89.891	1681660	1681660

Tab. 4 Upper and lower bounds in the BDA.

1	-1530997.881	147689.355
2	-1530994.881	147689.355
3	-1530991.881	147689.355
4	-702139.658	147689.355
5	147675.755	147689.355
6	147678.155	147689.355
7	147680.555	147689.355
8	147682.555	981232.928
9	147682.555	564974.573
10	147684.555	562903.376
11	147684.955	147887.473
12	147686.955	562905.776
13	147686.955	147686.955

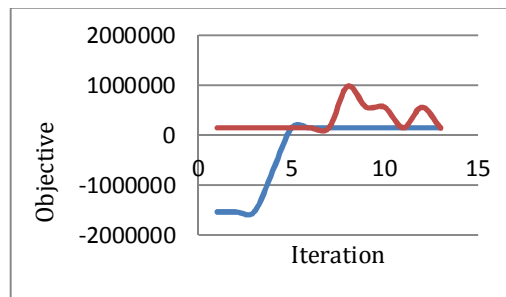


Fig. 3. Convergence of upper and lower bounds

5-1. Empirical results of the scenario-based BTO model

The scenario-based model with cost and quality uncertainties is reformulated by a robust optimization scenario-based method, as stated by Lalmazlounian et al. [5]; then, GAMS software and the Benders decomposition algorithm are used to solve it. The related results are shown in Table 5. According to this table, some test problems are generated with different importance weights solved by CPLEX.

The convergence progress of the BDA for one instance of the robust model is shown in Fig.4.

The upper and lower bounds in each iteration are given in Table 7. The same parameters are used for Tables 1 and 5, the only difference between which is that Table 5 lies in the results of the robust model. It is noticeable that the lead time object has the same values in both tables. It is reasonable since the parameters and decision variables of the lead time object are deterministic. In addition, the same parameters are used for Tables 3 and 6. The robust model has a longer size than the deterministic model because of scenarios; therefore, the iteration of the robust model is smaller than that of the deterministic

model because of using the BDA. It shows that the BDA is capable of being used for solving

large-sized problems.

Tab. 5. Computational results under different importance weights of the robust model.

Importance weight of OFs (w_1, w_2, w_3)	Aggregated OF value	OF values		
		Cost	Quality	Lead time
(1, 0, 0)	868751.219	868751.219	44852.962	224.000
(0.8, 0.1, 0.1)	690550.079	868751.219	44852.962	236.000
(0.6, 0.15, 0.25)	514608.787	868751.219	44852.962	254.000
(0.4, 0.2, 0.4)	338667.495	868751.219	44852.962	272.000
(0.37, 0.3, 0.33)	308039.146	869077.957	45444.061	263.600
(0.3, 0.3, 0.4)	247219.697	869635.202	46028.210	272.000
(0.17, 0.03, 0.8)	146617.318	868751.219	44852.962	320.000
(0.1, 0.6, 0.3)	59164.497	870287.125	46612.360	260.000

Tab. 6. Implementing BDA of the robust model.

Problem size $ J \times N \times R \times K \times M \times S \times W \times T $	Importance weight of OFs	BDA		Lower bound	Upper bound
		No. of iterations	of CPU time (second)		
$2 \times 2 \times 3 \times 2 \times 1 \times 1 \times 1 \times 1$	(0.8, 0.1, 0.1)	3	02.528	448508.3	448499.9
	(0.8, 0.05, 0.15)	3	02.614	449168.7	449156.1
$2 \times 3 \times 4 \times 2 \times 1 \times 2 \times 2 \times 1$	(0.8, 0.1, 0.1)	4	03.134	472058.5	471993.6
	(0.8, 0.05, 0.15)	4	03.328	472536.4	472439.0
$2 \times 2 \times 3 \times 2 \times 1 \times 1 \times 1 \times 2$	(0.8, 0.1, 0.1)	6	04.325	932833.9	932782.4
	(0.8, 0.05, 0.15)	7	05.253	898057.6	898012.9
$2 \times 2 \times 3 \times 2 \times 1 \times 2 \times 2 \times 2$	(0.8, 0.1, 0.1)	8	06.123	898573.1	898530.2
	(0.8, 0.05, 0.15)	10	07.231	898065.9	898012.9
$3 \times 4 \times 5 \times 2 \times 1 \times 1 \times 1 \times 2$	(0.8, 0.1, 0.1)	11	08.754	1514519.	1514214.
	(0.8, 0.05, 0.15)	12	09.130	1488796.	1488475.

Tab. 7. Upper and lower bounds in the BDA of the robust model.

Iteration	Lower bound	Upper bound
1	-1690939.73338473	167223.741915819
2	-1690936.73338473	167223.741915819
3	-774796.449969711	167223.741915819
4	167207.141915819	167223.741915819
5	167209.541915819	167223.741915819
6	167211.541915819	167223.741915819

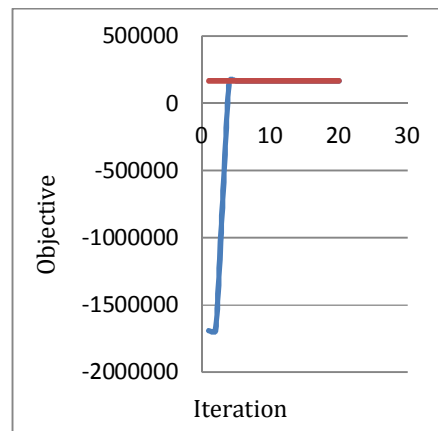


Fig. 4. Convergence of upper and lower bounds

6. Conclusions

In this study, a new BTO supply chain model was proposed. The model was formulated as a multi-objective mixed-integer non-linear programming model to minimize the cost and the lead time and maximize the quality. The problem was modeled under deterministic and scenario-based approaches with cost and quality uncertainties. The scenario-based model was reformulated by a robust optimization scenario-based approach. The return policy and outsourcing were new issues in BTO systems, which were addressed in this paper. The utility function method was used to convert the multi-objective problem to a single-objective problem. Since the model was a large-sized problem and had a decomposed structure, the Benders decomposition algorithm was used to solve the linearized model. At last, the results of deterministic and robust models were analyzed. In addition, a sensitivity analysis was carried out for a demand parameter. The comparison of BTO and MTO models and influence on all the objective functions will be considered for future research. The queue theory can be used in future research. The normal form of game theory that will use one criterion (such as pricing) or multiple criteria (e.g., pricing and quality) with two or multiple players (between two or multiple manufacturers) is another issue for future studies. Another extension can be found in solution method. Double Benders decomposition or nested Benders decomposition or accelerated Benders decomposition can be used for solving the problem.

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