Coordination and Competition in a Duopoly with Two Manufacturers and Two Retailers with a Wholesale Price Contract and Demand Uncertainty

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ABSTRACT

This study analyzes a supply chain involving two competing manufacturers that sell their products through two common competing retailers. The manufacturers' products are the same, yet come out with different brands in the market. The retailers face stochastic demand where demand is the decreasing function of price with an additive uncertain part. Manufacturers compete on supplying orders where retailers compete on selling price. Each manufacturer sets a wholesale price contract with retailers similarly. In this study, the supply chain coordination with the wholesale price contract under competition and demand uncertainty is examined. The analytical results show that, under coordinated conditions, manufacturers do not obtain any positive profit and, consequently, the retailers intend to increase wholesale prices. On the other hand, manufacturers can increase wholesale prices until the retailers' profit becomes zero. Hence, with a numerical study for actual cases, it is found that changing demand sensitivity and competition intensity affects the optimal ordering and pricing decisions. Moreover, increasing competition sensitivity increases supply chains' efficiency, stocking level, and selling price. The concluding remarks show that further investigations are required for exploring the possibility of coordination under competition by other contractual mechanisms.

KEYWORDS: *Supply chain coordination, Competition,Pricing, Wholesale price contract, Supply chain efficiency.*

1. Introduction

Supply chain is defined as a set of economic institutions that work with each other to provide values for end customers in terms of goods or services. Under such collaboration, the chain members want to optimize their profits without considering the others. Thus, the members naturally face deviations from synchronization and coordination between goals and benefits, which is often called the problem of principal-agent in economics [1]. Regarding the literature, the analysis of coordination in supply chain structures focuses on examining how partners can achieve their best profits parallel to providing the maximum level of chain's efficiency [2].

Coordination policy is useful for aligning members' objectives. Supply chain coordination puts the members' objectives in an integrated direction and increases the supply chain's efficiency and market share. There are four main coordination mechanisms: information sharing, joint optimization, information technology, and contracts [3]. Coordination contracts are common in use because they provide simultaneous analysis of quantitative decision-making with qualitative strategic policies. Moreover, they are frequently used in business and industries when they consider aspects of profit and risk-sharing between supply chain partners [4]. The main coordination contracts, which are used in business, are wholesale-price, revenue sharing, quantity discount, buyback, sales rebate, and quantity flexibility contracts [2]. Regarding the main results in the literature, wholesale-price

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contracts result in double marginalization and cannot coordinate supply chains. However, they are frequently used in practice for their simplicity, and many manufacturers seek to set this contract through the channel.

On the other hand, a major portion of the literature concentrates on vertical chains with one or multiple partners. However, actual chains may have one, two, or more echelons with multiple agents at retailer or manufacturer levels. Therefore, for such supply networks, the analysis of competition appears significant besides coordination issues. Jeuland and Shugan (1983) are the first researchers to have considered channel competition in a supply chain. Recently, the intertwined issue of competition and coordination has received much attention by researchers [5]. This stream is partly applicable because of a growth in the new and widespread appearance of economic firms in private sectors. Krishnan and Winter (2012) classified different structures of supply chains under contracting and reviewed the literature basically in two main trends: 1 supplier-n retailer (1-n) and n supplier-1

retailer (n-1) frameworks [6]. They finally analyzed two main supply structures: 1-2 and 2-1. Concerning their classification, the majority of studies in the literature concentrate on either 1-n or n-1 structures.

As an exception, Li et al. (2013) studied a supply chain with two manufacturers and two retailers (2-2) and assumed that price was a function of deterministic demand. In both echelons, there is competition between retailers and manufacturers. They considered wholesale price contract and quantity discount contract for supply chain coordination [7].

Herein, we consider the supply chain with two retailers and two manufacturers (Fig.1). Each echelon has two members; therefore, there is competition between retailers and manufacturers. Two manufacturers produce a similar product with different brands. When their products are quite similar, the competition between manufacturers is quite high. On the other hand, the retailers compete on the price of a product to increase market demand.
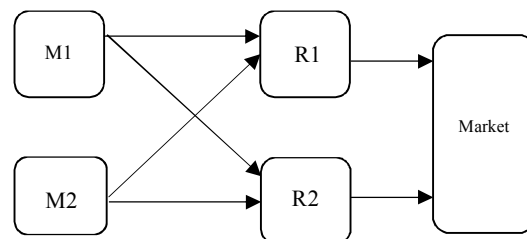

Fig. 1. Structure of competitive supply chains

In addition, price, quality, lead time, and ordering quantity are various decision variables for which supply chain's members compete. Thus, the current study assumes that stochastic or deterministic demand behaviors result in different responses with respect to competition and coordination. Therefore, uncertain demand forms are assumed here, which is the decreasing price.

This paper examines the conditions of competition and coordination in a 2-2 supply network under additive demand uncertainty. This analysis provides highlights for joint pricing and ordering decisions in a more actual structure. A wholesale-price contact is set through channels. Each manufacturer sets the same wholesale price with two retailers, where each retailer can choose different selling prices for each product.

In many cases, competition takes supply chains far away from coordination. Thus, the main objective of the research is to find the limitation

of the wholesale-price contract to provide simultaneous conditions for coordination under competition in a 2-2 structure. Finally, the level of channels' efficiency under competition and demand uncertainty is evaluated.

Therefore, the contribution of the research relies on the investigation of three aspects of supply chain structures: first, analyzing competition and coordination in a more actual supply network structure (2-2), which has not received enough attention in the literature (except [7], the case of deterministic demands); second, assuming joint pricing and ordering decision-making under additive price-demand uncertainty for developing models under competition; third, evaluating the power of contracts, in particular wholesale-price contract, to illustrate the actual limitations of coordination for competing supply chains.

The rest of the paper is as follows: Section 2 reviews the literature of competition and

coordination analysis in supply chains. Section 3 constructs the model under demand uncertainty. Section 4 deals with optimizing the agents' profit functions with competition and provides mathematical conditions. Section 5 analyzes the possibility of coordination under competition for the wholesale-price contract. Section 6 analyzes the limitations of the contract to achieve higher channel efficiency under competition with a numerical study. Finally, Section 7 concludes the analytical and managerial remarks.

2. Literature Review

The main issue of the supply chains, especially in the era of competing chains, is conflicting partners' objectives through the channel. Coordination is a perfect tool to align the members' objectives [8] with different mechanisms such as contracts and joint decision-making [3]. According to [2], there are different contracts to coordinate supply chain including wholesale-price, revenue sharing, quantity discount, buyback, quantity flexibility, and sales rebate contracts.

In fact, coordination helps supply chain members to be closer and centralized to each other [9]. In a coordinated system, decision variables such as pricing, production planning, and stocking can be integrated at both manufacturer and retailer levels [10]. Moreover, the coordinating strategy improves the performance of a supply chain in upstream and downstream to synchronize ordering quantities with total profit chain optimized level under demand fluctuations [11; 2]. Thus, achieving coordination for chain partners not only protects their individual profits but also shapes a win-win region in terms of decision variables, resulting in the total chain maximum profit level. This condition facilitates better channel strength in order to absorb the maximum level of market shares and competitive advantage with respect to the existing and emerging competing chains in the market.

For this reason, there is a dilemma under a competitive atmosphere for coordinating channels that should be resolved: Are there possible conditions to establish coordination mechanisms under actual competition? The main pioneer of analyzing competition in supply chain is the work of Jeuland and Shugan (1983) [5], who studied competing supply chains with deterministic price-dependent demand.

Regarding the classification done by the referenced study [6], the main trends for analyzing coordinating contracts under competition are divided into the following: 1 supplier-n retailer (1-n) and n supplier-1 retailer (n-1) structures. They argued that the majority of studies in the literature concentrate on 1-n or n-1 structures.

Herein, this study reviews related recent works in the literature based on the structural classification of Krishnan and Winter [6]. Li et al. [7] studied a supply structure with one retailer and multiple manufacturers (1-n), where demand function is stochastic. They analyzed the wholesale-price contract as a coordination mechanism when manufacturers compete on supplying order.

Chen [13] analyzed a two-echelon supply chain with one manufacturer and one retailer (1-1). Manufacturers can sell their products in the market with a direct channel. Accordingly, retail channel and direct channel compete with each other for pricing under deterministic price-dependent demand. Mahmoodi Eshghi [14] modeled two independent supply chains in which they set the wholesale-price contract for coordination and competition for pricing. Xiao et al. [15] considered two integrated supply chains with deterministic demand (1-1). They competed on price and lead time. Each supply chain has one manufacturer and one retailer. In their study, the wholesale price contract and quantity discount contract were compared for achieving coordination. A similar structure was analyzed by Esmaeili et al. [16] where pricing, advertising, and servicing for a two-echelon supply chain were considered.

In a study by Chakraborty et al. [17], two manufacturers compete on pricing with wholesale-price and revenue sharing contracts and deterministic demand (2-1). Chakraborty et al. [18] additionally developed such a structure with a cost-sharing mechanism, and showed that this type of contract could achieve coordination under competition and deterministic demand pricing.

Glock and Kim [19] considered a supply chain with one manufacturer and multiple retailers (1-n) that compete on price with price-dependent deterministic demand. Xie [20] analyzed a supply chain with two competing suppliers and one manufacturer (2-1). Two suppliers compete on providing better energy efficiency to protect environmental stability. Huang et al. [21] studied a supply chain with two competing retailers and one manufacturer (1-2). The manufacturer

chooses the wholesale-price contract with price-dependent deterministic demand.

Saghaeeian et al. [22] analyzed the optimization of pricing, production, and transportation decisions for two horizontal competing supply chains with one retailer. Using a bi-level nonlinear programming, they analyzed the effect of self-price and cross-price effects in the competition on the channels' profits. Asl-Najafi et al. [23] investigated the coordinated conditions of a bi-level closed-loop supply chain. They showed that coordination could be achieved where economical and green transportation modes were considered. Jena et al. [24] developed a model for two competing manufacturers and one retailer in a closed-loop supply chain. They showed that the collaboration of the retailer with a high-brand manufacturer was more profitable.

Lan et al. [25] analyzed the effect of competition in the upstream including two competing manufacturer-distributer in a three-echelon supply chain where the retailer in the downstream faced demand uncertainty with fixed pricing. They concluded that an increase in uncertainty that exceeds the threshold results in coordination under competition.

The work of Li et al. [7], which is quite closely related to the proposed model of ours, studied a supply chain structure with two manufacturers and two retailers (2-2). They studied different streams including two competing supply chains with one manufacturer, one exclusive retailer, and additionally one manufacturer with two common retailers. The manufacturer produces a competitive product and sells it through common or exclusive retailers. Two products are the same, yet produced by two different manufacturers (brands). When the retailers are common between two manufacturers, each retailer faces the pricing of two products with different brands. They consider the contract choice game between wholesale-price and quantity discount contracts to coordinate the supply chain. They assume that each manufacturer can select a contract that is the same with, or different from, another manufacturer's contract. For example, the manufacturer m1 can choose a quantity discount contract or a wholesale price contract, in which the manufacturer m2 can choose a quantity discount contract or a wholesale price contract. Considering the contract choice game, they found that retailers and manufacturers were more inclined to choose what kind of contract. As a

result, they found analytical conditions in which the supply chain could achieve coordination under competition.

According to the literature of competitive supply chains, there has been no study on competition in two-echelon supply chains with two manufacturers and two retailers by considering demand uncertainty. Therefore, This study analyzes the effect of competition on coordinating the structure of the supply chain (Fig. 1) with demand uncertainty.

3. The Model

In this study, a supply chain with two manufacturers and two retailers that compete on ordering and pricing decisions through supply chains are considered. The two manufacturers produce two products that are substitutable in the market, and perfect substitutability is considered when competitive intensity at the manufacturer level is high (around 1).

3-1. Assumptions, parameters, and decision variables

The basic assumptions about the model are presented as follows:

- Demand is stochastic and price dependent.
- Retailers sell leftover inventory at the salvage value
- Manufacturers should pay shortage cost for each lost/failed sale.
- Each manufacturer offers the same wholesale price to both retailers.
- Selling price of each product for any retailer would be set different.

In addition, the variables and parameters are presented below:

i The index for products/manufacturers
j The index for retailers
ij The combined index for item ij which is referred to product i when it is sold through retailer j
D_{ij} Demand for item ij
a_{ij} The constant of demand function for item ij
v_{ij} Salvage value for item ij
g_{ij} Lost sales' penalty cost for item ij
c_i The manufacturer cost for product i
a_{ij} Market volume for item ij
b_{ij} Price sensitivity of demand for item ij
x_i Competitive intensity at the manufacturer level
θ_i Competitive intensity at the retailer level

q_{ij}	Ordering quantity for item ij (decision variable)
p_{ij}	Retailing price for item ij (decision variable)
w_i	Wholesale price of manufacturer i (decision variable)
ε_{ij}	Random demand part for item ij
z_{ij}	Stocking level for item ij
F_{ij}	Cumulative distribution function for item ij
f_{ij}	Density function for item ij
μ_{ij}	Mean of uncertain demand for item ij

3-2. Demand function

The demand function is uncertain with additive uncertainty [26]. There is also similar research in this field for a 1-2 structure with deterministic demands (In this study, unlike [7]). With additive demand uncertainty and duopoly structure, demand for item ij is assumed as a decreasing function of the retail price for retailer j. Because of competition between retailers and manufacturers, the price of product i at retailer $3 - j$ and price of product $3 - i$ at retailer j and $3 - j$ have a positive effect on the demand for product i at retailer j. The competition between manufacturers decreases the effect of retailers' competition and, therefore, demand function of product i for retailer j is as follows:

$$D_{ij} = a_{ij} - b_{ij}p_{ij} + (1 - \theta_i)x_i p_{i,3-j} + \quad (1)$$
$$\theta_i(1 - x_i)p_{3-i,j} + \theta_i x_i p_{3-i,3-j} + \varepsilon_i$$

According to Pettruzi and Dada (1999), the following equations are used to simplify the model:

$$D_{ij} = y_{ij}(p) + \varepsilon_{ij} \quad (2)$$
$$z_{ij} = q_{ij} - y_{ij}(p) \quad (3)$$

The random part of demand, ε_{ij}, has probability distribution function, $f(\cdot)$, and cumulative distribution function, $F(\cdot)$, on $[A_{ij}, B_{ij}]$

3-3. The supply chain partners' profit functions with a wholesale-price contract

Each manufacturer sets a wholesale price contract with both retailers independently. Each retailer buys a product from two manufacturers with a specified wholesale price. Demand is uncertain and retailer faces shortage or leftover during the selling period. Thus, the j retailer's profit and expected profit functions are written as follows:

$$\pi_j(q_{ij}, p_{ij})$$
$$= \begin{cases} \sum_{i=1}^{2} p_{ij}(D_{ij}) - w_i Q_{ij} + v_{ij}(Q_{ij} - D_{ij}) \quad (4) \\ \sum_{i=1}^{2} p_{ij}(Q_{ij}) - w_i Q_{ij} - g_{ij}(D_{ij} - Q_{ij}) \end{cases}$$

$$E(\pi_j((z_{ij}, p_{ij}))) = \quad (5)$$
$$\sum_{i=1}^{2} \{ \int_{A_{ij}}^{z_{ij}} [p_{ij}(y_{ij}(p) + u_{ij}) - w_i(z_{ij} + y_{ij}(p)) + v_{ij}(z_{ij} - u_{ij})]f(u)du + \int_{z_{ij}}^{B_{ij}} [p_{ij}(z_{ij} + y_{ij}(p)) - w_i(z_{ij} + y_{ij}(p)) - g_{ij}(u_{ij} - z_{ij})]f(u)du \}.$$

The equation can be simplified as follows:

$$E(\pi_j((z_{ij}, p_{ij}))) = \sum_{i=1}^{2} \{ \psi_j(p_{ij}) - \quad (6)$$
$$L_j(p_{ij}, z_{ij}) \}$$

where the risk-less profit functions, (7), and the loss functions, (8), can be decomposed (referring [26]) for duopolistic chains:

$$\psi_j(p_{ij}) = (p_{ij} - w_i)[y_{ij}(p) + \mu_{ij}] \quad (7)$$
$$L_j(p_{ij}, z_{ij}) = (w_i - v_{ij})\Lambda_j(z_{ij}) + \quad (8)$$
$$(p_{ij} - w_i + g_{ij})\Theta_j(z_{ij})$$

In the above decomposition, $\Lambda_j(z_{ij})$ addresses the probability of leftover and $\Theta_j(z_{ij})$ is the probability of shortages:

$$\Lambda_j(z_{ij}) = \int_{A_{ij}}^{z_{ij}} (z_{ij} - u_{ij})f(u)du \quad (9)$$
$$\Theta_j(z_{ij}) = \int_{z_{ij}}^{B_{ij}} (u_{ij} - z_{ij})f(u)du \quad (10)$$

Moreover, the manufacturers' profit function is written as follows:

$$\pi_i(q_{ij}) = \sum_{j=1}^{2} (w_i - c_i) Q_{ij} \quad (11)$$

It is obviously assumed that $w_i \geq c_i$ because of profitability at the manufacturer level.

4- Supply Chain Optimization Under Competition

Herein, the optimal ordering and pricing decisions for total chains, retailers, and manufacturers are determined.

4-1. The retailers' problem optimization

Two retailers compete on price and ordering quantity. Thus, the optimal pricing decisions, p_{ij}^*, and optimal ordering quantity, q_{ij}^*, are calculated for any item ij. In addition, according to (3), the socking decisions, z_{ij}^*, can be optimized instead of q_{ij}^*. Therefore, the profit maximization problem of retailer j is as follows:

$$\text{Maximize} \underset{st. p_{ij}, z_{ij}}{E(\pi_j((z_{ij}, p_{ij})))}$$

Theorem 1 shows the optimal pair of ordering and pricing solutions for the retailers' profit function.

Theorem 1. The optimal pair of ordering and pricing decisions for the retailer j in a duopolistic structure is (q_{ij}^*, p_{ij}^*), where $q_{ij}^* = y(p_{ij}^*) + z_{ij}^*$:

$$1 - F(z_{ij}^*) = \frac{(w_i - v_{ij})}{p_{ij} - v_{ij} + g_{ij}} \tag{12}$$

$$p_{ij}^* = [a_{ij} + \mu_{ij} + (1 - \theta_i)x_i p_{i,3-j} + \theta_i(1 - x_i)p_{3-i,j} + \theta_i x_i p_{3-i,3-j} + b_{ij}w_i + \Theta_j(z_{ij})]/2b_{ij} \tag{13}$$

Proof. The first optimality conditions for the retailer's profit maximization problem are as follows:

$$\frac{\partial E(\pi_j((z_{ij}, p_{ij})))}{\partial p_{ij}} = \sum_{i=1}^2 a_{ij} - 2b_{ij}p_{ij} + (1 - \theta_i)x_i p_{i,3-j} + \theta_i(1 - x_i)p_{3-i,j} + \theta_i x_i p_{3-i,3-j} + \mu_{ij} + b_{ij}w_i + \Theta_j(z_{ij}) = 0 \tag{14}$$

$$\frac{\partial E(\pi_j((z_{ij}, p_{ij})))}{\partial Z_{ij}} = \sum_{i=1}^2 -(w_i - v_{ij}) - (p_{ij} - v_{iJ} + g_{ij})(1 - F(z)) = 0 \tag{15}$$

In addition, the second optimality conditions show that:

$$\frac{\partial^2 E(\pi_j((z_{ij}, p_{ij})))}{\partial p_{ij}} = -2b_{ij} < 0 \tag{16}$$

$$\frac{\partial^2 E(\pi_j((z_{ij}, p_{ij})))}{\partial Z_{ij}} = \sum_{i=1}^2 -(p_{ij} - v_{iJ} + g_{ij})f(z_{ij}) < 0 \tag{17}$$

The second optimality conditions confirmed that the retailers' profit maximized with the optimal pair solutions, which are computed through (14) and (15). ∎

4-2. The supply chain's problem optimization

Optimal decisions concerning the supply chain must be made to perform coordination analysis.

In this research, there are two different supply chain structures: vertical supply chain and total supply chain. In the vertical supply chain, one manufacturer interacts with two retailers. Therefore, there exist two vertical supply chains. Furthermore, in terms of the total supply chain, there are two manufacturers and two common retailers.

The profit function of vertical supply chains is as follows ($for\ i = k$):

$$\pi_{sk} = \pi_k + \sum_{j=1}^2 \pi_j \tag{18}$$

Thus, the optimal pair of stocking and pricing solutions for vertical supply chain k can be developed through the first conditions as follows:

$$\frac{\partial \pi_{sk}}{\partial z_{ij}} = \sum_{i=1}^2 \sum_{j=1}^2 [(p_{ij} - w_i + g_{ij}) - F(z_{ij})(p_{ij} - v_{ij} + g_{ij})] + 2(w_i - c_i) = 0 \tag{19}$$

$$\frac{\partial \pi_{sk}}{\partial p_{ij}} = \sum_{i=1}^2 \sum_{j=1}^2 [\mu_{ij} - 2b_{ij}p_{ij} + a_{ij} + (1 - \theta_i)x_i p_{i,3-j} + \theta_i(1 - x_i)p_{3-i,j} + \theta_i x_i p_{3-i,3-j} + b_{ij}w_i - \varphi_{ij}(z_{ij})] - \sum_{j=1}^2 b_{ij}(w_i - c_i) \tag{20}$$

Although the concept of coordination can be evaluated by comparing the isolated firms' optimal decisions with vertical chains, herein it is developed for the total supply chain. In this respect, it is assumed that there is a central decision-maker that considers all aspects of the collaboration at the same time. The profit function of the total chain is defined as follows:

$$\pi_{st} = \sum_{i=1}^2 \sum_{j=1}^2 \pi_i(q_{ij}) + E(\pi_j((z_{ij}, p_{ij}))) \tag{21}$$

Theorem 2 presents the optimal solutions of total chains' profit with respect to pricing and ordering decisions.

Theorem 2. The optimal pair of pricing and ordering decisions for the total supply chain is developed by (q_{ij}^0, p_{ij}^0), where $q_{ij}^0 = y(p_{ij}^0) + z_{ij}^0$:

$$p_{ij}^0 = [a_{ij} + \mu_{ij} + (1 - \theta_i)x_i p_{i,3-j} + \theta_i(1 - x_i)p_{3-i,j} + \theta_i x_i p_{3-i,3-j} + b_{ij}c_i + \Theta_{Rj}(z_{ij})]/2b_{ij} \tag{22}$$

$$1 - F(z_{ij}^0) = \frac{(c_i - v_{ij})}{p_{ij} - v_{ij} + g_{ij}}. \tag{23}$$

Proof. The first optimality conditions for the supply chain's profit maximization problem with respect to pricing and stocking variables are as follows:

$$\frac{\partial \pi_{st}}{\partial z_{ij}} = \sum_{j1}^{2} \sum_{i=1}^{2} -(w_i - v_{ij}) - (p_{ij} - v_{iJ} \qquad (24)$$
$$+ g_{ij})(1 - F(z))$$
$$+ (w_i - c_i) = 0$$

$$\frac{\partial \pi_{st}}{\partial p_{ij}} = \sum_i \sum_j \mu_{ij} - 2b_{ij}p_{ij} + a_{ij} + \qquad (25)$$
$$(1 - \theta_i)x_i p_{i,3-j} + \theta_i(1 - x_i)p_{3-i,j} +$$
$$\theta_i x_i p_{3-i,3-j} - \varphi_{ij}(z_{ij}) + b_{ij}(w_i - c_i) = 0$$

Concerning the second optimality conditions, it is obvious that:

$$\frac{\partial^2 \pi_{st}}{\partial p_{ij}} = -2b_{ij} < 0 \qquad (26)$$

$$\frac{\partial^2 \pi_{st}}{\partial z_{ij}} = \sum_{i=1}^{2} -(p_{ij} - v_{iJ} + g_{ij})f(z_{ij}) < 0 \qquad (27)$$

Thus, the second conditions in pricing and ordering show that the supply chain's profit is maximized with an optimal pair of pricing and stocking levels in (25) and (26).

5- Supply Chain Coordination

Supply chain coordination is the result of systematic collaboration and decision-making by all partners of the chain. Under coordination, the chain partners decide on strategic and tactical measures simultaneously. Thus, collaborative mechanisms, such as contracts, make these agreements easier. Herein, a basic wholesale-price contract is set for the chains and, therefore, the results of simultaneous optimal decision-making in pricing and ordering for isolated firms and also the chains (both vertical and total chains) are evaluated.

5-1. Vertical chains' coordination
Herein, it is assumed that each vertical supply

$$w_i = c_i \qquad (28)$$

chain has one manufacturer and two retailers. In this case, when the vertical supply chain is coordinated, the retailers' optimal pricing and stocking decisions should be equal to k supply

chain's optimal decisions. Theorem 3 represents a sufficient condition for vertical chain coordination.

Theorem 3. The vertical supply chains achieve coordination if the manufacturers' marginal profit becomes zero, i.e.,

Proof. The sufficient equalities for coordination, $z_{ij}{}^* = z_{ij}{}^{sk}$, $p_{ij}{}^* = p_{ij}{}^{sk}$, from Relations (12), (13), (22), and (23) result in (28).∎

Regarding the coordinated condition, the wholesale price of manufacturers should be equal to manufacturers' marginal cost in two vertical supply chains. Thus, for achieving coordination in vertical chains, the manufactures should neglect their profitability, which is not rational for them.

5-2. Total supply chain coordination
Total supply chain coordination can be achieved by similar equalities in Theorem 3 that is considered by total chains' optimal decisions. Theorem 4 represents the coordinated condition for the total supply chain.

Theorem 4. The total supply chain achieves coordination if the manufacturers' marginal profit becomes zero, i.e.,

$$w_i = c_i \qquad (29)$$

Proof. The proof is similar to that of Theorem 3 considering $p_{ij}{}^* = p_{ij}{}^{st}$ and $z_{ij}{}^* = z_{ij}{}^{st}$. ∎

Theorems 3 and 4 conclude that, with the wholesale-price contract, the marginal profit of manufacturers becomes zero for both cases of vertical and total chain coordination. Thus, the retailers obtain all supply chain's profit. Therefore, retailers attempt to utilize the wholesale-price contract, where manufacturers seek to leave it because of double marginalization. Nevertheless, the actual cases show the frequent application of the wholesale price contract for its simplicity [4]. Thus, in the next section, the actual supply chains' efficiencies are evaluated under the wholesale-price contract and demand uncertainty for a duopolistic structure.

6- Numerical Study

In this section, a numerical study for the case of additive demand uncertainty with uniform demand random part is designed. It is assumed here that all parameters for two retailers and two manufacturers are similar, i.e., $z_{ij} = z$, $p_{ij} = p$, $a_{ij} = a$, $b_{ij} = b$, $v_{ij} = v$, $g_{ij} = g$, $c_i = c$,

and $w_i = w$. This assumption is considered by [26] for a supply network. Accordingly, the demand function is simplified as follows:

$D = a + Lp$
where
$L = -b + x + t - tx$.

Thus, optimal pricing and stocking solutions for retailers can be rewritten as follows:

$p = \frac{a + \mu - Lw - 50 + z - \frac{z^2}{200}}{-2l}$

$F(z) = \frac{(p - w + g)}{(p - v + g)}$

Since retail prices should be positive, we have $L < 0$. Therefore, the following constraint should be satisfied in numerical cases:

$b > x + t - tx$ (30) .

The parameters of the problem are set as follows:
$a = 5000, c = 500, v = 450, g = 700, x = 0.5, t = 0.5, \varepsilon \in [0,100]$, $f(\varepsilon) = \frac{1}{100}$, and $F(\varepsilon) = \frac{\varepsilon}{100}$. Considering constraint (30), it should be considered that $b \geq 0.75$. Hence, we set $b = 0.8$ through the study.

In $w = c = 500$, the supply chain is coordinated. The solutions of the model for a Stackelberg game under the coordinated condition are summarized in Table 1. The solution is developed using Matlab 2016.

Tab. 1. The optimal solutions of the supply chains' partners for the wholesale price contract under coordination ($w = 500, x = 0.5, t = 0.5, b = 0.8$)

p	50749.99
z	99.90
q	2562.40
π_R	252501254.90
π_M	0
π_{SC}	505002509.80

The optimal solutions in Table 1 show that, in a coordinated supply chain, manufacturers do not obtain positive profits and all profits belong to retailers. Therefore, in competitive supply chains, retailers desire to set a wholesale price contract where manufacturers attempt to increase the wholesale price level from marginal costs for manufacturers. Thus, higher wholesale prices decrease retailers' profits. Moreover, manufacturers can increase their wholesale prices until the retailers' profit becomes zero. Consequently, the optimal wholesale price level is computed as follows:

$\max w$
$s.t.$
$\pi_R \geq 0 , \ q \geq 0 , \ \pi_{SC} \geq 0$ (31)

where π_R, π_M, and π_{SC} are the supplier, retailer, and supply chain profits in the symmetric design of this numerical study, respectively.

In the study, the optimal wholesale price for two manufacturers becomes $w^* = 87408$ according to Constrain (31). By considering w^*, the other decision variables are changed those in Table 2.

The results of Tables 1 and 2 show two extreme points for the spectrum of competitive supply chain collaborations with the wholesale price contracts. The results of Table 1 show the solutions for the coordinated chain, where manufacturers obtain zero profits. Conversely, the results of Table 2 show profitable manufacturers with non-profitable retailers, who leave the coordinated conditions.

Tab. 2. The chains' partners optimal decisions with profitable manufacturers ($w^* = 500, x = 0.5, t = 0.5, b = 0.8$)

p	99192.84
z	1.54
q	41.90
π_R	5.29
π_M	8201072.83
π_{SC}	3159402.42

However, in reality, there exist possible cases between two of these extreme points. Therefore, changing x, t, and b under (30) provides optimal coordinated and non-coordinated supply chains in order to investigate the possible efficiencies.

For this investigation, this study considers two values for competition sensitivity between manufacturers, $x \in (0.25, 0.5)$, and competition sensitivity between retailers, $t \in (0.25, 0.5)$. According to (30), price sensitivity demand, b, can be calculated for different assumed levels of x and t. The results are developed in Table 3.

The maximum amount of the wholesale price is set for each case of x, t, and b by manufacturers. Then, in the second round, retailers determine their optimal pricing and stocking decisions. Finally, the supply chain's profit can be calculated by the partners' optimal decisions. It is known that the obtained values do not achieve coordination. Thus, the optimal values for isolated partners can be compared with the profit levels of the ideal (coordinated) supply chain (Table 1). Therefore, the supply chains' efficiencies can be reported in the right column of Table 3.

Tab. 3. Coordination analysis with respect to changing competition sensitivity factor between manufacturers x and retailers t and price sensitivity demand factor b

Parameters			Uncoordinated supply chain				Coordinated supply chain			
x	t	b	p	z	w_{max}	π_{sc}	p^0	z^0	π_{sc}	Ef
		0.5	79279.9	1.82	78531	14615459.	40649.99	99.87	402995637.22	3.62
	0.25	1	8666.32	10.54	8426	4303637.3	4738.88	98.99	40418503.0	10.64
0.2		1.5	4553.17	18.07	4385	2802751.5	2626.46	98.26	19208151.76	14.59
		0.7	66011.1	2.08	65328	13291044.	33916.66	99.85	334992097.96	3.96
	0.5	1	13053.8	7.50	12755	5506809.9	6983.33	99.30	63040485.79	8.73
		1.5	5542.23	15.33	5354	3221325.8	3135.70	98.52	24304611.96	13.25
		0.8	28149.9	4.02	27706	8464487.4	14678.56	99.66	140712354.92	6.01
	0.25	1	13053.8	7.50	12755	5506809.9	6983.33	99.30	63040485.79	8.73
0.5		1.5	5542.23	15.33	5354	3221325.8	3135.70	98.52	24304611.96	13.25
		0.9	32876.3	3.56	32396	9200622.2	17083.33	99.71	164994195.51	5.57
	0.5	1	19651.4	5.37	19282	6950287.7	10349.99	99.52	97012547.16	7.16
		1.5	6478.19	13.45	6273	3575120.4	3616.66	98.70	29130962.64	12.27

The results show that increasing price sensitivity of demand makes the retail prices go lower, which is due to the decrease of supply chain's demands by increasing selling prices. In addition, high values of b result in low wholesale prices. This is because of demand increase where the retailers face market demands, which make them order more and use the price-sensitivity profits when wholesale prices are lower.

Furthermore, by increasing the competition sensitivity between manufacturers, x, the selling pricing levels and the wholesale prices increase. This also occurs because of the duopolistic situation of the supply chains, where it is anticipated that, by participating in new chains in the market, the pricing levels go lower than duopolistic levels.

According to Table 3, by increasing the demand sensitivity of prices, b, market demand decreases at high retail prices. Thus, retailers attempt to decrease the selling prices. Changing directions of selling prices and wholesale prices are in the same direction. When wholesale price increases,

retailers increase their selling price in order to prevent lower marginal profits.

In addition, supply chain efficiencies increase by increasing competition sensitivity at both chain levels, because, under competition, chain partners focus on achieving more market share instead of promoting channels' efficiencies.

In terms of contract analysis, with the wholesale price contract, the manufactures select high wholesale prices to earn positive marginal profits. This increases the supply chain's ordering quantity to more than retailers' ordering level. Therefore, the supply chain will not be coordinated with the high wholesale price. Moreover, the efficiency levels show that wholesale price contracts provide not strong enough orders to make powerful economic relationships. Thus, for further research, another contractual mechanism should be analyzed under competition and demand uncertainty.

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Coordination and Competition in a Duopoly with Two Manufacturers and Two Retailers with a Wholesale Price Contract and Demand Uncertainty

7- Conclusion

This study considered a competitive supply chain with two manufacturers and two retailers under demand uncertainty. There was competition at both manufacturer and retailer levels. In order to coordinate supply chains, the wholesale price contract was used. After finding the optimizing decisions for maximizing retailers' and manufacturers' profits, supply chains' optimal pair of ordering and pricing was developed in order to conduct analysis coordination.

The results showed that, in both cases of vertical and total supply chain coordinations, retailers earn all supply chains' profit and manufacturers are left with no profits. In addition, retailers seek to set a wholesale price contract with manufacturers; however, manufacturers may want to increase the wholesale prices to maximize their profit.

For actual cases between two extreme points of profitable manufacturers and retailers, a numerical study was designed with uniform demand uncertainty. For two values of competition sensitivity and tree values of price sensitivity demand, the maximum wholesale price for manufacturers was computed. The results showed that when wholesale price increased, the order quantity of retailers became less than that of supply chain's and the supply chain efficiency decreased. Additionally, by increasing the competition sensitivity, an increase in the supply chain efficiency, stocking level, and selling price increased.

Furthermore, when price sensitivity demand increased, the selling price and maximum wholesale price decreased and stocking level increased; however, supply chain's profit decreased, because, for increasing demand, retailers decrease selling price and, also, decrease partners' profit.

For future research, revenue sharing and quantity discount contracts can be used for supply chain coordination under competition. In addition, the manufacturers can select different contract types for each retailer regarding actual cases.

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