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RESEARCH PAPER

A Fuzzy Multi-objective Model for Order Allocation to Suppliers under Shortfall and Quantity Discounts

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ABSTRACT
For new strategies of purchase and production process, suppliers play a key role in achieving competitive capabilities for large-sized companies. The selection of suitable suppliers is a critical component of this strategy. The problem of allocating order to suppliers is a multi-objective one that includes fuzzy parameters; in addition, suppliers usually consider discount in the case of different levels of purchase amount. Since there is no multi-objective fuzzy model for order allocation in the literature to consider discount and shortfall simultaneously in the planning horizon of multiple products, this research proposes a new model that includes minimization of costs, delays, and the percentage of defective parts as objective functions. Price, demand, delay, and percentage of defective parts are considered fuzzy parameters. Since the model is NP-hard, the two metaheuristic algorithms, NSGAII and MOPSO, have been used for solving the problem with tuned parameters using Taguchi method. According to the results of numerical problems, the proposed algorithms can provide a good approximation of the optimal solutions.

KEYWORDS *Supplier selection, Order allocation, Discount, Genetic algorithm, Particle swarm optimization algorithm.*

1. Introduction

In recent decades, many companies have faced the challenge of purchasing management in a supply chain; therefore, the need for accessing a suitable level of global competition in supply chain has increased substantially. Since suppliers have a significant impact on the success or failure of a company, purchasing management, which was almost considered as a merely technical tool, is considered a strategic task (Van der Vlist et al., 1997). The decisions about supplier selection involve answering questions such as (a) what type of a supplier should be selected as a source purchase and (b) how order values should be allocated among the selected suppliers.
Among many other issues in the real world, uncertainty is an important part of the supplier selection problem. To make an effective and harmonic configuration of the supply chain, the ambiguity of existing information coming from implicit and explicit factors in suppliers' selection issue must be considered. Peidro et al. (2009) conducted the classification and a detailed review of qualitative techniques of supply chain planning under uncertainty.
In general, the selection of suppliers is expressed in two forms: single-source and multi-source. In the single-source form, a supplier can satisfy all buyers' needs including number, quality, delivery time, etc. In the second type of supplier selection, none of the suppliers will be able to meet all the needs of the buyers. Therefore, some restrictions such as the capacity of the buyer, quality, etc. must be investigated in the supplier selection process. In such terms, management divides his order quantity between different suppliers (Demirtas and Ustun, 2008).
In this research, supplier selection and order allocation are considered as the multi-criteria decision-making problems along with uncertain information. Since there are some uncertainties concerning price, demand, delays, and percentage of defective parts, they are considered fuzzy. The approach presented by Jimenez (2007) was used to make these parameters defuzzied. In addition, a comprehensive approach along with three objectives of minimizing price, delays, and defective parts is considered for the proposed model.

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In the following, first, the respective literature will be reviewed. Then, in Section 3, fuzzy multi-objective supplier selection is described in detail. Then, this model is converted to a deterministic model using Jimenez (2007) approach. In Section 4, NSGAII and MPSO approaches are developed to solve the problem, and the way these parameters are set using the Taguchi approach is described. Finally, in Section 5, the results of the implementation of two algorithms are compared with Epsilon constraints using a numerical example; in this regard, suggestions are recommended for future study.

2. Literature Review

The problem of supplier selection is proposed in two forms: single-source and multi-source. An appropriate method to ensure the functionality of supplyed services by the manufacturer is multiple-supply policy. In this condition, a company buys an item from several suppliers. Even if this choice requires greater flexibility of the company, it is attractive for some reasons such as discount or possible constraints in the field of making capacity, quality, delivery, price, etc., in case the provider fails to meet the customer demands. In this situation, mathematical programming is the best technique for formulating the problem of supplier selection and order allocation. Gaballa was the first author to use this technique for supplier selection in real cases. In 1974, he applied fractional integer programming to formulate the decision-making problem for Australia Post (Ghodsypour and O'Brien, 2001).

Tab. 1. Literature review of supplier selection and order allocation

Author(s)	Year of publication	Problem type		Objective type		products		period		discount		Inventory control		Solving method	
		Selecting supplier	Selecting and allocating order	Single-objective	Multi-objective	Single-product	Multi-product	Single-time	Multi-time	without	With	Yes	No	exact	Heuristic
Ebrahimi R. M. et al.	2009				■	■		■		■			■	■	
Amid et al.	2011		■		■			■		■			■	■	■
Kara	2011			■		■		■		■			■	■	■
Glock, C.H. et al.	2012		■		■	■		■			■		■	■	
Torabi et al.	2015	■		■		■		■		■		■		■	
Moghaddam	2015				■	■		■					■	■	
Memon et al.	2015				■	■		■					■	■	
Paydar et al.	2016		■		■		■		■	■		■		■	
Qin et al.	2017		■		■		■		■	■		■		■	
Torabi & Boostani	2018	■		■		■		■		■			■	■	
Ekhtiari et al.	2018		■		■		■		■	■			■		■
Abdel-Basset et al	2018		■		■	■		■		■			■	■	■

Chaudry et al. (1993) used linear programming for both single-source and multiple-source problems to minimize the order cost for individual items during the horizon time. The authors sought a number of objectives such as establishment of continuity in production capacity, suitable performance of delivery, quality, and a small discount. To generalize this objective, different models have been proposed. Besides the quantitative and commercial discounts, the third strategy of discount is related to multi-item models. In fact, in this method, the price of each item depends on the quantity order of other items. This problem occurs when two or more related items are sold. Resenthal et al. (1995) were the first researchers to use this method for supplier selection. They applied a linear programming method to minimize the ordering cost and create a continuous chain of supply capacities, customer satisfaction, and product quality and delivery simultaneously. The authors recommended developing and generalizing the export products in the form of EOQ.

Literature review of supplier selection and order allocation is presented in Table 1. According to the table, the most recent research studies have focused on supplier selection and order allocation under single-objective and single-product states. In addition, in the multi-objective researches, the discount and deficiency are not considered at the same time in the planning time horizon. Therefore, this study proposes a fuzzy multi-objective model with discount and deficiency policy and uses meta-heuristic methods to solve the model.

In this paper, a fuzzy multi-objective and multi-product mathematical model is represented with incremental and volume discounts approaches. Three criteria are considered for supplier selection as follows: the amount of defective parts, the percentage of delay time, and the costs that include the purchase costs based on the volume and incremental discounts. To solve the fuzzy model, constraints and objective function are integrated using Zimmerman method.

3. Mathematical Model

In this section, the assumptions, parameters, decision variables, and the proposed model are presented for the supplier selection and orders allocation in the production environment.

3-1. Assumptions

1. Supplier wants to optimize necessary products at various interval times.
2. In this model, the incremental and volume discounts are considered.
3. It is not necessary for the suppliers to provide all products in all time periods along with a range of discounts and both different types.
4. Fuzzy objective functions and constraints of demand are defined as a triangular fuzzy number. Other constraints are defined as the definitive one.
5. Inventory and shortage are permitted. The imposed cost of the inventory transferred to the next period is only the cost of holding inventory and the cost of shortage that is equivalent to the lost sale per unit of shortage.

3-2. Indices, parameters, and variables

i : Supplier's index
n: Total number of suppliers
t: Period index
m: Total number of periods
v: Index of product's type
o: Total number of product's types
k: Index of discount interval
$\tilde{\mathbf{P}}_{\mathbf{itvk}}$: The price of the ordered product to supplier *i* at the time interval *t* in the range of discounts *k* for product *v* (fuzzy)
$\mathbf{x}_{\mathbf{itvk}}$: The order numbers of supplier *i* at time interval *t* in the range of discounts *k* for product *v*
$\mathbf{y}_{\mathbf{itvk}}$: If product *v* is granted to the supplier *i* at time interval *t* in discounts *k*, $Y_{\text{itvk}} = 1$, otherwise $Y_{\text{itvk}} = 0$
$\mathbf{K}_{\mathbf{itv}}$: The last discount range of supplier *i* in time interval *t* for product *v*
$\mathbf{U}_{\mathbf{itvj}}$: The upper limit of the range discounts of supplier *i* in time interval *t* for product v
$\mathbf{L}_{\mathbf{itvk}}$: The Lower limit of the range discounts of supplier *i* in time interval *t* for product *v*
$\tilde{\mathbf{d}}_{\mathbf{iv}}$: The defective percentage of supplier *i* for product *v* (fuzzy)
$\tilde{\mathbf{h}}_{\mathbf{iv}}$: Supplier delay *i* for product *v* (fuzzy)
$\tilde{\mathbf{D}}_{\mathbf{tv}}$: Product demand *v* in time interval *t* (fuzzy)
$\mathbf{C}_{\mathbf{iv}}$: Supplier production capacity *i* for product *v*
$\mathbf{H}_{\mathbf{v}}$: The cost of holding products in the transition of time interval
$\mathbf{G}_{\mathbf{v}}$: The cost of shortage product *v* in the transition of time interval
$\mathbf{I}_{\mathbf{tv}}$:The inventory transferred from period *t* to *t + 1*
$\mathbf{S}_{\mathbf{tv}}$: Lacking amount of period *t*
$\mathbf{l}_{\mathbf{i}}$: If the discount is incremental, it is equal to 1, otherwise, it is 0.

3-3. Problem model

The objective functions and constraints of the problem are as follows:

3-3-1. Cost function

The developed cost function based on the incremental and volume discount takes into account multi-product and multi-period models, as well as the cost of holding inventory as follows:

$$\min Z_1 = \left(\sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i}\widetilde{P}_{itvk}(x_{itvk} - y_{itvk}U_{itv(k-1)}) + y_{itvk}\sum_{j=1}^{k-1}\widetilde{P}_{itvj}\left(U_{itvj} - U_{itv(j-1)}\right)\right) * l_i + \left(\sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i}\widetilde{P}_{itvk} * x_{itvk}\right) * (1 - l_i) + \sum_{v}^{o}\sum_{t}^{m} H_v(I_{tv}) + \sum_{v}^{o}\sum_{t}^{m} G_v(S_{tv}) \quad (1)$$

3-3-2. Defective parts function

Defective parts impose some cost on the buyer including the cost of improper planning, the cost of using low-quality components in the manufacturing and assembly, and the cost of reworking and warranty. The following function can express the objective of the buyer.

$$\min Z_2 = \sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i}\tilde{d}_{iv}x_{itvk} \quad (2)$$

3-3-3. Delay in delivery function

Due to the cost of delay in the delivery of the parts for the manufacturer, many companies tend to buy from suppliers who have the minimum delay in delivery of parts. Most of the costs are imposed because of failing to deliver products to the customer on time and, subsequently, due to the delay and the lost sales cost and also invisible costs such as loss of customer and loss of reputation or brand of manufactured product along with the cost of scheduling changes in production line. The following function provides this objective.

$$\min Z_3 = \sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i}\tilde{h}_{iv}x_{itvk} \quad (3)$$

There are several constraints for the proposed model of supplier selection as follows:

3-3-4. Inventory and demand constraints

For each product *v* at each time interval *t*, there is a demand constraint. This constraint states that the total number of orders *v* at time interval *t* per different suppliers plus the transferred inventory from the previous period minus the transferred inventory to the next period should be equal to demand in each period ($\widetilde{D}_{tv}$). In the proposed model, the demand rate for each product in different periods is expressed as a triangular fuzzy number.

$$\sum_{i=1}^{o}\sum_{k=1}^{K_i} x_{itvk} + I_{(t-1)v} - I_{tv} + S_{tv} \cong \widetilde{D}_{tv} \quad \text{for all t \&v} \quad (4)$$

$$I_{T\,v} = 0 \quad \text{for all } v \quad (5)$$

3-3-5. Range of discounts constraints

The number of order to supplier *i* at time interval *t* for product *v* in the range discount *k* should be between the upper limit and lower limit of the discounts range.

$$L_{itvk}y_{itvk} \leq x_{itvk} \leq U_{itvk}y_{itvk} \quad (6)$$

In addition, order is just accrued to supplier *i* at time interval *t* for product *v*.

$$\sum_{k=1}^{K_i} y_{itvk} \leq 1 \quad \forall\, i = 1, \dots, n \ \& \ t = 1, \dots, m \ \& \ v = 1, \dots, o. \quad (7)$$

3-3-6. Capacity constraints

It is stated that the number of orders to supplier *i* for product *v* must be less than or equal to production capacity of the supplier for product *v*.

$$\sum_{k=1}^{K_i} x_{itvk} \leq C_{iv} \quad \forall i = 1, \dots, n \& t = 1, \dots, m \& v = 1, \dots, o. \quad (8)$$

3-4. Finalizing the proposed model

Fuzziness of the model results from uncertainty in some parameters such as product demand, delay, product quality, price. Constraints and objective functions of the proposed model are defuzzied by changing the structure of the model. This paper uses the method proposed by Jimenez et al. (2007) to defuzzy the parameters. According to this method, the degree that shows a fuzzy number "a" is greater than or equal to number "b", expressed as follows:

$$\mu_M = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \dfrac{E_2^a - E_1^a}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b]) \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \quad (9)$$

where $[E_1^a, E_2^b]$ and $[E_1^a, E_2^b]$ are the expected intervals of $\tilde{a}$ and $\tilde{b}$. If $\mu_A(\tilde{a}, \tilde{b}) = 0.5$, numbers "a" and "b" will be equal. When $\mu_A(\tilde{a}, \tilde{b}) \geq \alpha$, $\tilde{a}$ in α degree will be greater than or equal to $\tilde{b}$, as shown by $\tilde{a} \gg \alpha\tilde{b}$.
Definition: for a constraint,

$$\tilde{a}_i x \geq \alpha\tilde{b}_i, \quad i = 1, \dots, m \quad (10)$$

According to the above description, it can be shown that

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geq \alpha \quad \text{OR} \quad [(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \quad i = 1, \dots, m \quad (11)$$

Finally, the overall model of objective function is as follows:

$$\min Z_1 \left(\sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i} \frac{P^1{}_{itvk} + 2P^2{}_{itvk} + P^3{}_{itvk}}{4} (x_{itvk} - y_{itvk}U_{itv(k-1)}) + y_{itvk} \sum_{j=1}^{k-1} \frac{P^1{}_{itvk} + 2P^2{}_{itvk} + P^3{}_{itvk}}{4} (U_{itvj} - U_{itv(j-1)}) \right) * l_i + \left(\sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i} \frac{P^1{}_{itvk} + 2P^2{}_{itvk} + P^3{}_{itvk}}{4} * x_{itvk} \right) * (1 - l_i) + \sum_{v}^{0}\sum_{t}^{m} H_v(I_{tv}) + \sum_{v}^{0}\sum_{t}^{m} G_v(S_{tv}) \quad (12)$$

$$\min \tilde{Z}_2 = \sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_i} \frac{d^1{}_{iv} + 2d^2{}_{iv} + d^3{}_{iv}}{4} x_{itvk} \quad (13)$$

$$\min \tilde{Z}_3 = \sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{v=1}^{o}\sum_{k=1}^{K_{itv}} \frac{h^1{}_{iv} + 2h^2{}_{iv} + h^3{}_{iv}}{4} x_{itvk} \quad (14)$$

St.

$$\sum_{i=1}^{o}\sum_{k=1}^{K_i} x_{itvk} - I_{tv} + I_{(t-1)v} + S_{tv} = \alpha\left(\frac{D^1{}_{tv} + D^2{}_{tv}}{2}\right) + (1-\alpha)\left(\frac{D^3{}_{tv} + D^2{}_{tv}}{2}\right) \quad \text{for all } t \,\&v \quad (15)$$

$$I_{T\,v} = 0 \quad \text{for all } v \quad (16)$$

$$L_{itvk}y_{itvk} \leq x_{itvk} \leq U_{itvk}y_{itvk} \quad (17)$$

$$\sum_{k=1}^{K_i} y_{itvk} \leq 1 \qquad \forall i = 1, \dots, n \& t = 1, \dots, m \& v = 1, \dots, o. \quad (18)$$

$$\sum_{k=1}^{K_i} x_{itvk} \leq C_{iv} \qquad \forall i = 1, \dots, n \& t = 1, \dots, m \& v = 1, \dots, o \quad (19)$$

4. The Proposed Solution Method

In order to solve the problem, two meta-heuristic solution methods have been proposed. The proposed solution methods are described in detail.

4-1. Non-recessive sorting genetic algorithm

One of the most efficient and best-known multi-objective optimization algorithms is non-recessive sorting genetic algorithm (NSGA-II), which was proposed by Deb et al. (2002). This algorithm is one of the fastest and most powerful optimization algorithms that feature less operational complexity than other methods and can obtain Pareto solutions using the dominance principle and congestion distance calculation that have appropriate extent in the changes area of objectives function and allow a user to choose his/her preferred solution through Pareto solutions. NSGA-II can keep elitism and dispersal simultaneously. In this method, a population of children is considered and, by using parents population, the size of the two populations is N. These two populations are merged and classified using the non-recessive sorting; finally, the new population is obtained including the best N members. Any classified population is called a front solution.

4-2. Multi-objective particle swarm optimization algorithm (MOPSO)

Particle swarm optimization algorithm was developed by a social psychologist named James Kennedy et al. (1995) for the first time using previous experiences related to modeling of the social behaviors that can be seen in many types of birds (particle). A significant number of these algorithms attracted researchers at the time; however, these two researchers greatly stressed the significance of models that were created by biologist Frank Heppner. According to Heppner model (a point that was different from the other models), it is possible for birds to have a high incentive to descend rather than stay among the team while descending. In PSO algorithm, members of groups exchange information about the best-found place. In addition, the best-found place in the current stage among a neighborhood of members is exchanged among members of the neighborhood. This information is used to update the position and speed of members in each stage (Kennedy and Eberhart, 2001).

Figure 1 shows the structure of the proposed multi-objective PSO algorithm in this study. In fact, since the proposed problem space is in the binary form, binary MOPSO developed by Eberhart and Kennedy was used in the current study. In order to update the archive of Pareto solutions, roulette wheel operator was used. Most evolutionary meta-heuristic methods use a random approach to generate initial solutions. Here, the same approach was adopted to generate original solutions (Benitez et al., 2005).

Pseudo code of MOPSO algorithm

$A := \emptyset$ — Initially empty archive
$\{x_n, v_n, G_n, P_n\}_{n=1}^{N} := initialise()$ — Random locations and velocities
$for\ t := 1 : G$ — G generations
$for\ i := 1 : N$
$for\ d := 1 : D$ — Update velocities and positions
$v_{id}(t) = wv_{id}(t-1) + c_1 r_1 (p_{id} - x_{id}(t-1)) + c_2 r_2 (p_{gd} - x_{id}(t-1))$
$x_{id}(t) = x_{id}(t-1) + v_{id}(t)$
end
$x_n := EnforceConstraints(x_n)$
$y_n := f(x_n)$ — Evaluate objectives
$if\ x_n \nprec u\ \forall u \in A$ — Add non-dominated $x_n\ to\ A$
$A := \{u \in A \mid u \nprec x_n\}$ — Remove points dominated by x_n
$A := A \cup x_n$ — Add x_n to A
end
end
$if\ x_n \leq p_n \vee (x_n \nprec p_n \wedge p_n \nprec x_n)$ — Update personal best
$p_n := x_n$
end
$G_n := SelectGuide\ (x_n, A)$
end

Fig. 1. The pseudo-code of the proposed algorithm MOPSO

4-3. Solution code

In the proposed model, the model's solution is composed of two matrices. The first matrix shows volume of products sent from suppliers at different time intervals. This matrix has a number of N rows (suppliers). Further, the number of columns shows the number of time intervals, and the number of the third dimension shows the number of products. As a result, dimensions of this matrix are (N * T * V). Figure 2 shows a flow matrix of products for the problem with 3 suppliers, 2 products, and 4 periods.

		Periods			
Suppliers		1	2	3	4
	1	1000	0	0	4500
Product 1	2	0	1500	0	0
	3	2000	0	1400	0
	1	0	0	0	1000
Product 2	2	0	0	0	0
	3	1000	0	1300	0

Fig. 2a. Solution code for 4 periods, 2 products, and 3 distribution centers

Supplier 1	Supplier 2	Supplier 3	Supplier 4
1	0	0	1

Fig. 2b. Coding for suppliers

Here, a sample solution to the first matrix is considered. As is clear, the rows and columns are associated with distribution centers and time intervals, respectively. Further, since the third dimension means that products are of two types, the number of matrixes is 2, in which the first 3 rows and the second 3 rows correspond to the first and second products, respectively. For example, the first supplier in the first period and in the fourth period sends 1000 units and 4500 units for the first product, respectively. In addition, the third supplier of Product 2 sends 1000 and 1300 units in the first and third periods, respectively.

The second part is a matrix in which the number of its cells is equal to that of suppliers. Numbers of each cell are zeros and ones; 1 and 0 show incremental discount and volume discount, respectively. For example, a matrix with 4 suppliers is shown in Figure 2. The first and fourth suppliers use incremental discounts, and the second and third suppliers use volume discounts.

4-4. Initial population

For each product, in each period, a random sequence of providers is generated. Then, the product is allocated to suppliers based on their capacity constraint. This process continues until the demand for the product is fully satisfied. Afterwards, the satisfaction of product demand will go to the next period. After allocating products to all periods, we go to the next product, and this process continues for all products.

4-5. Parameter setting

To set the parameters of the proposed algorithm, Taguchi method is used. For NSGAII method, 4 factors are considered: initial population (npop), the maximum number (max_it), mutation coefficient (Mutation), and displacement coefficient (Crossover). Pareto solution's number is considered as a standard criterion, and 3 levels for each factor are presented as follows:

Initial population: 20, 30, 50
Maximum iteration: 50, 75, 100
Crossover rate: 0.6, 0.7, 0.8
Mutation rate: 0.2, 0.3, 0.4

In Taguchi method, Signal/Noise (S/N) criterion is used. This criterion shows changes in the response variable. For each factor, the optimum level makes high S/N. Therefore, the second levels (75), (30), (0.7), and (0.3) are the best values for Maximum iteration, Initial population, Crossover rate, and Mutation rate, respectively, according to Fig. 3. Final results of parameters setting are shown in Table 2:

Tab. 2. Results of parameters setting for NSGA2

Mutation rate	Crossover rate	Initial population	Maximum iteration
0.3	0.7	30	75

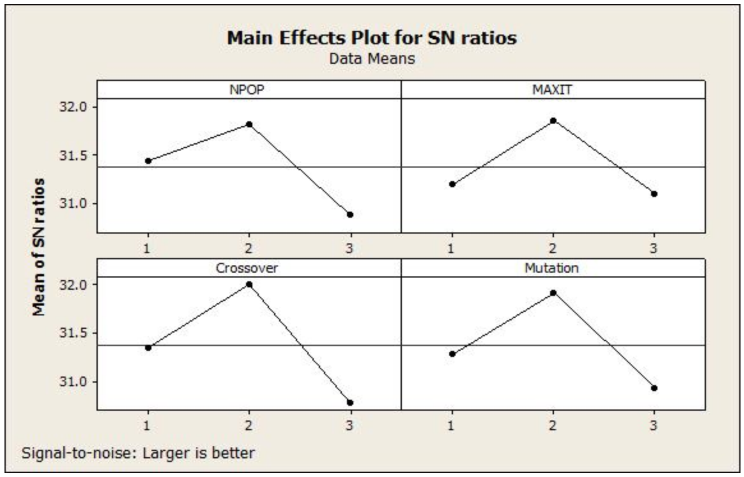

Fig. 3. Chart of S/N rate for NSGA2 coefficients

In PSO process, four factors are considered: initial population (npop), the maximum iteration number (max_it), acceleration coefficient 1 (C1), and the acceleration coefficient 2 (C2). Number of Pareto solution (NOS) is considered as the response variable.
Initial population: 30, 40, 50
Maximum iteration number: 100, 150, 200
Acceleration coefficient 1: 1/1, 2, 2/8
Acceleration coefficient 2: 1/1, 2, 1/3
According to Taguchi method, optimal values for 4 factors, i.e., npop, max_it, C1, and C2, belong to the second level (40), the first level (100), the third level (2.8), and the first level (1.1), respectively, as illustrated in Fig. 4.

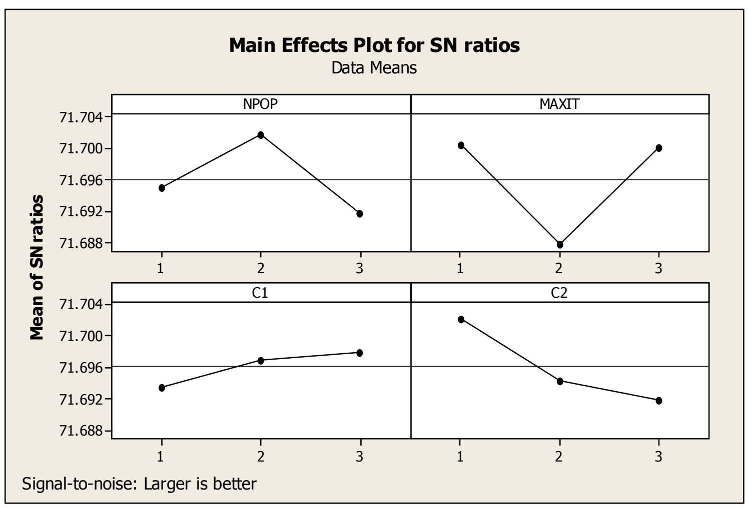

Fig. 4. S/N rate for PSO algorithm coefficients

Tab. 3. Results of setting particle swarm optimization algorithm

Acceleration coefficient 2	Acceleration coefficient 1	Initial population	Number of iteration
1.1	2.8	40	100

4-6. Comparison indexes

There are two metric groups to evaluate the performance of multi-objective meta-heuristic algorithm:

1. Convergence metrics
2. Distribution metrics

In the current research, five indexes that involve the combination of two main groups are used to compare the results. The first metric group includes Pareto solution number metric, distance of ideal solution metric, and set coverage metric. The second metric group includes spacing metric and diversification metric. In addition, executing time metric is considered to compare computational needs.

4-6-1. Spacing metric

This metric proposed by Schott calculates relative distance between consecutive solutions using the following equation.

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2} \quad (20)$$

where $d_i = \underbrace{\min}_{k\in n \wedge k\neq 1} \sum_{m=1}^{2}|f_m^i - f_m^k|$, $\bar{d} = \sum_{i=1}^{n}\frac{d_i}{|n|}$.

The measured spacing corresponds to the minimum sum of absolute difference of objective function values between the i^{th} solution and the final non-recessive solutions.

4-6-2. Number of pareto solution (NOS)

NOS metric represents the number of Pareto solutions (NOS) in each algorithm. Figure 5 is represented to calculate NOS metric.

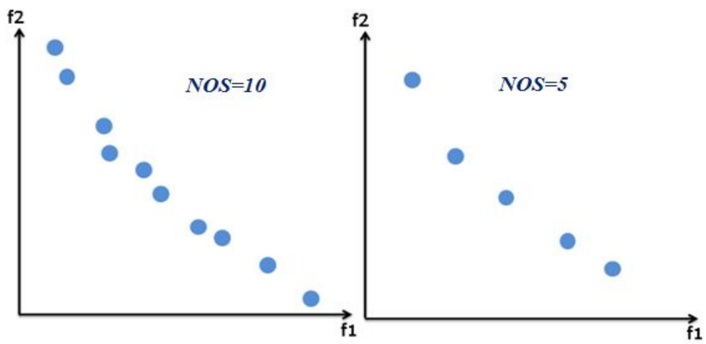

Fig. 5. Parto solutions in multi-objective function

4-6-3. Mean ideal distance (MID)

Mean Ideal Distance (MID) is a simple metric that measures the average of distances from an ideal point. This metric is used to calculate the average distance of Pareto solutions from a coordination point. According to the following relation, the less metric there is, the higher the algorithm efficiency can be.

$$MID = \frac{1}{NOS}\sum_{i=1}^{NOS} c_i \quad where \quad c_i = \sqrt{\sum_{j=1}^{m} f_{ji}^2} \quad (21)$$

where "f" is the objective function value of each Pareto solution. Since one objective is placed at a minimum distance from the coordination center in Pareto approach of the multi-objective optimization, this metric calculates the distance from the best population (Zitzler and Thiele, 1999). Figure 6 shows MID metric schematically. *n* is the number of Parto solutions whose mean distance to ideal point (c_i) is obtained as MID metric.

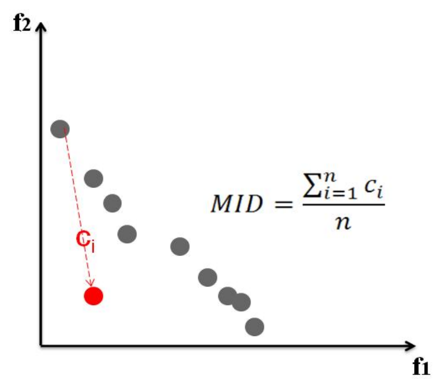

Fig. 6. MID for multi-objective optimization

4-6-4. Executing time

Executing time is also considered as a quality evaluation criterion of the algorithm.

4-6-5. Diversification metric (DM)

This measure calculates the diversity between non-recessive solutions in a set, and its equation is as follows:

$$D = \sqrt{\sum_{i=1}^{n} \max(\left||x_i' - y_i'|\right|)} \quad (22)$$

where $\left||x_i' - y_i'|\right|$ is the direct distance between non-recessive solutions x_i and y_i; thus, the more this criterion and the same lower solutions exist, the greater diversification in solutions will be.

5. Computational Results of The Proposed Algorithm

In this research, 15 problems are used as shown in Table 4. In addition, the problem parameters are considered based on the distribution functions that are presented in Table 5.

Tab. 4. Characteristics of the generated problems

item	supplier	product	period
1	2	5	2
2	3	5	2
3	3	10	3
4	4	10	3
5	5	15	4
6	7	15	4
7	8	20	6
8	9	22	7
9	10	25	8
10	10	30	8
11	10	40	10
12	15	50	10
13	20	70	10
15	25	90	12
15	30	100	12

Tab. 5. Distribution functions for making parameters

PARAMETER	RANGE
P_{ITVK}	Uniform (30,50)
U_{ITVK}	L_{itvk} + Uniform (700,100)
L_{ITVK}	U_{itvk-1}+1
D_{IV}	Uniform (0.05, 0.25)
H_{IV}	Uniform (0.5, 2.5)
D_{TV}	Uniform (200, 400)
C_{IV}	Uniform (150, 400)
H_{V}	Uniform (10,50)
G_{V}	Uniform (50,100)

Before explaining the results of each algorithm, it is noted that the highest values are more appropriate for two metrics: non-recessive solutions (NOS) and diversification criterion. Since the objective function is of minimization type, a greater value for MID criterion is better. In addition, the lower value for other criteria such as execution time and spacing criterion will be more appropriate. It is shown in Table 6.

Tab. 6. Comparative results of values of NSGA- II and MOPSO

No	NSGA- II MID	NOS	Time	SPACING	Diversification	MOPSO MID	NOS	Time	SPACING	Diversification
1	365.1	35	5.29	0.684	5723.6	1.32	55	4.74	0.397	4938.87
2	1.64	23	4.94	0.656	4661.3	1.29	32	4.81	0.453	4300.77
3	1.45	53	9.67	0.639	13182.9	1.51	26	7.1	0.603	5518.84
4	1.58	44	9.9	0.658	12128.7	1.65	36	10.16	0.423	6903.78
5	1.94	28	17.25	1.107	15781.2	1.78	43	17.6	0.55	11520.49
6	1.74	54	22.98	0.925	19472.0	1.72	35	22.31	0.48	10712.80
7	1.63	63	45.2	0.975	29213.2	1.78	40	47.44	0.44	15620.83
8	1.74	44	61.5	0.96	28284.5	1.9	29	69.13	0.42	15367.5
9	1.89	57	87.6	1.32	37979.7	1.9	27	102.02	0.635	16150.07
10	1.9	42	103.07	1.25	37658.9	1.77	30	115.79	0.624	19721.77
11	1.64	81	168.35	0.93	61848.2	2.01	22	215.38	0.834	21031.57
12	2.03	45	291.9	1.59	55912.9	2.137	19	415.26	0.935	20143.42
13	2.06	27	816.2	0.448	22673.3	1.97	20	732.45	0.874	27823.42
14	2.10	43	1572.3	0.574	36602.8	2.10	20	1515.88	0.739	33051.37
15	2.17	34	2141.7	0.573	27905.9	2.14	24	2065.86	1.16	38508.28

In this research, 15 problems are generated with different sizes of suppliers (2-30), time interval (2-12), and products (5-100). Based on the presented results in columns 2 and 7 of Table 6, the performance of the two is the same for MID metric. The next comparison metric is NOS; in Columns 3 and 8, the performance of NSGAII is better in most sizes. The third metric shows the computational time of two algorithms, and in Columns 4 and 9, the performance of NSGAII is slightly better than that of MOPSO algorithm. The fourth metric is spacing, which, based on Columns 5 and 10 of Table 6, is lower for MOPSO algorithm of all sizes except 3 large sizes. The last metric shows diversification such that NSGAII has a better performance than MOPSO in all sizes except 3 large sizes.

Figure 7 shows the comparison of the values of the first objective function for two algorithms, NSGAII and MOPSO, for 4 problems (1-4). Axis X shows the value of the first objective function and Axis Y shows the number of sample problems and algorithm. This graph shows the maximum, minimum, and average of the first objective function obtained from each algorithm and each problem test. As shown in Figure 7, the value of the first objective function in 4 problems for MOPSO algorithm is more optimized than NSGAII, meaning that the value of the objective function is lower, since this objective function is aimed at minimization.

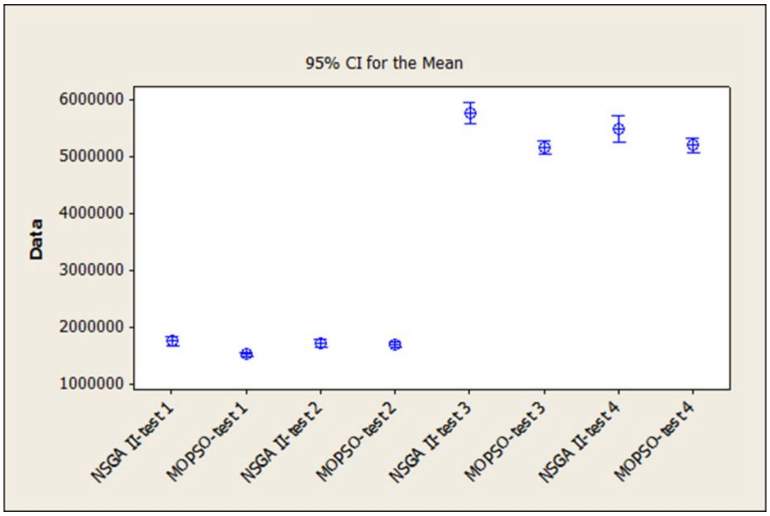

Fig. 7. The value of the first objective function for two algorithms

Figure 8 has compared the value of second objective function for two algorithms NSGAII and MOPSO for 5 problems (1-5). The value of the second function in 5 problems for MOPSO algorithm is more optimized than that for NSGAII.

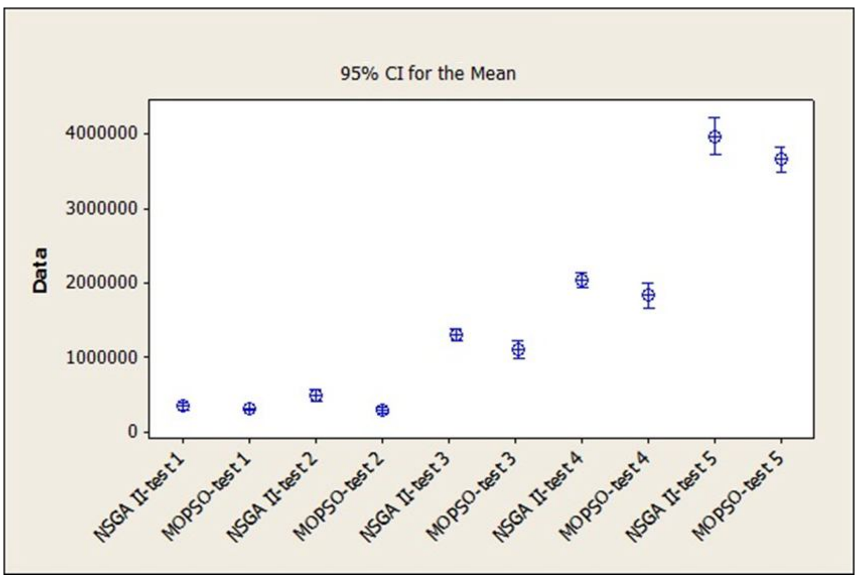

Fig. 8. The value of the second objective function for two algorithms

Tab. 7. Results of the comparison between methods of NSGAII and ϵ constraint

Size	∈-Constraint				NSGA II				RG(Error)		
	Object 1	Object 2	Object 3	time	Object 1	Object 2	Object 3	time	Object 1	Object 2	Object 3
1	1421413	870	8510	80	1442734	870	8510	5.29	1.5%	٠	٠
2	1494303	13891	13558	230	1539138	14446	13666	4.94	3%	4%	0.8%
3	4739387	11041	118395	435	5071144	11593	122420	9.67	7%	5%	3.4%
4	4658689	12263	130717	705	5147818	13366	141174	9.9	10.5%	9%	8%

Tab. 8. Results of the comparison between MOPSO and ε-constraint

Size	ε-Constraint				MOPSO				RG (Error)		
	Object 1	Object 2	Object 3	time	Object 1	Object 2	Object 3	time	Object 1	Object 2	Object 3
1	1421413	870	8510	80	1427098	870	8510	4.74	0.4%	·	·
2	1494309	13891	13558	230	1505292	13974	13585	4.81	1%	0.6%	0.2%
3	4739387	11041	118395	435	4881568	11339	119578	7.1	3%	2.7%	1%
4	4658695	12263	130717	705	4984765	12998	135945	10.16	7%	6%	4%

A previously conducted study on the time of problem solution for ∈-constraint with NSGAII and MOPSO shows a great increase in this time for constraint ∈-method. In addition, the highest average errors obtained in three objective functions for NSGAII and MOPSO compared with the exact method are 10%, 9%, 8%, 7%, 6%, and 4%.

6. Conclusions

This research presents a mathematical model for supplier prioritization and order allocation considering horizon time and discount. According to the literature review, for the first time, the supplier selection problem has been studied considering cost functions (with discount and lost sale), quality, and on-time delivery with shortfall in a multi-product, multi-period and fuzzy condition simultaneously. This model was solved by NSGA2 and MPSO. According to the complexity of this model, two meta-heuristic algorithms of MOPSO and NSGAII were developed to solve the model. These algorithms can represent a good approximation of efficient optimal solutions for the proposed objectives. According to the results, NSGAII makes more NOS and more diverse Pareto solutions; however, MOPSO makes highly regular and optimized Parto line. For future researches, the option of local and international suppliers that definitely increases the complexity of the model can be added. On the other hand, based on the model, it is assumed that buyer's demand, delivery rate, and price are definite. For the future, they can be related to market condition and other competitions.

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