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An Inventory Routing Problem for Perishable Products with Stochastic Demands and Direct Deliveries

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ABSTRACT

In any supply chain, distribution planning of products is of great importance to managers. By effective and flexible distribution planning, mangers can increase the efficiency of time, place, and delivery utility of whole supply chain. In this paper, inventory routing problem (IRP) is applied to distribution planning of perishable products in a supply chain. The studied supply chain is composed of two levels; a supplier and number of customers. Customers are geographically dispersed around the supplier location and their demands are uncertain and follow independent probability distribution functions. The product has pre-determined fixed life and is to be delivered to the customers via a fleet of homogenous vehicles. The supplier uses direct routes for delivering products to customers. The objective is to determine when to deliver to each customer, how much to deliver to them, and how to assign them to vehicle and routes. The mentioned problem is formulated and solved using a stochastic dynamic programming approach. Also, a numerical example is described to illustrate the applicability of the proposed approach.

1. Introduction

The inventory routing problem (IRP) is a specific form of vehicle routing problem (VRP). In the simplest form, IRP is concerned with supplying one or many products from one origin (supplier) to one or many destinations (customers) in a finite or infinite planning horizon, while considering inventory and routing in the problem simultaneously. In IRP, the following must be decided:

- When to visit customers
- How much to deliver to each customer when it is visited
- How to select routes in order to visit customers

The class of IRP is broad and its solution approaches are even broader. But all the IRPs have common basic characteristics. All of them consider situation in which products are transported using fleet of capacitated vehicles from supplier to customers, while transportation cost is usually calculated based on distance. This

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refers to word routing. But the element that differentiates this type of problem from VRP is inventory. The supplier has to manage inventory levels of customers in order to prevent causing shortage. Obviously, the inventory element in IRP makes it more complex than VRP. The objective of IRP is to minimize the distribution costs during the planning horizon without causing shortages at any of the customers (Bertazzi et al. [1]).

The IRP defined above is deterministic, because we assumed customers' demands are known and certain. Obviously in real life applications, this assumption is not credible. Therefore, an important variant of the IRP is the stochastic inventory routing problem (SIRP). The SIRP differs from the IRP in that the future demands of customers are uncertain. In the SIRP, the probability distributions of the demands of customers are given throughout planning period. Also because demand is uncertain, there is a probability that a customer faces shortage. Here, the objective is to find a distribution plan that minimizes the total costs over the planning horizon including shortage costs (Coelho et al. [2]).

In this paper, a stochastic inventory routing problem is formulated for perishable products. The perishable product has pre-determined life time. To solve the problem and find optimal solutions, a stochastic dynamic program is presented.

The remainder of this paper is as follows; in the next section, a review of literature concerning stochastic inventory routing problem is given. The problem is defined formally in section 3 and then in section 4, the proposed stochastic dynamic programming approach is described. In order to clarify the steps of proposed method, a numerical example is presented in section 5. Finally, concluding remarks and future research directions are given.

2. Literature Review Toward Stochastic Inventory Routing Problem

There is a vast literature toward IRP. For detailed information regarding classifications and characteristics of inventory routing problem, the reader may refer to Coelho et al. [2] and Andres on et al. [3]. Hence in this section, we constrain our attention on researches relating to SIRP.

In the SIRP, the supplier knows customers' demands only in an uncertain way. As mentioned before, demand uncertainty means that shortages may occur. In order to prevent them, system incurs a penalty cost when a customer faces shortage. Shortage is usually considered as lost sale, and hence, lost sale cost is calculated as "unit lost sale cost × lost sale amount". The SIRP objective is similar to IRP objective; find a distribution plan that minimizes total costs throughout the planning horizon. It should be noted that in some cases, minimizing total discounted costs is considered too.

With respect to modeling/ solution method of SIRP, three classes can be identified in the literature: heuristic methods, stochastic dynamic programming, and robust optimization. Among above classes, heuristics methods have been applied more than others and dynamic programming been applied less than others.

Usually studies on the SIRP assume that the probability distribution of demand is known before. Note that both heuristics and dynamic programming methods are applicable only when complete information about probability distributions of the parameters are available, but when no information is available, robust optimization framework is more appropriate to deal with uncertainty. In this paper, a stochastic dynamic programming framework is used to formulate and solve the SIRP.

A number of recent SIRP papers are classified and presented in table1. Criteria of classification are time horizon, structure, routing, fleet type, fleet size, and solution method. In table1, time horizon can be finite or infinite. The number of suppliers and customers can be different in cases, and therefore the structure can be one-to-one when there is only one supplier and one customer, one-to-many when there are one supplier and several customers, and many-to-many with several suppliers and several customers. Routing can be direct when there is only one customer in a route or multiple when there are several customers in a route. The last two criteria are fleet composition and fleet size. The fleet can be homogeneous or heterogeneous, and the number of available vehicles can be one, many, or unconstrained.

In this research, we study a variation of inventory routing problem in which products are perishable. In light of perishable products, there two separated trends in literature. In the first one, researchers have focused on extending economic order quantity (EOQ) models for perishable products. The reader may refer to Bakker et al. [4] for latest review of the topic. In another line of research, researchers have focused on developing VRP models for perishable items. For example Hsu et al. [5], Osvald and Stirn [6], Chen et al. [7] and Gong and Fu [8] applied VRP with time windows to distribution planning of perishable products.

There are only few researches in the literature that combine inventory and distribution decisions for perishable products. Federgruenet al. [9] worked on an integrated inventory and distribution problem. They considered three types of costs in their model: shortage cost, out-of-date cost and transportation cost. They developed a heuristic algorithm to tackle the problem. Le [10] proposed an inventory routing problem for perishable products with expiration date and deterministic demand. He developed a column generation based heuristic to solve the problem and find good solutions.

In the current paper, we propose a SIRP formulation for distribution planning of perishable products. To best of our knowledge, there is no similar research in the literature that has such characteristics. Moreover, full characteristics of the current paper are given in the last row of Table 1.

Tab. 1. Classification of SIRP literature

	Time Horizon		Structure			Routing		Fleet Type		Fleet Size			Solution Method		
Reference	Finite	Infinite	One-to-one	One-to-many	Many-to-many	Direct	Multiple	Homogeneous	Heterogeneous	Single	Multiple	Unconstrained	Heuristic	DP	Robust Opt.
Berman and Larson [11]		*		*			*	*		*			*		
Kleywegt et al. [12]		*		*		*		*				*		*	
Adelman [13]		*		*			*	*			*			*	
Kleywegt et al. [14]		*		*			*	*			*			*	
Aghezzaf [15]		*		*			*	*			*				*
Hvattum and Lokketangen [16]		*		*			*	*			*		*		
Hvattum et al. [17]		*		*			*	*			*		*		
Huang and Lin [18]		*		*		*		*			*		*		
Geiger and Sevaux [19]	*			*			*	*				*	*		
Liu and Lee [20]	*			*			*	*			*		*		
Solyalh et al. [21]	*			*			*	*		*					*
Current paper	*			*		*		*			*			*	

3. Problem Definition

In this section, the considered problem is thoroughly defined; we consider a supply chain consisting of a supplier and number of customers. In this supply chain, a perishable product is to be distributed from central depot of the supplier to customers' sites. Planning horizon is divided into discrete equal time intervals where each time interval is a planning period. So, we have finite planning horizon. The perishable product has fixed life time that is defined in term of number of planning periods, i.e., if life time of the product is 5, that means that it is storable only for the next 5 planning periods and after that, it must be thrown away. The product quality remains unchanged during its life time. Demand of each customer in each planning is uncertain and follows a known probability distribution function. This distribution function is fixed during the planning horizon and is known to the supplier. Moreover, the demand of each customer is independent from the other customers. The customers can store the product in their on-house warehouses. Their storage capacity is limited and known. Supplier is responsible for managing inventories in whole supply chain. It is aware of customers' inventory levels in the beginning of planning periods and can decide on whether which customers should be replenished, how much should be delivered to each customer, and how to allocate customers into vehicles and routes. The distribution is done using a fleet of homogenous vehicles. Capacity of vehicles is limited and known. Distribution is done based on direct routes. A vehicle in a direct route starts from the supplier, visits only one customer, and then returns to the supplier. Each vehicle can make at most one delivery per planning period. Distribution costs include fixed transportation costs and variable distribution costs. Because the demand is not certain, some customers may face shortage in some planning periods. Shortages are considered as lost sale. System costs include inventory holding costs at customers, probable shortage costs, distribution costs, and probable perishing cost. The objective is to find a distribution plan in which total costs are minimized within a finite planning horizon.

4. The Proposed Stochastic Dynamic Programming Approach

In this section, a backward stochastic dynamic programming approach is described to formulate the problem and find optimal distribution plan for the above mentioned problem.

Dynamic programming is a standardized method for solving a complex problem by breaking it down into a collection of simpler sub problems. Every dynamic programming contains elements that should be defined. These elements are stage, state, action, return, transition probability, and recursive function. In dynamic programming, sub problems are called stage. Every stage includes several states. In each state, there are several actions to do that one of them must be chosen. In fact, actions are decision variables. Return is what the system makes or produces in a stage and can be profit, cost, etc. In every dynamic programming problem, we should use a recursive function to calculate optimal value of a state that is basic rule for solving dynamic programming problems. With recursive function, the optimal action in each stage can be determined.

Here, we define the elements for our problem:

Stage

Stage is the planning periods within planning horizon and is depicted with t.

State

State is the vector of customers' inventory levels at the beginning of planning period t and is depicted with (t, I);

$(t, I) = \{t, (i_1, i_2, \ldots, i_n)\}$

Action

Action is the vector of delivered products to customers in planning period t and is depicted with (t, K).

$(t, K) = \{t, (k_1, k_2, \ldots, k_n)\}$

Return

Return is the expected net return in each planning period with respect to state and action vectors and is depicted with $r(t, I, K)$. It should be noted that the return is equal to inventory holding costs + inventory shortage costs + inventory transportation costs + inventory perishing costs.

$r(t, I, K) = r\{t, (i_1, \ldots, i_n), (k_1, k_2, \ldots, k_n)\}$

Transition Probability

Transition probability is the probability of that with delivering K to customers, their inventory level that is I at the beginning of planning period t, reaches to J at the beginning of planning period t+1 and is depicted with $\Pr(t, I, J, K)$.

$\Pr(t, I, J, K) =$
$\Pr\{t, (i_1, \ldots, i_n), (j_1, \ldots, j_n), (k_1, k_2, \ldots, k_n)\}$

Optimal Value of a State

Optimal value of a state is the average return gained from system transition from state (t, I) into end of planning horizon and is depicted with $f(t, I)$.

$f(t, I) = f\{t, (i_1, \ldots, i_n)\}$

Recursive Function

Recursive function is composed of two parts; one is the expected net return and the other is the value of next stages.

$$f(t,I) = f\{t,(i_1,i_2,\ldots,i_n)\} = \underset{K}{\text{Min}}\left[r(t,I,K) + \sum_J p(t,I,J,K).f(t+1,J) \right] = \underset{K}{\text{Min}}\left[\begin{array}{l} r\{t,(i_1,\ldots,i_n),(k_1,\ldots,k_n)\} \\ +\sum_J \begin{array}{l} p\{t,(i_1,\ldots,i_n),(j_1,\ldots,j_n),(k_1,\ldots,k_n)\} \\ .f\{t+1,(j_1,\ldots,j_n)\} \end{array} \end{array} \right]$$

In proposed stochastic dynamic programming approach, the followings must be considered; Assume that $I_{n,t}$ denotes inventory level of customer n in planning period t, $X_{n,t}$ denotes the amount of product delivered to customer n in planning period t, $d_{n,t}$ denotes the demand of customer n in planning period t, and LT denotes the life time of the product. So, we have the amount of product consumed by customer n in

planning period t equal to min{$I_{n,t}$+ $X_{n,t}$, $d_{n,t}$}, shortage in customer n in planning period t equal to max{$d_{n,t}$ - [$I_{n,t}$+ $X_{n,t}$] , 0}, and the inventory level of customer n in planning period t+1 equal to $I_{n,t+1}$ = max{$I_{n,t}$+ $X_{n,t}$ - $d_{n,t}$, 0}. Also, as demand is uncertain, some of products would perish and perishing cost must be considered for them. The expected perishing cost is equal to:
unit perishing cost × (inventory level at the end of period – average of demand of next LT periods).
Finally, the return is equal to some of transportation costs, inventory holding costs, lost sales costs, and perishing costs. Cost parameters are depicted in table 2.

Tab. 2. Cost parameters of the problem

Symbol	Description
h_i	Unit inventory holding cost
b_i	Unit shortage (lost sale) cost
f_i	Unit fixed transportation cost
t_i	Unit variable transportation cost
p_i	Unit perishing cost

In each stage, the expected return is calculated according to below procedure; if no product is delivered to customers, the expected return is equal to inventory holding costs + inventory shortage costs + perishing costs:

$$r(t,I,0) = h_i.\sum_{n=1}^{N}\sum_{x=1}^{i_n}\Pr\{X < x\} + b_i.\sum_{n=1}^{N}\sum_{x=i_n}^{c_n}\Pr\{X > x\} + p_i.P_i \quad (1)$$

If vector K is delivered to customers, the expected return is calculated using relation (1) that the transportation costs is added to it:

$$r(t,I,K) = r(t,I,0) + f_i + t_i.\sum_{n=1}^{N}k_n \quad (2)$$

Transition probabilities are calculated according to below procedure; if no product is delivered to customers and inventory levels at the beginning of next planning period is above zero:

$$\Pr\{t,I,J,0\} = \Pr\{X = I - J\} = \Pr\{x_1 = i_1 - j_1,\ldots,x_N = i_N - j_N\} = \prod_{n=1}^{N}\Pr\{x_n = i_n - j_n\} \quad (3)$$

If no product is delivered to customers and inventory levels in the next planning period is zero:

$$\Pr\{t,I,0,0\} = \Pr\{X \ge I\} = \Pr\{x_1 \ge i_1,\ldots,x_N \ge i_N\} = \prod_{n=1}^{N}\Pr\{x_n \ge i_n\} \quad (4)$$

And finally, if vector K is delivered to customers:

$$\Pr\{t,I,J,K\} = \Pr\{t,I,J-K,0\} \quad (5)$$

Finding optimal solution is done as follows; start from the last planning period and calculate expected returns and transition probabilities using relation (1) to (5) for every possible state. Then use the recursive function to calculate values and optimal value of each state. Then determine the optimal action based on the optimal value. The steps of proposed approach are depicted in figure 1 in more details.
In next section, the steps of approach are described within a numerical example.

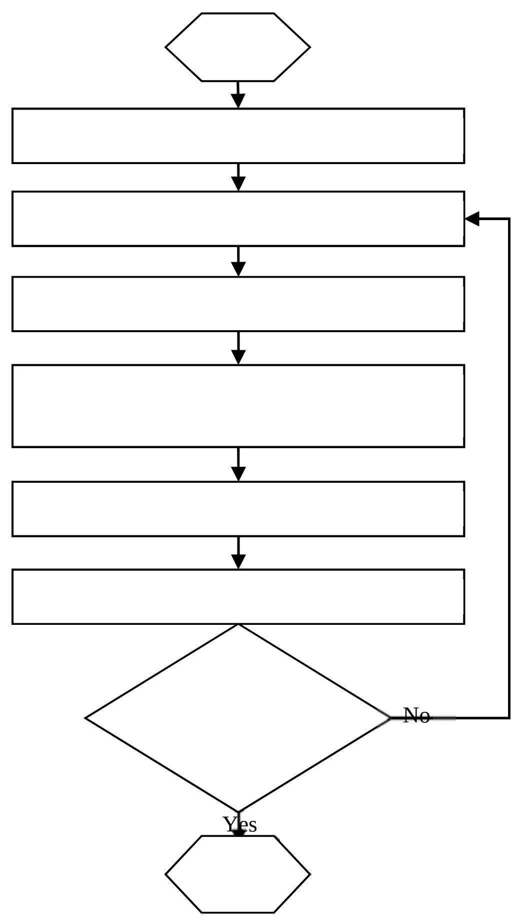

Fig. 1. Steps of the proposed approach

4. Numerical Example

In order to illustrate applicability of proposed method, a numerical example is described in current section. Parameters of numerical example are given in table 3.

Tab. 3. Parameters of numerical example

Parameter	Value
Number of planning periods	3
Number of customers	2
Number of vehicles	1
Customers storage capacity	2, 3
Vehicles capacity	2
Demand distribution function	uniform {0, 1, 2, 3}
Product life time (periods)	2
Unit holding cost	2
Unit shortage (lost sale) cost	8
Unit fixed transportation cost	4
Unit variable transportation cost	1
Unit perishing cost	4

Since the probability distribution function of demand is fixed throughout planning horizon, we can calculate expected returns and transition probabilities independent from planning periods. Here, we present the procedure done for calculating expected returns and transition probabilities when initial inventories are (0,2) that is given in table 4.

It should be noted that:

(1) Since the vehicles are able to make at most one delivery in each planning period, some of combinations would be infeasible,

(2) Since the shortages are considered as lost sales, negative inventories at the end of planning periods would be considered as zero inventories at the beginning of next planning periods.

Tab. 4. Calculating expected returns when initial inventories are between (0,2)

Demand quantity		Amount of products delivered to customers					
		To customer 1 (Initial Inventory = 0)			To customer 2 (Initial Inventory = 2)		
		0	1	2	0	1	2
0	Final Inventory	0	1	2	2	3	4
1		-1	0	1	1	2	3
2		-2	-1	0	0	1	2
3		-3	-2	-1	-1	0	1
0	return	0	7	12	6	17	infeasible
1		8	5	8	2	11	
2		16	13	6	0	7	
3		24	21	14	8	5	
Expected return		12	11.5	10	4	10	-

Based on above table, we can calculate expected returns:

r (t, I, K) = r {t, (0,2), (0,0)} = 12 + 4 = 16

r (t, I, K) = r {t, (0,2), (1,0)} = 11.5 + 4 = 15.5

r (t, I, K) = r {t, (0,2), (2,0)} = 10 + 4 = 14

r (t, I, K) = r {t, (0,2), (0,1)} = 12 + 10 = 22

Also, we can calculate transition probabilities:

Pr (t, I, J, K) = Pr {t, (0,2), (0,0), (0,0)} = 1 × 0.5 = 0.5

Pr (t, I, J, K) = Pr {t, (0,2), (0,1), (0,0)} = 1 × 0.25 = 0.25

Pr (t, I, J, K) = Pr {t, (0,2), (0,2), (0,0)} = 1 × 0.25 = 0.25

Pr (t, I, J, K) = Pr {t, (0,2), (0,0), (1,0)} =

0.75 × 0.5 = 0.375

Pr (t, I, J, K) = Pr {t, (0,2), (0,1), (1,0)} = 0.75 × 0.25 = 0.1875

Pr (t, I, J, K) = Pr {t, (0,2), (0,2), (1,0)} = 0.75 × 0.25 = 0.1875

Pr (t, I, J, K) = Pr {t, (0,2), (1,0), (1,0)} = 0.25 × 0.5 = 0.125

Pr (t, I, J, K) = Pr {t, (0,2), (1,1), (1,0)} = 0.25 × 0.25 = 0.0625

Pr (t, I, J, K) = Pr {t, (0,2), (1,2), (1,0)} = 0.25 × 0.25 = 0.0625

Pr (t, I, J, K) = Pr {t, (0,2), (0,0), (2,0)} = 0.5 × 0.5 = 0.25

Pr (t, I, J, K) = Pr {t, (0,2), (0,1), (2,0)} = 0.5 × 0.25 = 0.125

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Pr (t, I, J, K) = Pr {t, (0,2), (0,2), (0,1)} = 1 × 0.25 = 0.25

Pr (t, I, J, K) = Pr {t, (0,2), (0,3), (0,1)} = 1 × 0.25 = 0.25

After expected return and transition probabilities of all states related to a planning period are calculated, the optimal actions can be selected. Results are presented in table (5) to (7).

5. Problem Modeling

Modularity and layout design of a system in an optimum way, affecting the design factors for maintainability, results in boosting the maintainability of the system. Optimum system modularity, by affecting accessibility increase, simplification, ergonomics, identification of failure places, and by using DSM technique, increases system maintainability.

The modular and layout designimpacton maintainability factors isspecified in Fig. 1.

Tab. 5. Optimal actions at the beginning of third planning period

Optimal value	Amount	Chosen action	Optimal value	Amount	Chosen action
f [3, (0,0)]	22	(0,2)-(2,0)	f [3, (1,2)]	10.5	(0,0)
f [3, (0,1)]	16.5	(2,0)	f [3, (1,3)]	11.5	(0,0)
f [3, (0,2)]	14	(2,0)	f [3, (2,0)]	14	(0,2)
f [3, (0,3)]	15	(2,0)	f [3, (2,1)]	10.5	(0,0)
f [3, (1,0)]	16.5	(0,2)	f [3, (2,2)]	8	(0,0)
f [3, (1,1)]	13	(0,0)	f [3, (2,3)]	9	(0,0)

Tab. 6. Optimal actions at the beginning of second planning period

Optimal value	Amount	Chosen action	Optimal value	Amount	Chosen action
f[2, (0,0)]	40.625	(0,2)-(2,0)	f[2, (1,2)]	28.75	(1,0)
f[2, (0,1)]	34	(2,0)	f[2, (1,3)]	28.25	(1,0)
f[2, (0,2)]	29.75	(2,0)	f[2, (2,0)]	29.75	(0,2)
f[2, (0,3)]	29.25	(2,0)	f[2, (2,1)]	28.75	(0,1)
f[2, (1,0)]	34	(0,2)	f[2, (2,2)]	23.75	(0,0)
f[2, (1,1)]	32.945	(0,1)-(1,0)	f[2, (2,3)]	23.25	(0,0)

Tab. 7. Optimal actions at the beginning of first planning period

Optimal value	Amount	Chosen action	Optimal value	Amount	Chosen action
f[1, (0,0)]	58.25	(0,2)-(2,0)	f[2, (1,2)]	45.79	(0,0)
f[1, (0,1)]	51.8	(2,0)	f[2, (1,3)]	44.92	(0,0)
f[1, (0,2)]	47.23	(1,0)	f[2, (2,0)]	47.23	(0,2)
f[1, (0,3)]	46.04	(2,0)	f[2, (2,1)]	45.79	(0,0)
f[1, (1,0)]	51.79	(0,2)	f[2, (2,2)]	41.23	(0,0)
f[1, (1,1)]	50.30	(0,2)	f[2, (2,3)]	40.04	(0,0)

6. Conclusion and Future Research Directions

In this paper, a stochastic dynamic programming approach is proposed to find optimal distribution plan in a supply chain. The studied supply chain consists of two levels; a supplier and number of customers. The supplier aims to deliver a perishable product from its place to customer sites. The delivery fleet includes number of homogenous vehicles and distribution is done through direct routes. Demand of customers follow probability distribution functions and they may face shortage in some planning periods. Optimal distribution plan must determine when to serve customers, how much to deliver to them, and assign them to vehicles and routes. In order to illustrate the applicability of the proposed approach, a numerical example is given.

The current paper can be extended in several ways; first, considering other distribution strategies such as feasible delivery routes or routing. These strategies give more flexibility to supply chain and make it agile into accommodate to changes. Second, considering other types of modeling uncertainty such as fuzzy sets. Estimating probability distribution function for demand requires complete information about demand behavior that sometimes is not available. Whereas estimating fuzzy sets is easy and can be done even with partial information. Third, as number of states grows when problem size grows, the proposed stochastic dynamic program becomes slow in finding the optimal solutions. Therefore, an interesting future research opportunity would be using state reduction methods in the proposed stochastic dynamic program.

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