Multiple Response Optimization with Probabilistic Covariates Using Simultaneous Equation Systems

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KEYWORDS

ABSTRACT

The control of production processes in an industrial environment needs the correct setting of input factors, so that output products with desirable characteristics will be resulted at minimum cost. Moreover, such systems have to meet set of quality character is tics to satisfy customer requirements. Identifying the most effective factors in design of the process which support continuous and continual improvement is recently discussed from different view points. In this study, we examined the quality engineering problems in which several characteristics and factors are to be analyzed through a simultaneous equations system. Besides, the several probabilistic covariates can be included to the proposed model. The main purpose of this model is to identify interrelations among exogenous and endogenous variables, which give important insight for systematic improvements of quality. At the end, the proposed approach is described analytically by a numerical example.

1. Introduction

Making decisions about complex problems involving process optimization and engineering design is strongly depended on well identification of effective factors. From the viewpoint of quality, a process should be designed so that the products could satisfy customer's needs. Quality engineering plans to find the interrelations between input parameters and output quality characteristics.

A common problem in product or process design is the determination of variable settings to optimize different outputs, which are often highly correlated. Several studies have presented approaches addressing multiple quality characteristics but few published papers have focused primarily on the existence of correlation.

When correl ations among quality characteristics are ignored, engineering designer may miss finding design variable settings which simultaneously improved the quality of all the responses. According to the literature, many works have been conducted on using Principal Components Analysis (PCA) to solve correlated multi response problems. PCA converts several correlated columns to independent components by linear transformations. These components are then substituted into multiple original responses. Another approach to solve this problem is based on prediction of the correlation as an individual response variable by Response Surface Methodology (RSM). Each of the mentioned approaches has specific benefits and limitations. PCA cannot provide proper directions for optimization of components. Moreover, if the number of selected components is less than the number of original responses, some information was lost. Consideration of correlation coefficientsas separate response variables requires multi-replicated design for

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experiments. Additionally, the accuracy of estimated correlation is strongly dependent on the number of replications. However, more experiment runs are more costly and time-consuming.

Furthermore, even though there are enough experimental runs, the statistical error in response regression is unavoidable. The last approach in solving multi response optimization problem is multivariate regression method (a.k.a. seemingly unrelated regression) that is very useful when response variables are correlated.

The rest structure of this paper is as follows. Section 2 provides a summary of MRO approaches. In section 3,we present information about the proposed model and its applications. Then, section 4illustrates the method by a numerical example and finally, conclusions are discussed in section 5.

2. Literature Review

Several methods have been presented to solve multi response optimization (MRO). In this section, the literature of multi response problems is reviewed according to the optimization techniques and modeling approaches separately.

2-1. Multi Response Optimization

MRO problems have been studied in several areas from different aspects. We can categorize all viewpoints in the literature into three general categories:

a- Desirability viewpoints: in this category, researchers try to aggregate information of each response to get one response and then conduct one of optimization methods on single objective called total desirability function. In this regard, Derringer and Suich applied a desirability function to optimize multi response problems in a static experiment[1]. Castillo et al.,[2] demonstrated the use of modified desirability functions for optimizing a multi response problem. Sensitivity of optimal solution with respect to major parameters of desirability functions has been recently studied and the effects of weights, range, and shape etc. have been investigated [3].

b- Priority based methods: some cases have responses with different importance. In such problems, we must consider most important response for optimization. Multiple Criteria Decision Making (MCDM) methods are examples in this category. Some of the related works are as follows:

Bashiri and Hejazi [4] used Multiple Attribute Decision Making (MADM) methods such as VIKOR, PROMETHEE II, ELECTRE III and TOPSIS in converting multi response to single response in order to analyze the robust experimental design. The main advantage of their method was to consider standard deviation that contributed to robust experimental design. Because of fitting only one response regression function, the proposed method also decreased statistical error. Tong et al.[5] used VIKOR method in converting Taguchi criteria to single response to find regression model and related optimal setting. Ramezani et al. [6] developed a goal programming-TOPSIS approach to multiple response optimization. The proposed method is benefited by goal programming to find set of non-dominated solutions, in which tradeoff between the responses has been done by prediction intervals, and TOPSIS method to rank those solutions.

c- Loss Function: in this category, all the response values are aggregated based on Taguchi's loss function and converted to a single one.

Many researchers have studied to develop and generalize Taguchi's loss function with respect to a special trait of its cases. In this regard, Pignatiello [7] utilized a variance component and a squared deviation-from-target to form an expected loss function to optimize a multiple response problem. This method is difficult to implement.

The first reason is that a cost matrix must be initially obtained, and the second reason is that it needs more experimental data. Ames et al.[8] presented a quality loss function approach in response surface models to deal with a multi response problem. The basic strategy is to describe the response surfaces with experimentally derived polynomials, which can be combined into a single loss function by known or desired targets.

Next, minimization of the loss function with respect to process inputs locates the best operating conditions. Tong et al.[9] developed a multi response signal to-noise (MRSN) ratio, which integrates the quality loss for all responses to solve the multi response problem. The conventional Taguchi method can be applied based on MSRN and the optimum factor/level combination can be obtained. Su and Tong [10] also proposed a PCA-based approach to optimize a multi response problem. Initially, the quality loss of each response is standardized and then PCA is applied to transform the primary quality responses into fewer quality responses. Finally, the optimum combination of parameters can be obtained by maximizing the summation of standardized quality loss.

2-2. Correlated Multi Response Modeling

In multi response modeling there are often three types of variables: Factors, nuisances and responses. Montgomery[11] categorized nuisance variables into the following three classes: i) Known and controllable variables, ii) Known and uncontrollable variables. iii) Unknown and uncontrollable variables.

When a significant degree of correlation exists among the variables, the standard methods cannot estimate the model precisely and also the optimum results may be unreliable. Chiao and Hamada [12]considered experiments with correlated multiple responses. Analysis of these experiments consists of modeling distributional parameters in terms of the experimental factors and finding factor settings which maximize the

probability of being in a specification region. Kazemzadeh et al. [13] proposed a general framework for multi response optimization problems based on goal programming. They studied some existing works and attempts to aggregate all of characteristics into one approach. Shah et al.[14] illustrated the seemingly unrelated regressions (SUR) method for estimating the regression parameters.

The method can be useful when response variables in MRS problem are correlated that leads to a more precise estimate of the optimum variable setting. Tong et al.[15] also considered correlation of responses and use PCA and TOPSIS methods to find the best variable setting. Antony[16] used PCA in combination of Taguchi's method. In this method, it is assumed that only those of components whose eigenvalues greater than one can be selected to form final response variables.

Thus, their method could not be applied if the problem has more than one component with such characteristic. Tong et al.[15] determined the optimization direction of each component based on corresponding variation mode charts. Furthermore, Wang[17] used TOPSIS to find an overall performance index as a criterion for optimizing the multiple quality characteristics. A summary of correlated multi response optimization methods and a comparison between them are reported in Table1.

In some situations, the experiment is affected by some covariates. To model this condition, Rahbar and Gardiner[18] proposed a non-iterative method of estimation where the response is linearly related to the covariate. Their method was nonparametric, with a single, integer-valued covariate. Wu[19]studied covariate effects in nonlinear mixed-effect models and used maximum likelihood to estimate the regression coefficients.

They confirm the reliability and stability of univariate or multivariable fractional polynomial models. Das et al. [20] proposed optimal structure for RBD, BIBD and CRD designs for better and more reliable analysis of covariates.

Hanson[21]reviewed important works on illegal migrations from Mexico to United States. For this purpose, time series covariates of illegal labor flows were considered as well as composition of legal and illegal populations. Hejazi et al. [22] represented a novel method based on goal programming to find the best combination of factors so as to optimize multi response-multi covariate surfaces with consideration of location and dispersion effects. Moreover, they considered covariate probable values as an objective function which should be maximized. Table 2 compares the studies about the existence of covariate in experimental design and optimization according to the following indices.

➢ Number of Covariates (NC)
➢ Correlation among covariates (CC)
➢ Interaction Effects among covariates (IEC)
➢ Correlation among responses (CR)
➢ Stochastic Covariate (SC)
➢ Focus on Statistical aspects (FSA)
➢ Focus on Optimization aspects (FOA)

Tab. 1. Comparisons of the researches in correlated response optimization.

Method	Solution Space	Location Effect	Dispersion Effect	Interaction Effect	Description
[13]	Continuous	●	●	●	Considers Correlation Coefficient as responses
[14]	Continuous	●		●	Considers Correlation before regressing the response surfaces
[10]	Discrete	●			Application of PCA only in Taguchi method More efficient with single PC),Analyzing by Factor Plot
[15]	Discrete	●			Analyzing by variation mode chart
[12]	Continuous	●	●	●	Consider Correlation Coefficient as responses
[16]	Discrete		●		Use first component and optimize by factor plot
[17]	Discrete	●	●		Use TOPSIS and relative closeness to the ideal solution index for optimization
[23]	Continuous	●		●	Use first PC and optimize the response surfaces
[24]	Continuous	●	●	●	Considers Correlation before regressing the response surfaces Allowing different weight (importance) of responses, gives Pareto optimum in mathematical model

Tab. 2. Comparison of existing methods that consider covariates

Methods	NC	CC	IEF	IEC	CR	SC	FSA	FOA	DE
MANCOVA	Multiple						✓		
[18]	Single						✓		
[20]	Multiple						✓		
[19]	Multiple	✓	✓			✓	✓		
[22]	Multiple	✓	✓	✓		✓		✓	✓
The proposed Method	Multiple	✓	✓	✓	✓	✓	✓	✓	

The proposed method aims to consider all of location effects and correlation among the responses. In addition, probabilistic covariates are included into the multi response model to reduce error terms and uncovered variance.

3. The Proposed Method

In design of experiments (DOE), several variables affect response vector assuming the effect size of input variables is not time-dependent. In this regard, if correlated covariates have meaningful effects on response variables, that assumption might be violated. On the other hand, ignoring the covariates would result into a less explanative model and loss of information. Another assumption in DOE is homogeneity of variance-covariance matrix across the experiment region. This phenomenon is usually related to sectional data. Hence, the present work tries to provide a modeling approach having the capability to cope with aforementioned problems when all types of data are to be considered in an integrated model.

In this study, the response surfaces are fitted through a Simultaneous equations system to optimize correlated multi response problems in which probabilistic covariates affect the experiment results. Consecutive steps of the proposed approach are as follows:

Step1: Identify input and outputs variables.

Step2: Select a proper design and run the experiments.

Step3: Develop a system of equations.

3) a. Define an equation for relations between each response and other variables.

3) b. Define equations presenting relations between covariates.

Step4: Estimate parameters of the system.

Step5: Construct multi-objective optimization model including response surfaces and the occurrence of covariates as objective functions.

Step6: Apply Global Criterion (GC) method to solve the multi-objective optimization model.

In Section4 these steps are discussed in details.

When the problems involve several equations with common variables, it is recommended to estimate the parameters through a system of equations simultaneously. Various methods have been proposed to solve such problems. Some of the more efficient methods are Ordinary Least Squares (OLS), Cross-Equation Weighting method, Seemingly Unrelated Regression (SUR), Two-Stage Least Squares (2SLS), Weighted Two-Stage Least Squares (WTSLS), Three-stage least squares (3SLS), Full Information Maximum Likelihood (FIML), and The Generalized Method of Moments (GMM).

Ordinary Least Squares minimizes the sum-of-squared residuals for each equation accounting for any cross-equation restrictions on the parameters of the system.

Cross-Equation Weighting method considers cross-equation heteroscedasticity by minimizing the weighted sum-of-squared residuals. The equation weights are the inverses of the estimated equation variances, and are derived from unweighted estimation of the parameters.

The seemingly unrelated regression (SUR) method, also known as the multivariate regression or Zellner's method, estimates the parameters of the system, accounting for heteroscedasticity and contemporaneous correlation in the errors across equations.

System Two-Stage Least Squares (STSLS) estimator is an appropriate technique when some of the right-hand side variables are correlated with the error terms and there is neither heteroscedasticity nor contemporaneous correlation in the residuals.

Weighted Two-Stage Least Squares (WTSLS) estimator is the two-stage version of the weighted least squares estimator. WTSLS is an appropriate technique when some of the right-hand side variables are correlated with the error terms and there is heteroscedasticity, but no contemporaneous correlation in the residuals.

Three-Stage Least Squares (3SLS) is the two-stage least squares version of the SUR method. It is an appropriate technique when right-hand side variables are correlated with the error terms, and there is both heteroscedasticityand contemporaneous correlation in the residuals.

Full Information Maximum Likelihood (FIML) estimates the likelihood function under the assumption that the contemporaneous errors have a joint normal distribution.

The Generalized Method of Moments (GMM) estimator belongs to a class of estimators, known as M-

estimators, defined by minimizing some criterion function. GMM is a robust estimator that does not require information of the exact distribution of the disturbances.

GMM estimation is based upon the assumption that the disturbances in the equations are uncorrelated with a set of instrumental variables. The GMM estimator selects parameter estimates so that the correlations between the instruments and disturbances are as close to zero as possible, as defined by a criterion function. In Table 3 aforementioned methods are compared with respect to the main characteristics.

Tab.3. Characteristics of the major methods of system estimation.

Method of estimation	Limiting assumptions				
	Normality	Homoscedasticity	IPE[1]	IET[2]	Instruments
OLS	-	*	*	*	No
Cross-Equation Weighting	-	-	*	*	No
SUR ([25])	-	-	-	-	No
2SLS ([26])	-	*	-	*	Yes
WTSLS	-	-	-	*	Yes
3SLS([27])	-	-	-	-	Yes
FIML([28])	*	-	-	-	No
GMM ([29])	-	-	*	-	Yes

1- Independency between Predictors and Errors 2- Independent error terms

3-1. Model Representation

A general multi response problem can be expressed as:

$$\min R(x) = \begin{pmatrix} \widehat{R_1}(x) \\ \widehat{R_2}(x) \\ \vdots \\ \widehat{R_p}(x) \\ f(c) \end{pmatrix} \tag{1}$$

Subject to:

$$l < x < u$$
$$lcl < c < ucl$$

Where $\widehat{R}(x)$ represents response surface vector for STB type quality characteristics, $f(c)$ is the probability function of covariate vector, x is vector of factors, and c is covariate vector. Furthermore, it is assumed that the process is statistically under control and the control range is (lcl, ucl).

3-2. Optimization Method (Global Criterion)

This method allows one to transform a multi-objective optimization problem into a single-objective problem. The function traditionally used in this method is distance. The multi-objective method can be written as follows:

$$\text{Optimize} \quad F'(x) = \left(\sum_i w_i \left| \frac{Z_i - \widehat{R}_i(x)}{d_i} \right|^r \right)^{\frac{1}{r}} \tag{2}$$

Subject to   The same constraints

where Z_i is the optimum value of problem objective function when only ith objective was considered, w_i is a value representing importance of each objective and d_i is the range of ith response within the observed experimental runs [30]. In this study GC method was applied to convert problem into single objective form.

Tab. 4.Results of designed experiments for numerical example.

Std	Run	Time(x_1)	Heat (x_2)	Catalyst (x_3)	Humidity (c_1)	Temp (c_2)	Conversion (R_1)	Activity (R_2)
1	1	-1.000	-1.000	-1.000	41%	16.7	74.000	53.200
2	2	1.000	-1.000	-1.000	55%	17.3	51.000	62.900
3	3	-1.000	1.000	-1.000	67%	19.3	88.000	53.400
4	4	1.000	1.000	-1.000	55%	12.3	70.000	62.600
5	5	-1.000	-1.000	1.000	12%	11.5	71.000	57.300
6	6	1.000	-1.000	1.000	95%	18.5	90.000	67.900
7	7	-1.000	1.000	1.000	65%	19.2	66.000	59.800
8	8	1.000	1.000	1.000	96%	16.5	97.000	67.800
15	9	0.000	0.000	0.000	30%	13.2	81.000	59.200
16	10	0.000	0.000	0.000	59%	14.0	75.000	60.400
17	11	0.000	0.000	0.000	46%	16.4	76.000	59.100
18	12	0.000	0.000	0.000	57%	16.4	83.000	60.600
9	13	-1.682	0.000	0.000	59%	13.5	76.000	59.100
10	14	1.682	0.000	0.000	33%	13.9	79.000	65.900

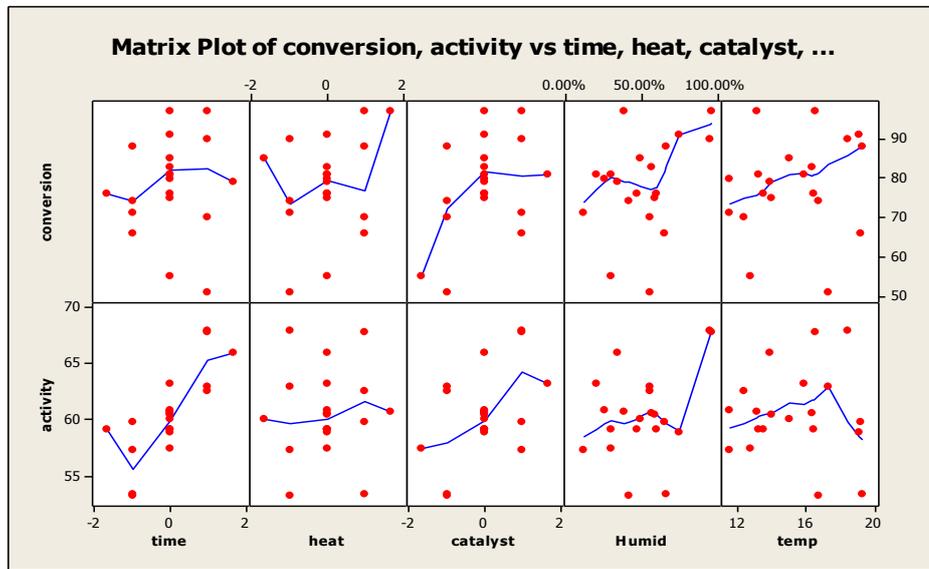| Std	Run	Time (x_1)	Heat (x_2)	Catalyst (x_3)	Humidity (c_1)	Temp (c_2)	Conversion (R_1)	Activity (R_2)
11	15	0.000	-1.682	0.000	48%	15.0	85.000	60.000
12	16	0.000	1.682	0.000	38%	13.1	97.000	60.700
13	17	0.000	0.000	-1.682	29%	12.7	55.000	57.400
14	18	0.000	0.000	1.682	20%	15.8	81.000	63.200
19	19	0.000	0.000	0.000	25%	11.5	80.000	60.800
20	20	0.000	0.000	0.000	75%	19.1	91.000	58.900

Fig. 1. Scatter plot for the experimental data

4. Numerical Example

This section is organized to demonstrate the computational steps of the proposed approach. For this purpose, a numerical example from the literature is considered with some modifications [11].

Step 1: A chemical experiment with three controllable variables and two covariates is analyzed by the proposed method. The outputs are conversion (Y1) and activity (Y2) levels. Humidity (c1) and environment temperature (c2) are considered as probabilistic covariates. Table 4 shows the results of experiments gathered by a Central Composite Design (CCD).

Step 2: First and foremost, the existing relationship between inputs and outputs should be assessed. To do so, the values of response variables are plotted across the input levels as illustrated in Fig. 1. It seems that the

Heat (x2) and Time (x1) did not significantly affect the Activity and Conversion respectively.

Understanding the strong effects helps us to fit better surfaces of response variables. To confirm the above conclusions gained from the matrix plot, the experiment is also analyzed by response surface methodology. The existing relationships assessed by RSM are:

$$\begin{aligned} Y_1 &\propto x_2, x_3, x_2^2, x_3^2, x_2 x_3, \\ Y_2 &\propto x_1, x_3 \end{aligned} \tag{3}$$

Step3: Moreover, Fig.1 shows that the covariates are likely to have meaning ful effects. But due to the collinearity among them, RSM should be performed through a simultaneous system of equations.

Tab. 5. Estimated equations in the system using FIML and ISUR method

Method	Estimated system
ISUR	$R_1(X,C,Y) = 81.8 + 4.15\,x_2 + 6.2 x_3 - 5.13 x_3^2$ $+ 11.45 x_2 x_3$ $R_2(X,C,Y) = 67.37 + 3.23 x_1 + 2.49 x_3 + 7.21\,c_1$ $- 0.47\,c_2 - 0.042\,Y_1$ $c_1(c_2) = -0.4 + 0.06\,c_2$
FIML	$R_1(X,C,Y) = 81.8 + 4.15 x_2 + 6.2 x_3 - 5.13 x_3^2$ $+ 11.45 x_1 x_3$ $R_2(X,C,Y) = 66.14 + 3.23 x_1 + 2.49 x_3 + 4.1\,c_1$ $- 0.29\,c_2 - 0.042\,Y_1$ $c_1(c_2) = -0.4 + 0.06\,c_2$

In this case, the problem is analyzed by Iterative Seemingly Unrelated Regression (ISUR) and Full Information Maximum Likelihood (FIML). The response surfaces regressed by the mentioned methods are given below.

Since, there is a significant linear relation between two covariates, it is reasonable to consider a bivariate distribution for their treatments. It observed that these two covariates follow a normal distribution with following parameters:

$$c_1 \sim N(0.5032,(0.2278)^2),$$
$$c_2 \sim N(15.30,(2.581)^2), \text{ and} \qquad (4)$$
$$\rho(c_1,c_2)=0.655$$

Consider the above distributions as marginal probability functions of c1 and c2. Therefore, the bivariate normal probability distribution for the covariates can be estimated as follows:

$$C \sim N_2 \left(\underbrace{\begin{pmatrix} 0.5032 \\ 15.30 \end{pmatrix}}_{\mu}, \underbrace{\begin{pmatrix} (0.2278)^2 & 0.391 \\ 0.391 & (2.581)^2 \end{pmatrix}}_{\sigma} \right)$$

$$\qquad (5)$$

$$f(C) = \frac{e^{-\frac{1}{2}\left[\binom{c_1}{c_2} - \binom{0.5032}{15.30}\right]^T \begin{bmatrix} 0.5190 & 0.391 \\ 0.3916 & 0.662 \end{bmatrix}^{-1} \left[\binom{c_1}{c_2} - \binom{0.5032}{15.30}\right]}}{2\pi \left(\left| \begin{matrix} 0.5190 & 0.391 \\ 0.3916 & 0.662 \end{matrix} \right| \right)^{1/2}}$$

Step4. Construct the mathematical program.

Two response surfaces and one probability function are to be optimized as objective functions with respect to input variables constrained by their specification limits. Therefore, the multi-objective mathematical program for this problem is developed in which the decision variables consist of three factors and two interdependent covariates.

$$\text{Max } F = \begin{pmatrix} f(C) \\ R_1(X,C,Y) \\ R_2(X,C,Y) \end{pmatrix} \qquad (6)$$

S.t,

$$\begin{pmatrix} -1.68 \\ -1.68 \\ -1.68 \end{pmatrix} \leq \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} \leq \begin{pmatrix} 1.68 \\ 1.68 \\ 1.68 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 8.43 \end{pmatrix} \leq \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \leq \begin{pmatrix} 1 \\ 22.17 \end{pmatrix}$$

Table 6 gives a summary of optimal solutions obtained by solving the above model for each objective functions separately.

Tab. 6. Trade off matrix for Global Criterion Method

Single Objective	Optimal Value			Method
	Z	R_1	R_2	
$Z(C)$	0	100	68.689	
$R_1(X,C,Y)$	-0.536	100	66.4479	ISUR
$R_2(X,C,Y)$	-3.628	100	68.968	
$Z(C)$	0	100	64.98	
$R_1(X,C,Y)$	-3.628	100	68.971	FIML
$R_2(X,C,Y)$	-3.628	100	68.971	
Best observed	-2.68	97	67.9	
Worst observed	-0.0061	51	53.2	

According to Table 6, the final multi-objective mathematical model using Global Criterion can be constructed by replacing the objective functions of the above multi-objective program as Equation (7).

$$\text{Min } GC = \left(\left(\frac{Z(C) - 0}{-2.68 + 0.0061} \right)^2 + \left(\frac{R_1(X,C,Y) - 100}{97 - 51} \right)^2 + \left(\frac{R_2(X,C,Y) - 68.968}{67.9 - 53.2} \right)^2 \right)^{1/2} \qquad (7)$$

Tab. 7. Optimal results of the numerical example

Method	X	C	$Z(C)$ $f(C)$	$R_1(X,C,Y)$	$R_2(X,C,Y)$
ISUR	$\begin{pmatrix} 1.519 \\ -1.68 \\ 1.68 \end{pmatrix}$	$\begin{pmatrix} 0.534 \\ 15.246 \end{pmatrix}$	-0.0201 0.355	100	68.93
FIML	$\begin{pmatrix} 1.519 \\ -1.68 \\ 1.68 \end{pmatrix}$	$\begin{pmatrix} 0.5305 \\ 15.22 \end{pmatrix}$	-0.018 0.356	100	68.83
[23]	$\begin{pmatrix} -1.68 \\ +1.68 \\ -1.68 \end{pmatrix}$	$\begin{pmatrix} 0.091 \\ 22.17 \end{pmatrix}$	-3.627 0.01	100	62.62

In this example, we consider the same important degrees for all objective functions. Table 7 shows the optimal solution and the related objective values for this example.

The results support the claim that the PCA based methods cannot correctly find optimization direction. It is also observed that most probable values of covariates would lead into the more reliable results. The PCA method reaches the target of first objective due to the large coefficient of first response in the first PC. It seems PCA is more useful for correlated predictors rather than correlated multi response problems.

5. Conclusion

This study presents a multi response optimization approach in which correlated covariates are to be considered as well as correlated responses. Most existing MRO works used PCA to gain uncorrelated responses, but they usually disregarded the proper direction of location effects. As a remedy, this study tries to model the multi response-multi covariate problem in a simultaneous system of equations and use the estimated equations to construct an optimization program. The proposed approach also has following features:

(1) The suggested method constructs a model for multi response-multi covariate problems having several relations among inputs and outputs.

(2) Covariates with known distribution function can be analyzed in this approach,

(3) It takes covariance structure among responses into account,

(4) It provides the capability to cope with Collin earity among covariates, and

(5) In contrast to the most PCA-base methods, the desired direction for optimization of responses doesn't change after modeling and optimization.

As future research, consideration of qualitative variables in proposed method is suggested. In this work, only the variances of observed values are considered. Therefore, considering the variances of predicted responses can be as another future research in this subject. Some decision making methods for extracting the weights of response variables may add benefits to the analysis procedures proposed in this research.

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