

# Capacitated Facility Location Problem with Variable Coverage Radius in Distribution System

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## KEYWORDS

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## ABSTRACT

*The problem of locating distribution centers (DCs) is one of the most important issues in design of supply chain. In previous researches on this problem, each DC could supply products for all of the customers. But in many real word problems, DCs can only supply products for customers who are in a certain distance from the facility, coverage radius. Thus, in this paper a multi-objective integer linear programming (MOILP) model is proposed to locate DCs in a two-echelon distribution system. In this problem, customers who are in the coverage radius of the DCs can be supplied. Moreover, we suppose that the coverage radius of each DC can be controlled by decision maker and it is a function of the amount of money invested on the DC. Finally, a random generated problem is used to verify the model and the computational results are presented.*

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## 1. Introduction

Efficient design of supply chain is one of the most important issues that have received considerable attention. Among various type of problem in supply chain management, facility location problem (FLP) has been significantly considered. In facility location problem the goal is to locate a set of new facilities such that the cost of establishing facilities at candidate location and transportation cost from factories (producers) to distribution centers (DCs) and from DCs to customers (demand points) are minimized. In this problem, the decision maker seeks to determine the number of facilities, the location of the facilities, capacities of the facilities, allocate products to facilities, and the flow of products between facilities [1]. Early mixed-integer models to formulate this problem have been proposed by Baumol and Wolfe [2] and Kuehn and Hamburger [3]. Some researchers have also considered the location of producers as decision

variable [4], [5], [6], and [7]. Some cases have investigated the uncapacitated facility location problem (UFLP) where no limit is assigned to facilities and producers [8], [9], [10], [4], and [11]. On the other hand, in some other cases, the problem has been formulated as capacitated facility location problem (CFLP) [12], [13], [14], [6], and [15]. Maximum storage capacity in DCs or maximum producing capacity for plant centers results in CFLP. Figure1. Shows a two-echelon distribution system in which products are produced in factories and shipped to customers through distribution centers. Various elements and characteristics of facility location problem in supply chain have been considered in the literature, such as:

- Number of products
- Type of products
- Planning horizon
- Type of facilities
- Location-allocation decisions
- Capacity of facilities
- Transportation channels
- Number of echelons
- Stochastic costs and demands
- Transportation and facility associated risks

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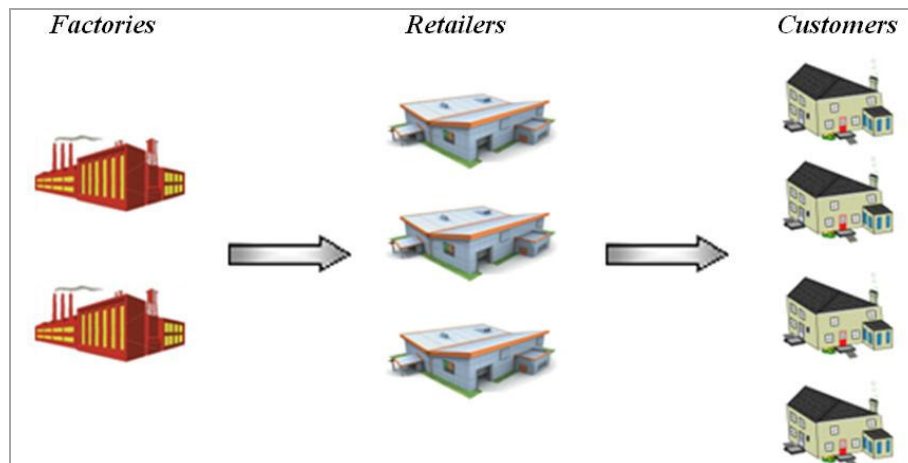


Fig. 1. A two-echelon supply chain

One of the most popular facility location problems is the Maximal Covering Location Problem (MCLP) proposed by Church and ReVelle [16]. A customer is called a covered customer if he/she is located within a specified distance  $R$ , coverage radius, away from the closest facility. In this problem,  $m$  facilities must be established according to a set of candidate facility location sites with the objective of maximization of the total weight of covered customers. For more information about applications and solution methods the reader is referred to [17], [18], and [19]. The proposed problem is solved by linear programming when the optimal solution is integer and if it is not integer, branch and bound algorithm is used to solve the resulted problem [20].

Another version of the problem was considered by Drezner [21] and [22] and by Watson-Gandy [23]. A more general family of set covering problems, of which MCLP is a special case, is also studied in Hochbaum and Pathria [24].

One main assumption in the Maximal Covering Location Problem is the fixed coverage radius of each facility. In other words, coverage radius is an input parameter to the MCLP and is not a decision variable, while in many real cases the physical characteristic of a facility may change the coverage radius of the facility. Berman et al. [25] investigated the covering problem with variable radius. They proposed a model in which the coverage radius of each facility is a function of facility establishing cost and is determined by the decision maker.

This problem has real world applications such as: locating light posts whose illuminating strength depends on the intensity of each bulb, locating radio station that by increasing the signal power of each station the covering area of the station increases, and locating retailers such that larger retailers has greater coverage area. Recently, Jabalameli et al. [26] relaxed all of the assumption of maximal covering location problem and developed a MCLP which combined the characteristics of gradual cover models, cooperative cover models, and variable radius models.

In many real world problems, distribution centers can only supply products for customers who are in certain distance (covering radius) from the DC. In addition, the coverage radius of each DC is related to the physical characteristics of that DC. Increasing the size of each facility, for instance, makes the coverage radius of the facility larger. Therefore, instead of considering a predefined coverage radius, it is more reasonable to consider available budget to locate the facilities in distribution system and the coverage radius of each facility (DC) is determined as a function of amount of money allocated to establish the facility.

This paper discusses a facility location problem with capacitated distribution centers in a two-echelon supply chain including factories, distribution centers and customers. The problem is formulated under the following assumptions:

- Each DC can supply a customer's demand if the customer is located in the covering radius of the facility.
- The available budget to locate the facilities is limited
- The coverage radius of facility enhances by increasing the size of the facility.
- The capacity of each facility is limited.

Therefore, the goal of the problem is to determine the number, location and covering radius of the facilities to:

- Maximize total covered demands
- Minimize the transportation costs in the supply chain.

The organization of the paper is as follows. The mathematical model of the problem is proposed in section 2. Section 3 provides the solution procedure. Computational example is reported in section 4 and finally, conclusions are discussed in section 5.

## 2. The Mathematical Formulation

### 2-1. The Proposed Mathematical Model

Assume that  $I$ ,  $J$ , and  $N$  are the set of indexes of factories, distribution centers and customers,

respectively. Following parameters and variables have been used to formulate the problem.

#### Parameters:

$c_{ij}$	unit transportation cost from factory $i$ to DC $j$
$c_{jn}$	unit transportation cost from DC $j$ to customer $n$
$d_{ij}$	distance between factory $i$ and DC $j$
$d_{jn}$	distance between DC $j$ and customer $n$
$D_n$	demand quantity of customer $n$
$H_j$	capacity of DC $j$
$G_i$	production capacity of factory $i$
$B$	total available budget
$F_j$	fixed cost of constructing a facility at location $j$
$\phi_j(r)$	variable cost of constructing a facility at location $j$ with coverage radius $r$

#### Decision Variables:

$Z_j = \begin{cases} 1 & \text{if a facility is located at location } j \\ 0 & \text{otherwise} \end{cases}$	
$Y_{jn} = \begin{cases} 1 & \text{if customer } n \text{ is assigned to facility } j \\ 0 & \text{otherwise} \end{cases}$	
$x_{ij}$	amount of products transported from factory $i$ to facility $j$
$r_j$	the unknown coverage radius of facility

It is assumed that the cost of constructing a facility with coverage radius  $r > 0$  is  $F_j + \phi_j(r)$ . Variable cost of constructing a facility is usually assumed to be a non-negative, non-decreasing function of  $r$  which represents the cost of locating a DC with coverage radius  $r$  at location  $j$ . Different coverage cost functions have been proposed in the literature, such as  $\phi_j(r) = cr^2$ ,  $\phi_j(r) = cr$ , and  $\phi_j(r) = c\sqrt{r}$  [25]. The coverage radius of each facility is determined according to the farthest assigned customer to the facility [25]:

$$r_j = \max_{n \in N} \{y_{jn} d_{jn}\}$$

Thus, the total cost of constructing facilities is:

$$\sum_{j \in J} F_j Z_j + \sum_{j \in J} (\phi_j(r_j))$$

Therefore, the mathematical model of the problem is as follows:

$$\min \sum_{i \in I} \sum_{j \in J} c_{ij} d_{ij} x_{ij} + \sum_{j \in J} \sum_{n \in N} D_n y_{jn} c_{jn} d_{jn}, \quad (1)$$

$$\max \sum_{j \in J} \sum_{n \in N} D_n y_{jn}, \quad (2)$$

$$s.t. \sum_{j \in J} F_j Z_j + \sum_{j \in J} (\phi_j(\max_{n \in N} \{y_{jn} d_{jn}\})) \leq B \quad (3)$$

$$\sum_{n \in N} y_{jn} D_n \leq Z_j H_j \quad j \in J, \quad (4)$$

$$\sum_{j \in J} x_{ij} \leq G_i \quad i \in I, \quad (5)$$

$$\sum_{i \in I} x_{ij} = \sum_{n \in N} y_{jn} D_n \quad j \in J, \quad (6)$$

$$\sum_{n \in N} y_{jn} \leq 1 \quad n \in N, \quad (7)$$

$$y_{jn}, Z_j \in \{0, 1\}, \quad (8)$$

$$x_{ij} \geq 0, \quad (9)$$

In this model, the first objective function minimizes total cost of products transportation from factories to DCs and from DCs to customers while the second objective function is to maximize total covered demand. Constraint (3) ensures that the total establishing cost of the facilities does not exceed the available budget.

Constraint (4) indicates that total transported products from facility  $j$  do not exceed the capacity of the facility. Similarly, constraint (5) represents that total transported products from factory  $i$  must be less than the production capacity of the factory. Constraint (6) ensures that the total amount of products enter facility  $j$  are equal to the total amount of products leave that facility; in that, the product balance equation for facility  $j$ . Constraint (7) indicates that each demand point must be assigned to at most one facility. Finally, constraint (8) and (9) show the type and range of the variables.

#### 2-2. Linearization of the Model

If  $\phi$  has a linear relation to  $r$ , non-linear constraint (3) could be changed to a linear constraint. By assuming that  $r_j = \max_{n \in N} \{y_{jn} d_{jn}\}$ , and adding the new constraint  $r_j \geq d_{jn} y_{jn}$ , the new form of the mathematical model is as follows:

$$\min \sum_{i \in I} \sum_{j \in J} c_{ij} d_{ij} x_{ij} + \sum_{j \in J} \sum_{n \in N} D_n y_{jn} c_{jn} d_{jn},$$

$$\max \sum_{j \in J} \sum_{n \in N} D_n y_{jn},$$

$$\begin{aligned}
s.t. \quad & \sum_{j \in J} F_j Z_j + \sum_{j \in J} (\varphi_j(r_j)) \leq B \\
& r_j \geq d_{jn} y_{jn} \quad j \in J, n \in N, \\
& \sum_{n \in N} y_{jn} D_n \leq Z_j H_j \quad j \in J, \\
& \sum_{j \in J} x_{ij} \leq G_i \quad i \in I, \\
& \sum_{i \in I} x_{ij} = \sum_{n \in N} y_{jn} D_n \quad j \in J, \\
& \sum_{n \in N} y_{jn} \leq 1 \quad n \in N, \\
& y_{jn}, Z_j \in \{0, 1\}, \\
& x_{ij}, r_j \geq 0,
\end{aligned}$$

### 3. Solution Procedure

To solve the proposed model, bounded objective function method has been used. In this method, the single most important objective function is optimized and all other objectives play the role of constraints [27]. Thus, the second objective function which maximizes total covered demands forms the additional constraint:

$$\sum_{j \in J} \sum_{n \in N} D_n y_{jn} \geq \alpha \left( \sum_{n \in N} D_n \right)$$

Therefore, the single objective form of the model is as follow:

$$\begin{aligned}
\min \quad & \sum_{i \in I} \sum_{j \in J} c_{ij} d_{ij} x_{ij} + \sum_{j \in J} \sum_{n \in N} D_n y_{jn} c_{jn} d_{jn}, \\
s.t. \quad & \sum_{j \in J} \sum_{n \in N} D_n y_{jn} \geq \alpha \left( \sum_{n \in N} D_n \right) \\
& \sum_{j \in J} F_j Z_j + \sum_{j \in J} (\varphi_j(r_j)) \leq B \\
& r_j \geq d_{jn} y_{jn} \quad j \in J, n \in N, \\
& \sum_{n \in N} y_{jn} D_n \leq Z_j H_j \quad j \in J, \\
& \sum_{j \in J} x_{ij} \leq G_i \quad i \in I, \\
& \sum_{i \in I} x_{ij} = \sum_{n \in N} y_{jn} D_n \quad j \in J, \\
& \sum_{n \in N} y_{jn} \leq 1 \quad n \in N, \\
& y_{jn}, Z_j \in \{0, 1\}, \\
& x_{ij}, r_j \geq 0,
\end{aligned}$$

where  $\alpha$  is the minimum percent of total demand that must be supplied.

Tab. 1. Parameters values to generate the problem

Parameter	Value
$D_n$	U(10,100)
$F_j$	U(3000,5000)
$\varphi_j(r)$	$\varphi_j(r) = c_j r$
$c_j$	U(100,300)
$H_j$	U(300,500)
$\{G_1, G_2, G_3\}$	$\{500, 700, 600\}$
$c_{ij}, c_{jn}$	0.5

### 4. Computational Example

To validate and examine the performance of the model, a problem consisting of 3 factories, 10 candidate location for DCs and 30 demand points have been generated. Demand quantity of each demand point is randomly generated between [10, 100]. Demand quantity of each customer is shown in Figure 2. The establishing cost and variable cost of each facility are random numbers between [3000, 5000] and [100, 300], respectively. Table 1.

Briefly shows the values of each parameter. As it was mentioned before, we have applied the Bounded Objective method to solve the model. As a result, the second objective function was changed to constraint. Let us suppose that the values of  $\alpha$ , which the decision maker is willing to consider are:  $\alpha \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ .  $\alpha=0.5$ , for instance, means that the decision maker needs half of the customers to be covered while when  $\alpha=1$ , all of the customers must be covered.

The problem was solved for different available budget sizes ( $B \in \{18000, 20000, 22000, 24000\}$ ). We used CPLEX to solve the generated problem. Table 2. and Figure 3. show the results of the problem for different combinations of available budget size and coverage percentage. The first column of each table is the value of  $\alpha$ . The second and third column gives the objective function values and total established DCs, respectively. The last column represents total number of covered demand points. Some facts are obvious in this table. For instance, by increasing the value of  $\alpha$ , the objective function value (total cost of transportation) increases. Besides, when  $B=18000, \alpha=1$ , there is no feasible solution for the problem; in that, when the available budget is 18000, it is impossible to cover all of the demand points. Also, in a certain level of  $\alpha$ , by increasing the amount of available budget, total transportation cost decreases. For example, when  $\alpha=1$ , transportation cost for  $B=20000, 22000$ , and  $24000$  are 10172, 9610, and 9372, respectively. The decision maker selects the best alternative according to the coverage percentage ( $\alpha$ ), total amount of available budget size, and transportation cost.

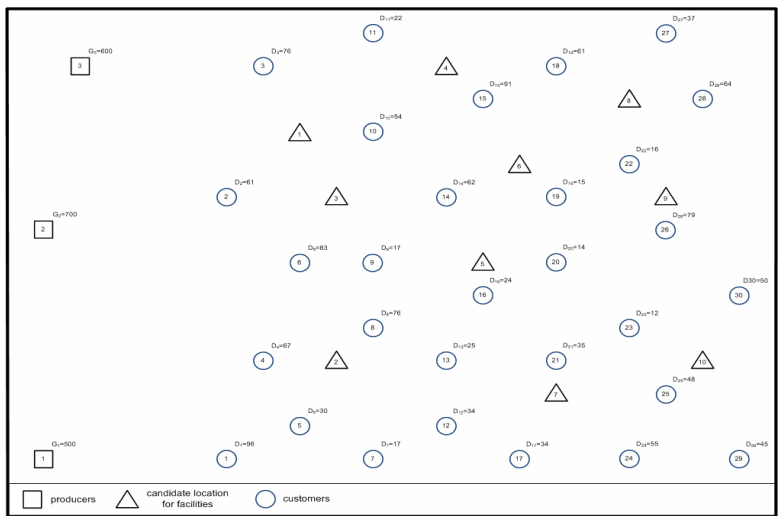


Fig. 2. Network of the problem

Tab. 2. Objective function value, number of constructed facilities and Total covered demands

B=18000				B=20000			
$\alpha$	Objective function	# of facilities	Total covered demands	$\alpha$	Objective function	# of facilities	Total covered demands
0.5	3909	4	715	0.5	3659	4	712
0.6	4682	3	850	0.6	4603	3	845
0.7	5891	3	994	0.7	5666	3	983
0.8	7433	3	1130	0.8	7252	3	1150
0.9	9697	3	1265	0.9	8865	4	1318
1	infeasible			1	10172	4	1400

B=22000				B=24000			
$\alpha$	Objective function	# of facilities	Total covered demands	$\alpha$	Objective function	# of facilities	Total covered demands
0.5	3651	4	715	0.5	3651	4	715
0.6	4661	4	872	0.6	4661	4	872
0.7	5663	4	990	0.7	5623	4	983
0.8	6745	4	1123	0.8	6618	4	1123
0.9	8499	4	1308	0.9	8244	4	1264
1	9610	4	1400	1	9372	4	1400

Figure 3 shows total cost of transportation for various values of coverage percentage ( $\alpha$ ) and available budget (B). By increasing the amount of available budget, total

transportation cost decreases. As an illustration, Figure 4. shows distribution details when B=20000 and  $\alpha=0.9$ .

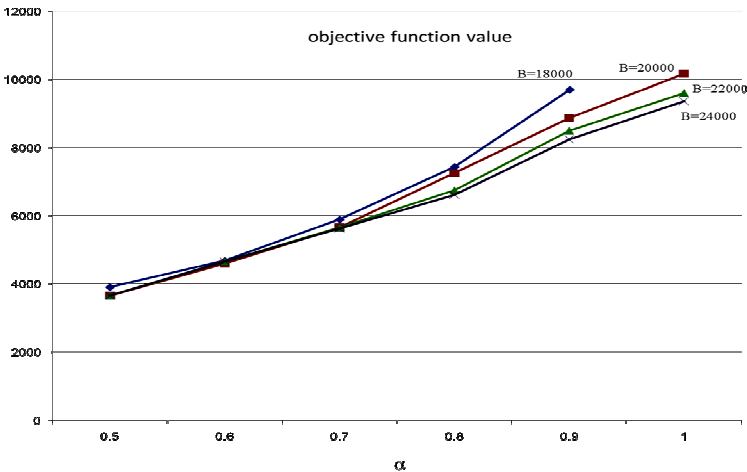
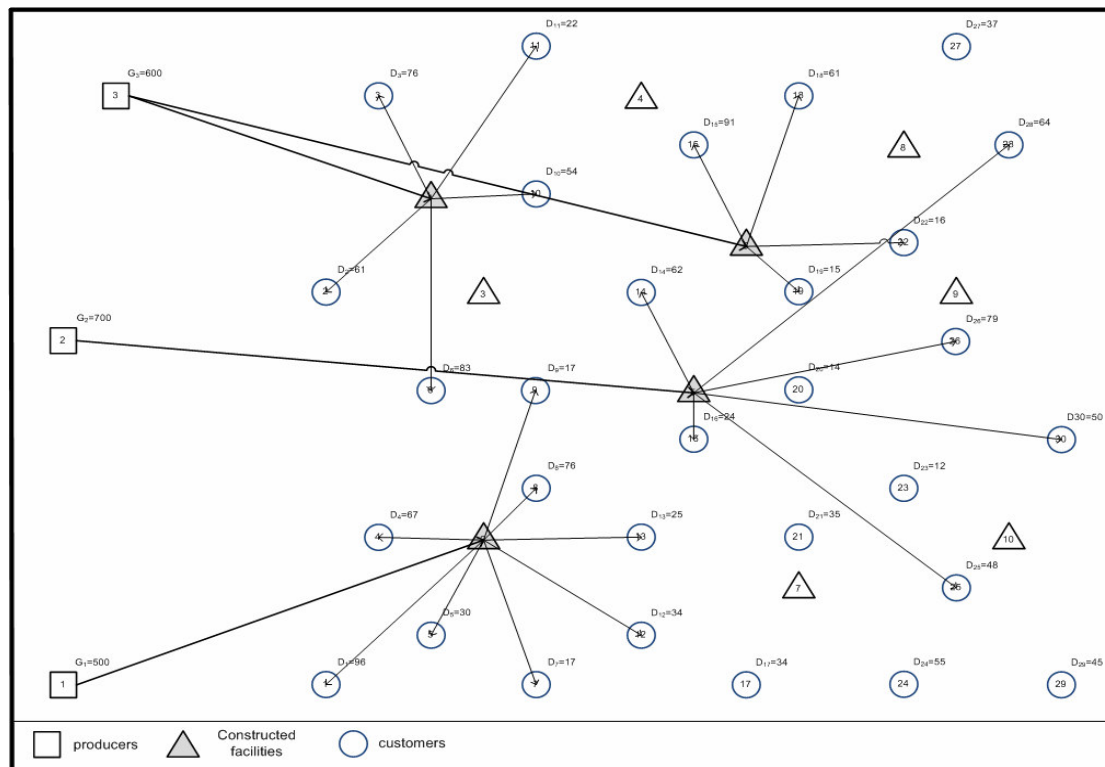


Fig. 3. Sensitivity analysis for  $\alpha$  and associated transportation cost in different available budget

Fig. 4. Details of the solution when  $B=20000$  and  $\alpha=0.9$ 

## 5. Conclusions

In this paper we proposed a mathematical model to determine the number of facilities along with place and coverage radius of each facility in a two-echelon distribution system. In this problem, customers are supplied by a facility if they are in the coverage radius of the facility which varies according to the amount of investment in establishing a facility at a set of candidate locations. The objectives of the model were: 1- minimization of total transportation cost, and 2- maximization of total covered demands. Bounded objective function method was used to solve the proposed model and the model was verified by a random generated problem. As an area for further study, the capacity of each facility may be considered as a variable and non-decreasing function of establishing cost of each facility. In other words, larger facilities with greater investment in construction have larger capacities.

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