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A new probabilistic particle swarm optimization algorithm for size optimization of spatial truss structures

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Abstract

In this paper, a new enhanced version of the Particle Swarm Optimization (PSO) is presented. An important modification is made by adding probabilistic functions into PSO, and it is named Probabilistic Particle Swarm Optimization (PPSO). Since the variation of the velocity of particles in PSO constitutes its search engine, it should provide two phases of optimization process which are: exploration and exploitation. However, this aim is unachievable due to the lack of balanced particles' velocity formula in the PSO. The main feature presented in the study is the introduction of a probabilistic scheme for updating the velocity of each particle. The Probabilistic Particle Swarm Optimization (PPSO) formulation thus developed allows us to find the best sequence of the exploration and exploitation phases entailed by the optimization search process. The validity of the present approach is demonstrated by solving three classical sizing optimization problems of spatial truss structures.

Keywords: *Particle swarm optimization, Probabilistic particle swarm optimization, Spatial truss structures.*

1. Introduction

The aim of the optimization of an engineering project in the design stage, known as *optimal design*, has become an important issue and it has been under the focus of researchers. In 1960s, the meta-heuristic optimization algorithms emerged and because of having fewer limitations compared to the mathematical programming approaches, these algorithms became popular among the researchers. Many meta-heuristic algorithms, were developed inspired by natural evolution such as *Evolutionary Algorithm* of Fogel et al. [1] Genetic algorithm by Holland [2], the laws of physics and mechanics such as Simulated annealing of Kirkpatrick et al. [3], and *Charged System Search* (*CSS*) of Kaveh and Talatahari [4] with example applications in [5,6], and natural interactions between animals and insects such as *Ant Colony Optimization* of Dorigo et al. [7] or Bees Algorithm of Pham et al. [8]

Particle Swarm Optimization (PSO) is another meta-heuristic algorithm which was inspired by social interactions among swarms and flocks of birds and was formulated by Eberhart and Kennedy [9] in order to be applied in engineering design optimization problems.

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In this algorithm, each particle moves towards the best located particle and the best location ever of the particle itself. As the optimization search progresses, the velocity of each particle is updated and used for defining the new position of that particle in the search space. The new position is evaluated in terms of cost function: for example, in minimization problems, the particle with less cost function value is considered better located.

From the aforementioned facts, one can conclude that the definition of the particles' velocity is the search engine of this algorithm and it should provide two phases of the optimization process including exploration (global search), and exploitation (local search). In the exploration phase, the whole search space is swept and the region of the global minimum is found. Then the optimization process is progressed by correction of the position of the particles in the found region. This leads the particles to move towards the global minimum of the cost function with an approximation.

However, the velocity updating scheme utilized in classical PSO has a steady form and hence does not distinguish the different contributions that exploration and exploitation give to the optimization process in the current iteration. To overcome this limitation, a new velocity updating scheme based on the probability of performing global or local search is proposed in the paper.

Based on this approach, in the process of the iterations of the algorithm, PSO will perform global or local search with a predefined probability and thus, provides two phases of search for the algorithm.

The best formulation of the new probabilistic PSO algorithm is obtained by comparing different variants. An important modification is the randomly varying inertia weight that provides simultaneous global and local search capability thus enhancing the overall performance of the algorithm.

Results obtained in three classical weight minimization problems of truss structures demonstrate the validity of the proposed approach.

2. Particle Swarm Optimization

PSO utilizes velocity vectors to update the current position of each particle in the swarm. The position of each particle in the swarm which adapts to its environment by flying in the direction of the best position of the entire particles and the best position of particle itself constitute the search of the PSO. The position of *i*th particle at iteration *k+1* can be calculated using the following relationship:

$$x_{k+1}^{i} = x_{k}^{i} + v_{k+1}^{i}.\Delta t \tag{1}$$

Where, x_{k+1}^{i} is the new position; x_{k}^{i} is the position at iteration *k*; v_{k+1}^{i} is the updated velocity vector of the *i*th particle; and Δt is the time step which is considered as unity. The velocity vector of each particle is determined by:

$$v_{k+1}^{i} = w.v_{k}^{i} + c_{1}.r_{1}.\frac{\left(p_{k}^{i} - x_{k}^{i}\right)}{\Delta t} + c_{2}.r_{2}\frac{\left(p_{k}^{g} - x_{k}^{i}\right)}{\Delta t} \tag{2}$$

Where, w is the inertia weight, which plays an important role in the PSO and distinguishes global and local search for the algorithm; V_{k}^{i} is the velocity vector at iteration *k*; r_1 and r_2 are two random numbers between 0 and 1; p_{k}^{i} represents the best ever position of the particle *i,* local best; p_{g}^{k} is the global best position in the swarm up to iteration *k*; c_1 is the cognitive parameter; and c_2 is the social parameter.

With the above description of PSO, the algorithm can be summarized as follows:

Initial position, x_{0}^{i}, and velocities, v_{0}^{i}, of the particles are distributed randomly in a feasible search space.

$$x_{0}^{i} = x_{\min} + r.(x_{\max} - x_{\min}) \tag{3}$$

$$v_{0}^{i} = \frac{x_{\min} + r.(x_{\max} - x_{\min})}{\Delta t} \tag{4}$$

Where, r is a random number uniformly distributed between 0 and 1; $x_{\min}$ and $x_{\max}$ are minimum and maximum possible variables for the *i*th particle, respectively.

From the previous discussion, it appears that particles' velocity provides the basis of the PSO search engine because it allows to update the position of each particle (and, hence, the design variables) throughout the optimization process. The definition of velocity should take care of two major phases entailed by optimization: (i) exploration or global search; (ii) exploitation or local search. In the exploration phase, particles should fly across the whole search space to find a limited region containing the global minimum. Velocity of particles should be large enough to allow particles to sweep the entire search space. After the region containing the global minimum has been found, the exploitation phase should begin. The position of each particle is corrected by taking small movements in the neighbourhood of the global minimum. By adopting the correct sequence of these two stages, it is possible to lead particles towards the global minimum**.**

The PSO velocity formula actually includes two contributions. The first part is $w.v_{k}^{i}$, deals with the exploration capability: if PSO is performing a global search, the value of w should be large. The second part is $c_{1}.r_{1}.\frac{\left(p_{k}^{i} - x_{k}^{i}\right)}{\Delta t} + c_{2}.r_{2}\frac{\left(p_{k}^{g} - x_{k}^{i}\right)}{\Delta t}$ is related to exploitation capability: the value of w should be small so to make local search predominate over global search.

However, the classical velocity updating scheme (2) implemented in classical PSO does not allow to dynamically alternate exploration and exploitation based on the current trend of the optimization process. This limitation will be solved in the present study.

3. Probabilistic Particle Swarm Optimization

This section presents a new variant of the PSO algorithm including probabilistic global and local search mechanisms. For that purpose, some probabilistic functions are added into the standard PSO formulation. These probabilistic functions are applied to the different parts of the velocity formula in order to control the search operations performed in the optimization process. By doing this, three different searches can be performed: (i) global search; (ii) local search towards the global best; (iii) local search towards the combination of global and local bests. The new optimization algorithm developed in this research is termed Probabilistic Particle Swarm Optimization (PPSO). In PPSO, the velocities of particles are updated as follows:

$$v_{k+1}^{i} = \alpha.w.v_{k}^{i} + \beta.c_{1}.r_{1}.\frac{\left(p_{k}^{i} - x_{k}^{i}\right)}{\Delta t} + \gamma.c_{2}.r_{2}\frac{\left(p_{k}^{g} - x_{k}^{i}\right)}{\Delta t} \tag{5}$$

Where α, β, and γ are probabilistic functions and are defined as:

$$\begin{cases} \alpha \neq 0, \quad \beta = 1, \quad \gamma = 1. \quad if \quad p < p_1 \\ \alpha = 0, \quad \beta = 1, \quad \gamma = 1. \quad if \quad p_1 \leq p \leq p_2 \\ \alpha = 0, \quad \beta = 0, \quad \gamma = 1. \quad if \quad p > p_2 \end{cases} \tag{6}$$

Where p is a random number in the interval $[0,1]$; and p_1 and p_2 are predefined levels of probabilities set by the user. β and γ are parameters for selection of the type of search. $\beta = 1$ provides local search towards local best and $\gamma = 1$ provides local search towards global best. Thus the values of β and γ were selected to be 0 or 1. On the other hand, α controls the amount of global search and it should be chosen from a range of real numbers rather of 0 or 1. Thus, in this formula, some feasible variants are considered for α to find the best one.

First, the constant value of 1 is considered for α; second, a linear varying value in the format of Eq. (7) is assigned to α:

$$\alpha = \left(1 - \frac{iter}{iter\max}\right) \tag{7}$$

Equation (7) was derived from the work of Shi and Eberhart [10]. The inertia weight is progressively reduced as we approach to the optimum design. Larger values of inertia weight selected in the optimization process provide the algorithm with exploration capability in the early iterations while local search associated to small values of inertia weight predominates in the final iterations.

The third strategy is to define α as a random number in the interval [0,1]. PPSO can simultaneously perform global and local searches. This approach is a step further the common belief that meta-heuristic algorithms should perform only global search in the initial iterations and only local search in the final iterations. However, if a particle is close to the global optimum already in the early stages of the optimization process, global search may force that particle to fly away from such a good position. Consequently, the optimization algorithm becomes less robust. By setting α as a random number, exploration and exploitation can simultaneously be performed in all iterations: global search will be performed for some particles while other particles will perform local search.

If $\beta = 0$ and $\gamma = 1$, local search towards the global best is performed. If $\beta = \gamma = 1$, local search towards the combination of global and local bests is performed. By properly integrating the global and local search strategies mentioned above it is possible to improve the overall performance of the optimization algorithm. The main steps of PSO and PPSO algorithms are briefly summarized in the following.

The main steps of PSO and PPSO algorithms are briefly summarized in the following.

3.1. Initialization of the optimization algorithm

Equation (3) is utilized to randomly generate particles so to cover the entire design space. Initial velocities are computed with Eq. (4).

3.2. Evaluate particles

The cost function is computed for each particle and the global best is determined. Local bests also are determined for each particle. Global best and local bests are respectively stored in two databases.

3.3. Update particles' positions

The position of each particle is updated with Eq. (1) based on the velocity and previous position of the particle. Should any particle fall outside of feasible design space (i.e. if optimization constraints are violated), it must be returned back to its previous location.

3.4. Update memory

If Step 3.3 results in improving the position of a particle, databases defined in Step 3.2 can be updated. The new position of the particle replaces the old position.

3.5. Update particles' velocities

The velocity of each particle is updated with Eqs. (5-6) based on the previous velocity of the particle, the global best and the local best. Exploration/exploitation search mechanisms are selected based on the current trend of the optimization history.

3.6. Stopping criterion

The optimization search terminates when a predefined number of iterations is reached.

4. Design Examples

In order to evaluate the efficiency and robustness of the PPSO approach proposed in this research, five algorithmic variants were implemented and then compared in three classical weight minimization problems of spatial truss structures: a 25-bar truss, a 72-bar truss, and a 120-bar dome. For each test problem, ten independent optimization runs were performed to evaluate the performance of each algorithm on a statistical basis. The following nomenclature was utilized: (i) *LPSO* denotes the classical PSO formulation with linearly varying inertia weight; (ii) *PPSO C-1* denotes the probabilistic PSO formulation with constant inertia weight; (iii) *PPSO C-2* denotes the probabilistic PSO formulation with randomly varying inertia weight; (iv) *LPPSO* denotes the probabilistic PSO formulation with linearly varying inertia weight.

For each problem 30 independent runs are performed to obtain some statistical data about each algorithm. Number of particles in all variants is 20 and value 1 selected as c_1 and c_2. Also, after some try and error, values of 0.6 and 0.8 were found to be suitable for p_1 and p_2, respectively. A penalty function approach used to deal with the constraints. In this paper, the following abbreviations are adopted: PSO with linear varying inertia weight as *LPSO*, Probabilistic PSO with constant inertia weight as *PPSO C-1*, Probabilistic PSO with random inertia weight as *PPSO C-2*, and Probabilistic PSO with linear varying inertia weight as *LPPSO*.

The number of analyses for PPSO is naturally higher than that of PSO because of additional searches required for better exploitation, and consequently avoiding the algorithm from being trapped in local optima (see Tables 5, 9 and 11).

Here the suitability of PPSO compared to PSO and some of its variants are shown, however, there may be other algorithms which produce slightly better results for some of the considered examples. In fact no sole algorithm is known which produces the best results for different optimization problems compared to the other ones.

4.1. A 25-bar spatial truss structure

The first example is a 25-bar spatial truss structure in the form of a transmission tower is considered as described by Schmit and Fleury [11], and shown in Fig. 1.

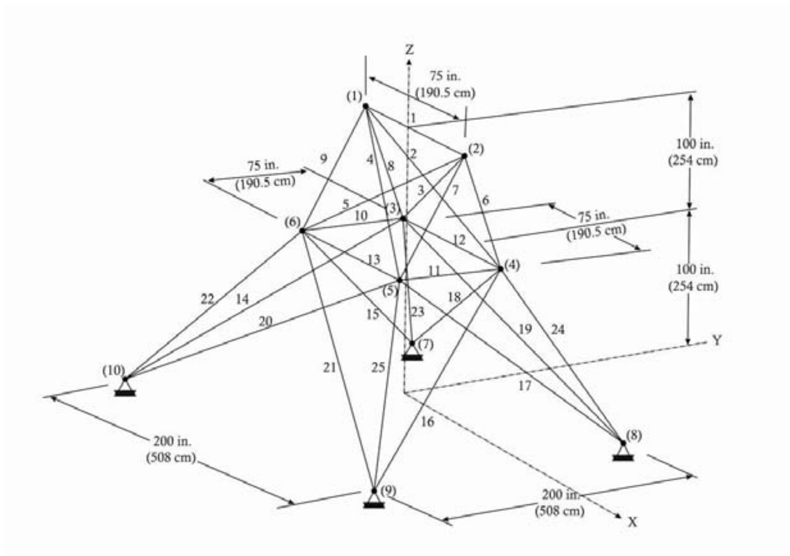

Fig. 1 Schematic of the 25-bar spatial truss structure

The design variables are the cross sectional areas of the members, which are categorized into eight groups presented in Table 1.

Table 1 Element grouping for the spatial 25-bar truss structure

Group	*Truss Members*
1	1
2	2~5
3	6~9
4	10~11
5	12~13
6	14~17
7	18~21
8	22~25

Loading of the structure is shown in Table 2. Constraints are imposed to cross sectional areas of the members between 0.1 in^2 to 3.4 in^2, and to the allowable stresses which are included in Table 3. The other considered constraint is the allowable displacement which is taken as ± 0.35 in for every direction. The modulus of elasticity and material density in this problem are 10,000 ksi and 0.1 lb/in^3, respectively.

Table 2 Loading conditions acting on the spatial 25-bar truss structure

Node	*F_x (lb)*	*F_y (lb)*	*F_z (lb)*
1	10,000	−10,000	−10,000
2	0	−10,000	−10,000
3	500	0	0
6	600	0	0

Table 3 Stress constraints for the spatial 25-bar truss problem

Element Group	*Allowable compressive stress ksi (MPa)*	*Allowable tensile stress ksi (MPa)*
1	35.092 (241.96)	40.0 (275.80)
2	11.590 (79.913)	40.0 (275.80)
3	17.305 (119.31)	40.0 (275.80)
4	35.092 (241.96)	40.0 (275.80)
5	35.092 (241.96)	40.0 (275.80)
6	6.759 (46.603)	40.0 (275.80)
7	6.959 (47.982)	40.0 (275.80)
8	11.082 (76.410)	40.0 (275.80)

Table 4 shows some optimized designs quoted in literature. The optimization results obtained for the five PSO variants compared in this study are listed in Table 5. The global optimization capability of each PSO variant was evaluated by computing the standard deviation of structural weights found in the 30 optimization runs: robustness and reliability of the optimization algorithm obviously increase as the standard deviation decreases. It can be seen that introducing linear variation of the inertia weight parameter α and probabilistic selection of particles' velocity updating scheme in the PSO formulation allowed designs to be significantly improved with respect to standard PSO. In particular, probabilistic PSO with randomly varying inertia weight and probabilistic PSO with linearly varying inertia weight converged practically to the same optimized weight corresponding to slightly better designs than LPSO and PPSO C-1. Since PPSO C-2 found the best design overall with the lowest standard deviation, it should be considered the most efficient optimization algorithm.

Table 4 Optimized designs quoted in literature for the spatial 25-bar truss structure

Element Group [in^2]	*Wu and Chow [12]*	*Zhu [13]*	*Hasancebi et al. [14]*	*Erbatur et. al [15]*	*Perez and Behdinan [16]*
1	0.1	0.1	0.1	0.1	0.1
2	0.5	1.9	0.3	1.2	1.0227
3	3.4	2.6	3.4	3.2	3.4
4	0.1	0.1	0.1	0.1	0.1
5	1.5	0.1	2.1	1.1	0.1
6	0.9	0.8	1.0	0.9	0.6399
7	0.6	2.1	0.5	0.4	2.0424
8	3.4	2.6	3.4	3.4	3.4
Best Weight (lb)	486.29	562.93	484.85	493.8	485.33
Average Weight (lb)	N/A	N/A	N/A	N/A	N/A
Heaviest Weight (lb)	N/A	N/A	N/A	N/A	534.84
SD (lb)	N/A	N/A	N/A	N/A	N/A

Table 5 Optimized designs obtained for different PSO variants in the spatial 25-bar truss problem

Element Group [in^2]	*PSO*	*LPSO*	*PPSO C-1*	*PPSO C-2*	*LPPSO*
1	1.83	0.18	0.10	0. 10	0.10
2	0.28	0.42	0.46	0.42	0.45
3	3.03	3.40	3.40	3.40	3.40
4	0.20	0.23	0.10	0. 10	0.10
5	0.91	2.02	1.99	1.92	1.84
6	1.45	0.97	0.98	0.96	0.95
7	1.21	0.45	0.42	0.48	0.48
8	3.19	3.40	3.40	3.40	3.40
Best Weight (lb)	537.50	486.67	484.31	484.06	484.07
Average Weight (lb)	565.68	487.37	484.57	484.07	484.14
Heaviest Weight (lb)	603.03	488.22	484.82	484.08	484.39
SD (lb)	18.18	0.52	0.17	0.01	0.10
Number of Analyses	480	6420	2220	2880	2340

Convergence curves are compared in Fig. 2. It can be seen that probabilistic PSO formulations are considerably faster than standard PSO with constant or linearly varying inertia weight. Convergence speed was practically the same for all probabilistic PSO variants. Convergence curves averaged over the ten optimization runs are compared in Fig. 3. Trends are similar to those shown in Fig. 2 but convergence data were smoothed out by averaging. LPSO is slower than probabilistic PSO because of the presence of a step in the first third of the optimization history.

Fig. 3 shows the average results of 30 independent runs for this example. This is performed to compare the convergence rate of each algorithm. All these curves are similar to those of as Fig. 1 but smoother. For example, it is concluded that the LPSO has a pause in the middle and this problem causes the reduction of convergence rate of this algorithm. Also, the PSO has a major problem in local search; and convergence rate of the PPSO C-1, PPSO C-2, and LPPSO are nearly the same and higher than PSO and LPSO.

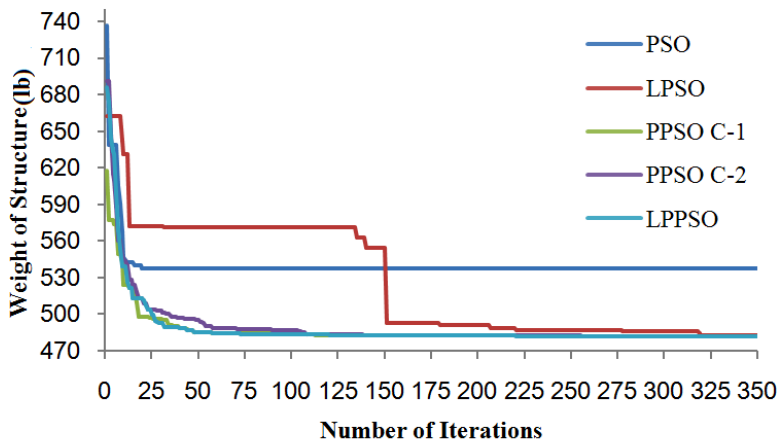

Fig. 2 Convergence curves obtained for different PSO variants in the spatial 25-bar truss problem

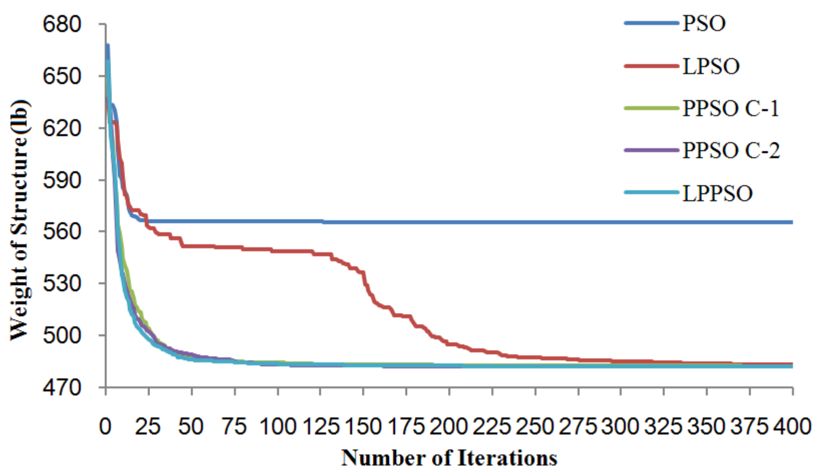

Fig. 3 Spatial 25-bar truss problem: convergence curves of different PSO variants averaged over 30 optimization runs

For most of the structural optimum design problems, the global minimum is located on or close to the boundary of a constraint. In other words, the constraints in the engineering problems determine the limits of the search space and often at least one constraint is active in the final optimum result [17]. In this problem, Fig. 4 is shown from the best result of the PPSO C-2. As seen, the active constraint is the nodal displacements which are shown for each Degree of Freedom (DOF) and, there are two DOFs the displacements of which are exactly on the constraint.

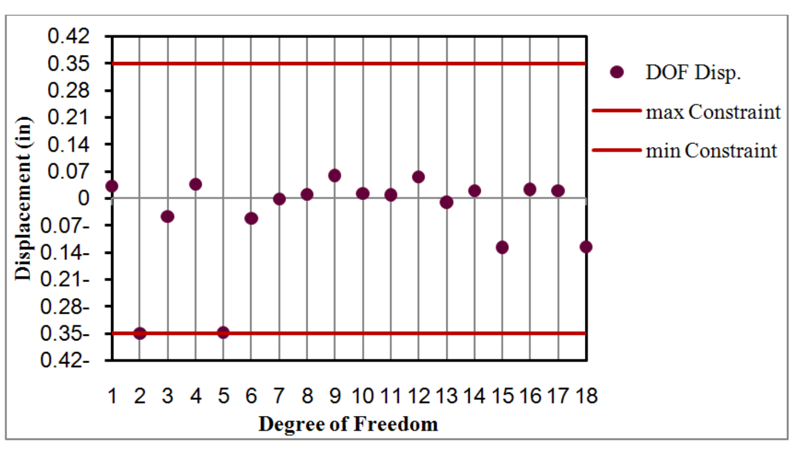

Fig. 4 Displacement vs. Degrees of Freedom for the 25-bar truss using results of the PPSO C-2

4.2. A 72-bar spatial truss structure

The second test case considered in this study was the 72-bar truss shown in Fig. 4. The cross sectional areas of elements were included as sizing variables. Because of structural symmetry, elements were grouped in sixteen groups (see Table 6) therefore, this test case has sixteen design variables.

The two independent loading conditions acting on the structure are listed in Table 7. Nodal displacements must be less than $\pm$ 0.25 in while element stresses must be less than $\pm$ 25000 psi and the material density is 0.1 lb/in^3 the modulus of elasticity is 10,000 ksi. The minimum cross sectional area for this problem is 0.1 in^2.

Table 8 shows some optimized designs quoted in literature. The optimization results obtained for the five PSO variants compared in this study are listed in Table 9.

Table 6 Element grouping for the spatial 72-bar truss structure

Group	***Truss Members***
1	1,2,3,4
2	5,6,7,8,9,10,11,12
3	13,14,15,16
4	17,18
5	19,20,21,22
6	23,24,25,26,27,28,29,30
7	31,32,33,34
8	35,36
9	37,38,39,40
10	41,42,43,44,45,46,47,48
11	49,50,51,52
12	53,54
13	55,56,57,58
14	59,60,61,62,63,64,65,66
15	67,68,69,70
16	71,72

Table 7 Loading conditions acting on the spatial 72-bar truss structure

	Case 1			**Case 2**		
Node	P_X *kips(kN)*	P_Y *kips(kN)*	P_Z *kips(kN)*	P_X	P_Y	P_Z *kips(kN)*
1	5.0 (22.25)	5.0 (22.25)	-5.0 (22.25)	0.0	0.0	-5.0 (22.25)
2	0.0	0.0	0.0	0.0	0.0	-5.0 (22.25)
3	0.0	0.0	0.0	0.0	0.0	-5.0 (22.25)
4	0.0	0.0	0.0	0.0	0.0	*-5.0 (22.25)*

Table 7 Optimized designs available in literature for the spatial 72-bar truss structure

Element Group [in^2]	***Zhu & Rozvany [18]***	***Venkayya [19]***	***Erbatur et al. [11]***	***Schmit & Farshi [20]***	***Gellatly & Berke [21]***
1	0.157	0.161	0.155	0.159	0.149
2	0.536	0.557	0.535	0.594	0.773
3	0.410	0.377	0.480	0.341	0.453
4	0.569	0.506	0.520	0.608	0.342
5	0.507	0.611	0.460	0.264	0.552
6	0.520	0.532	0.530	0.548	0.608
7	0.100	0.100	0.120	0.100	0.100
8	0.100	0.100	0.165	0.151	0.100
9	1.280	1.246	1.155	1.107	1.024

10	0.515	0.524	0.585	0.579	0.542
11	0.100	0.100	0.100	0.100	0.100
12	0.100	0.100	0.100	0.100	0.100
13	1.897	1.818	1.755	2.078	1.464
14	0.516	0.524	0.505	0.503	0.521
15	0.100	0.100	0.105	0.100	0.100
16	0.100	0.100	0.155	0.100	0.100
Best Weight (lb)	379.660	381.200	385.760	388.63	395.970

Table 8 Optimized designs obtained for the different PSO variants in the spatial 72-bar truss problem

Element Group [in^2]	***PSO***	***LPSO***	***PPSO C-1***	***PPSO C-2***	***LPPSO***
1	0.543	0.159	0.155	0.157	0.158
2	0.635	0.550	0.557	0.542	0.529
3	0.877	0.381	0.388	0.398	0.405
4	0.521	0.611	0.617	0.584	0.586
5	0.738	0.562	0.492	0.497	0.475
6	0.493	0.522	0.546	0.523	0.524
7	0.166	0.102	0.103	0.100	0.100
8	1.031	0.111	0.120	0.100	0.100
9	1.245	1.268	1.213	1.281	1.301
10	0.610	0.502	0.502	0.514	0.518
11	0.812	0.111	0.109	0.100	0.100
12	1.197	0.101	0.101	0.100	0.100
13	1.446	1.902	1.898	1.869	1.843
14	0.512	0.502	0.492	0.513	0.525
15	0.287	0.101	0.101	0.100	0.104
16	1.030	0.107	0.103	0.100	0.100
Best Weight (lb)	567.895	381.111	380.788	379.395	379.755
Average Weight (lb)	613.954	383.988	383.102	379.610	380.312
Heaviest Weight (lb)	661.255	389.610	387.365	379.932	381.939
SD (lb)	37.631	2.759	1.884	0.168	0.664
Number of Analyses	1240	19680	8580	10460	11260

PPSO C-2 again found the best weight with the lowest standard deviation. Convergence curves relative to the best designs and convergence curves averaged over the ten optimization runs carried out for each PSO variant are plotted in Fig. 6 and Fig. 7, respectively.

It can be seen that global optimization capability and convergence speed improved significantly in the case of probabilistic PSO. Standard PSO got stuck in a local minimum while standard PSO with linearly decreasing inertia weight showed a large step in the first part of the optimization history. The convergence history of PPSO can clearly be divided in two phases: in the first part, structural weight decreases sharply because global search predominates; in the last part, structural weight decreases much more slightly because local search predominates. Compared to standard PSO, probabilistic search allows improving significantly both phases mentioned above. The optimized design was critical with respect to nodal displacements.

Also, the active constraint of this problem is the displacements of the DOFs, which are shown in Fig. 8. From this figure, it is apparent that some displacements are on the border.

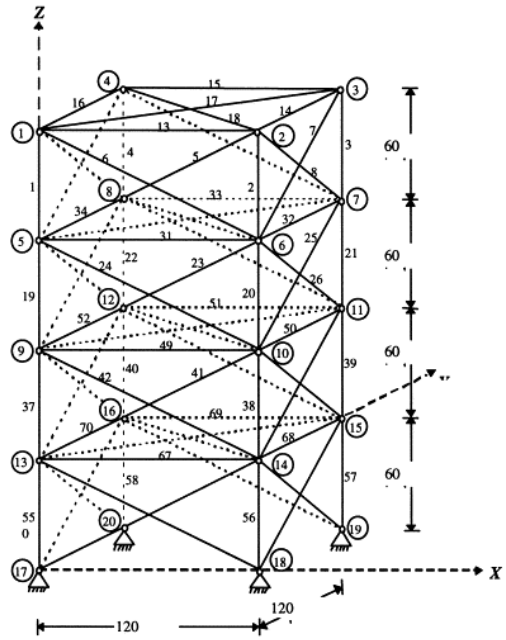

Fig. 5 Schematic of the spatial 72-bar truss structure

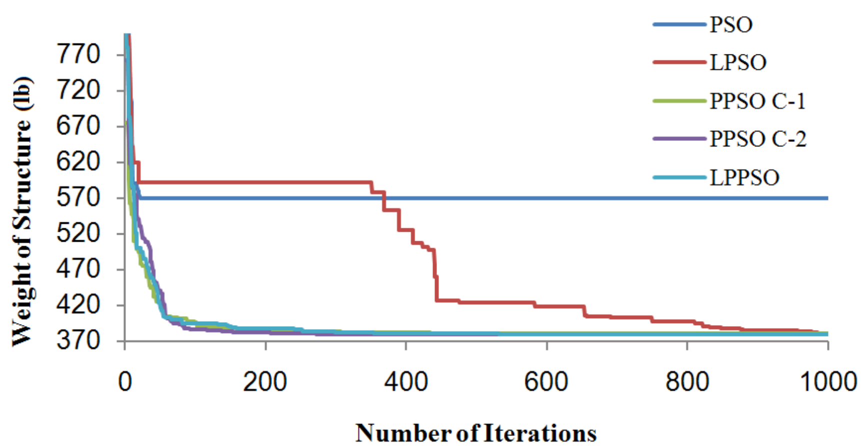

Fig. 6 Convergence curves obtained for different PSO variants in the spatial 72-bar truss problem

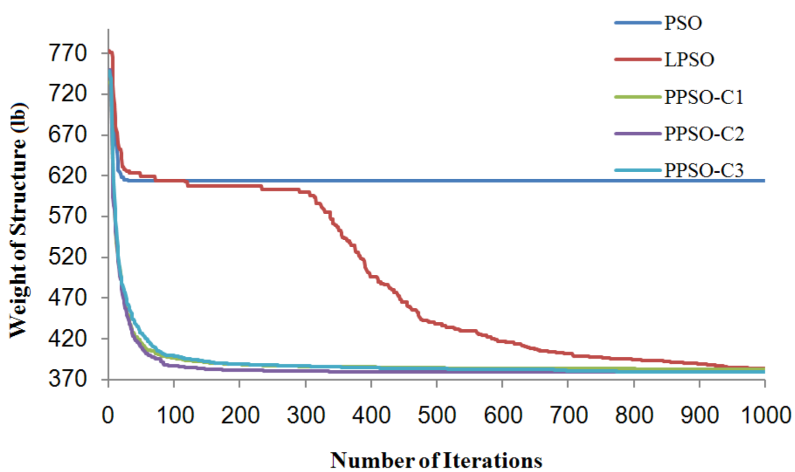

Fig. 7 Spatial 72-bar truss problem: convergence curves of different PSO variants averaged over 30 optimization runs

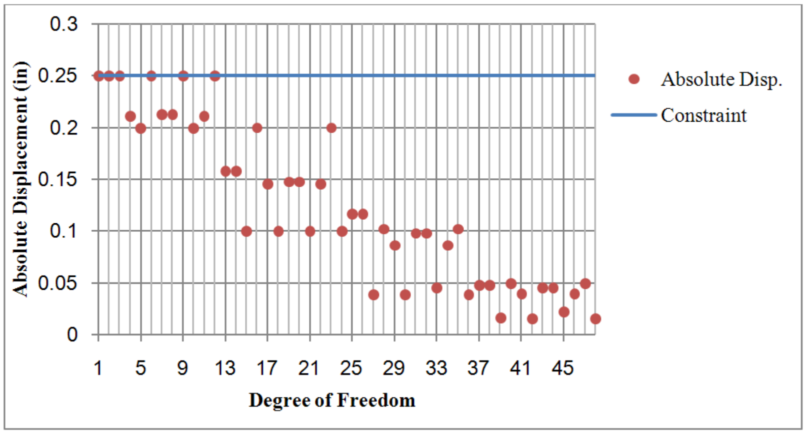

Fig. 8 Absolute displacement vs. Degrees of Freedom for the 72-bar truss using results of the PPSO C-2

4.1. A 120-bar spatial dome truss structure

Design of a 120-bar spatial dome truss, shown in Fig. 9, is considered as the last example to compare the practical capability of the proposed modification. This dome is utilized in literature to find size optimum design. The modulus of elasticity is 30,450 ksi (210,000 MPa), and the material density is 0.288 lb/in^3 (7971.810 kg/m^3). The yield stress of steel is taken as 58.0 ksi (400 MPa). This dome is considered to be subjected to vertical loading at all the unsupported joints. These loads are taken as −13.49 kips (−60 kN) at node 1, −6.744 kips (−30 kN) at nodes 2 through 14, and −2.248 kips (−10 kN) at the remaining nodes. The minimum cross sectional area of all members is 0.775 in^2 (2 cm^2) and the maximum cross-sectional area is taken as 20.0 in^2 (129.03 cm^2). The stress constraints of the structural members are calculated as per AISC (1989) specifications as illustrated in Eq. (8). The 120 bar spatial truss members are categorized into 7 groups as shown in Fig. 9.

$$\begin{cases} \sigma_i^+ = 0.6F_y & \textit{for} \quad \sigma_i \ge 0 \\ \sigma_i^- & \textit{for} \quad \sigma_i < 0 \end{cases} \tag{8}$$

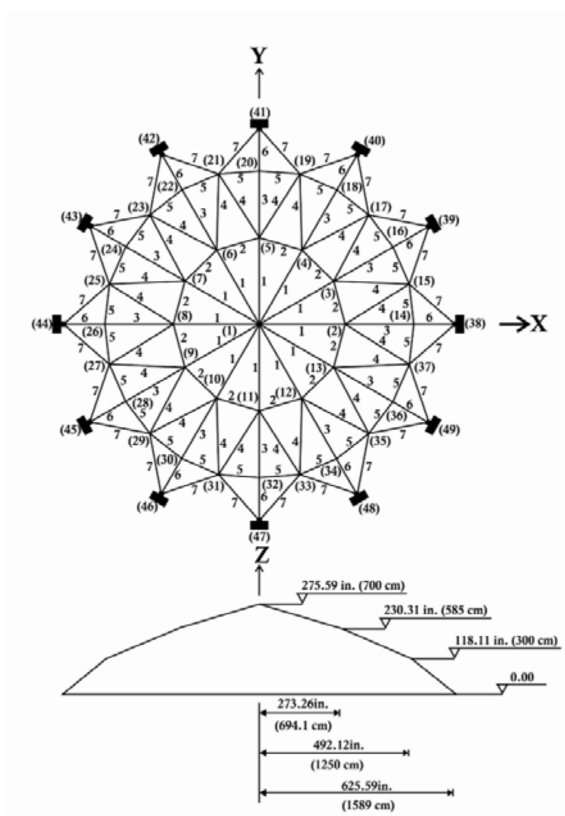

Fig. 9 Schematic of the spatial 120-bar truss structure and element group numbering

Where, σ_i^+ is the allowable tensile strength; F_y is the yield stress of steel; and σ_i^- is the compressive strength of the section and is calculated according to the slenderness ratio as follows:

$$\sigma_i^- = \begin{cases} \left[\left(1-\dfrac{\lambda_i^2}{2C_c^2}\right)F_y\right] \Big/ \left(\dfrac{5}{3}+\dfrac{3\lambda_i}{8C_c}-\dfrac{\lambda_i^3}{8C_c^3}\right) & for \quad \lambda_i < C_c \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & for \quad \lambda_i \geq C_c \end{cases} \tag{9}$$

Where, E is the modulus of elasticity; F_y is the yield strength of steel; C_c is the slenderness ratio which divides the elastic and inelastic buckling regions $\left(C_c = \sqrt{2\pi^2 E / F_y}\right)$; and λ_i is the slenderness ratio. The displacement constraint for this example is *0.1969 in* in every direction.

Fig. 10 shows the convergence curves for the best design found for each PSO variant. It can be seen that all probabilistic search schemes were faster than standard PSO. The optimization results reported in literature are shown in Table 10 while Table 11 presents the optimization results obtained in the present study for the different PSO variants. Probabilistic PSO again outperformed standard PSO with constant parameters and linearly decreasing inertia weight.

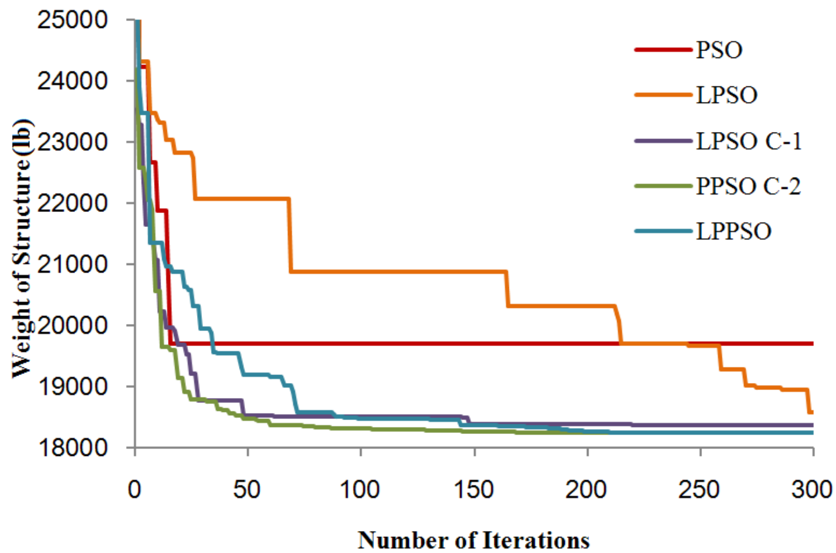

Fig. 10 Convergence curves obtained for different PSO variants in the spatial 120-bar dome problem

Table 9 Optimized designs quoted in literature for the spatial 120-bar dome structure

Element Group [in²]	*Lee and Geem [22]*			*Kaveh and Talatahari [23]*
	HS	***PSO***	***PSOPC***	***HPSACO***
1	3.295	3.147	3.235	3.311
2	3.396	6.376	3.37	3.438
3	3.874	5.957	4.116	4.147
4	2.571	4.806	2.784	2.831
5	1.15	0.775	0.777	0.775
6	3.331	13.798	3.343	3.474
7	2.784	2.452	2.454	2.551
Best Weight (lb)	19707.77	32432.9	19618.7	19491.3
Average Weight (lb)	N/A	N/A	N/A	N/A
Heaviest Weight (lb)	N/A	N/A	N/A	N/A
SD (lb)	N/A	N/A	N/A	N/A

Table 10 Optimized designs obtained for different PSO variants in the spatial 120-bar dome problem

Element Group	*PSO*	*LPSO*	*PPSO C-1*	*PPSO C-2*	*LPPSO*
1	3.185	3.113	3.042	3.019	3.019
2	3.724	3.881	4.059	3.894	4.049
3	3.672	3.324	3.330	3.238	3.240
4	2.273	2.243	2.245	2.236	2.236
5	1.821	1.660	1.496	1.624	1.537
6	3.733	2.530	2.515	2.482	2.487
7	2.302	2.371	2.319	2.301	2.301
Best Weight (lb)	19707.775	18589.917	18380.585	18248.994	18251.473
Average Weight (lb)	21260.009	18832.655	18447.719	18281.442	18284.989
Heaviest Weight (lb)	22364.558	19035.884	18551.380	18398.045	18329.027
SD (lb)	807.740	165.849	51.946	27.554	47.000
Number of Analyses	340	5980	4440	3480	4380

The analysis of convergence curves averaged over the ten optimization runs performed for each PSO variant revealed that standard PSO could complete only global search but failed in the local search (see Fig. 11). LPSO shows a large step in the middle part of the convergence history. Conversely, probabilistic PSO variants always completed global search in about 100 iterations and were able to further reduce structural weight in the local search phase. Fig. 12 shows that all nodal displacements, as active constrants in this example, are in the allowable range.

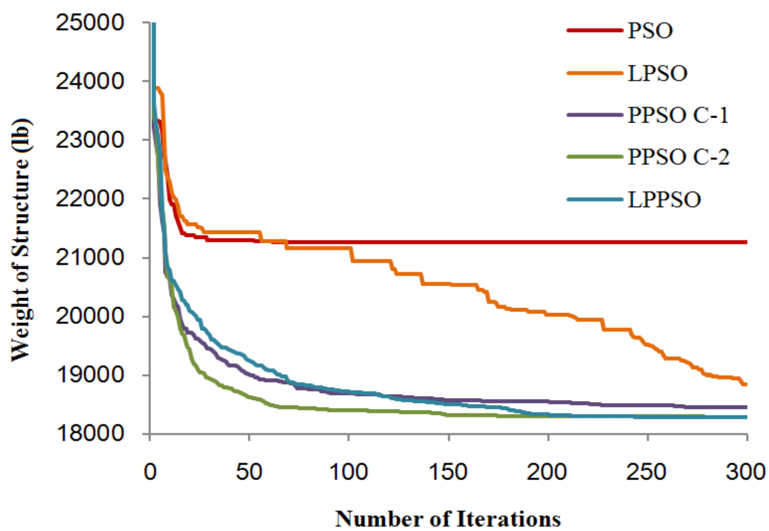

Fig. 11 Spatial 120-bar dome problem: convergence curves of different PSO variants averaged over 30 optimization runs

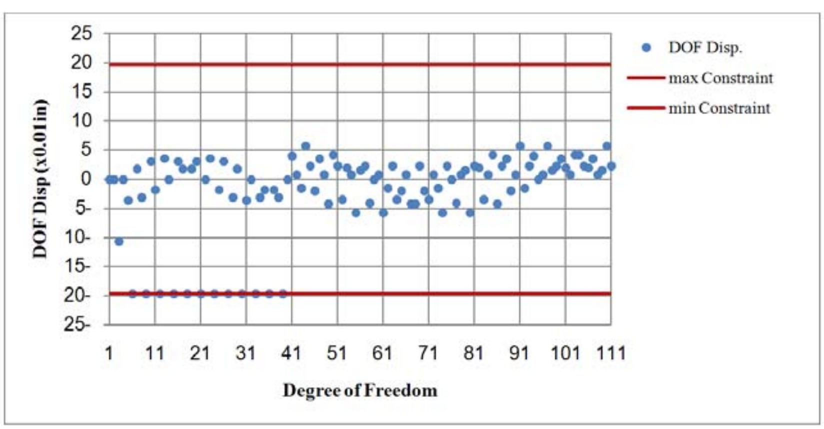

Fig. 12 Displacement vs. Degrees of Freedom for the 120-bar spatial dome truss using the results of the PPSO C-2

5. Concluding Remarks

A new probabilistic particle swarm optimization *(PPSO)* algorithm is developed in this research. The study is motivated by the fact that missing the best combination of the exploration and exploitation phases may significantly reduce the overall efficiency of optimization search. PSO with linearly varying inertia weight can take care in part of this issue but is not the best strategy as it forces the optimization search to be either global or local thus reducing the overall performance of the PSO algorithm. The concept of simultaneous global and local searches is introduced in this research by combining randomly variable inertia weight with probabilistic selection of the best global/local search mechanism.

The validity of the new approach is tested using three classical weight minimization problems of spatial truss structures. Optimization results indicate that the probabilistic PSO is more efficient than the standard PSO. Further investigations should be carried out in order to tailor probabilistic search schemes to other metaheuristic algorithms. This would allow global optimization capability, convergence behavior and reliability of the optimization algorithm to be enhanced.

Recently other enhanced versions of PSO are also developed for structural optimization, from which one may refer to those of Gholizadeh [24], and Kaveh and Zolghadr [25,26].

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