International Journal of Civil Engineering

Water-Hydrology

Unsteady flow in a porous medium between parallel plates in the presence of uniform suction and injection with heat transfer

H. A. Attia[1], W. A. El-Meged[1], W. Abbas[2], M. A. M. Abdeen[3,*]

Received: January 2012, Revised: June 2013, Accepted: January 2014

Abstract

The unsteady flow in porous medium of a viscous incompressible fluid bounded by two parallel porous plates is studied with heat transfer. A uniform and constant pressure gradient is applied in the axial direction whereas a uniform suction and injection are applied in the direction normal to the plates. The two plates are kept at constant and different temperatures and the viscous dissipation is not ignored in the energy equation. The effect of the porosity of the medium and the uniform suction and injection velocity on both the velocity and temperature distributions are investigated.

Keywords: *Unsteady flow, Viscous incompressible fluid, Heat transfer, Porous medium, Numerical solution.*

1. Introduction

The flow of a viscous electrically conducting fluid between two parallel plates has important applications as in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays [1]. The flow between parallel plates of a Newtonian fluid with heat transfer has been examined by many researchers in the hydrodynamic case considering constant physical properties [2-6]. The extension of the problem to the MHD case has attracted the attention of many authors [7-12].

In this paper, the transient flow with heat transfer through a porous medium of an incompressible viscous fluid between two infinite horizontal porous plates is investigated. A constant pressure gradient is applied in the axial direction and a uniform suction and injection is imposed in the direction normal to the plates. The flow through a porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [13-15].

* *Corresponding author: mostafa_a_m_abdeen@hotmail.com*

1 Department of Engineering Mathematics and Physics, Faculty of Engineering, Fayoum University, El-Fayoum-63415, Egypt

2 Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology, and Maritime Transport, Cairo, Egypt

3 Department of Engineering Mathematics and Physics, Faculty of Engineering, Cairo University, Giza 12211, Egypt

The two plates are maintained at two different but constant temperatures.

This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices.The linear partial differential equations of motion are solved analytically using the method of Laplace transform to obtain the velocity distribution as a function of space and time. The inhomogeneous energy equation is solved numerically considering the viscous dissipation using the method of finite differences and by applying the Crank-Nicolson implicit method. The effect of the porosity of the medium and the suction and injection velocity on both the velocity and temperature distributions are reported. The transient solutions proved that the steady state solution is approached as the asymptotic development of a time-dependent process and presented some interesting results.

2. Description of the Problem

The two parallel horizontal plates are located at the $y=\pm h$ planes and extend from $x=-\infty$ to ∞ and $z=-\infty$ to ∞ as shown in Fig. 1. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2>T_1$. The fluid flows between the two plates in a porous medium where the Darcy's model is assumed [13-15]. The motion is driven by a constant pressure gradient dp/dx in the x-direction, and a uniform suction from above and injection from below which are applied at $t=0$ with velocity v_o.Due to the infinite dimensions in the x and z-directions all quantities apart from the pressure gradient *dp/dx* which is assumed constant, are independent of the *x* and *z*-coordinates. The velocity vector of the fluid is given as

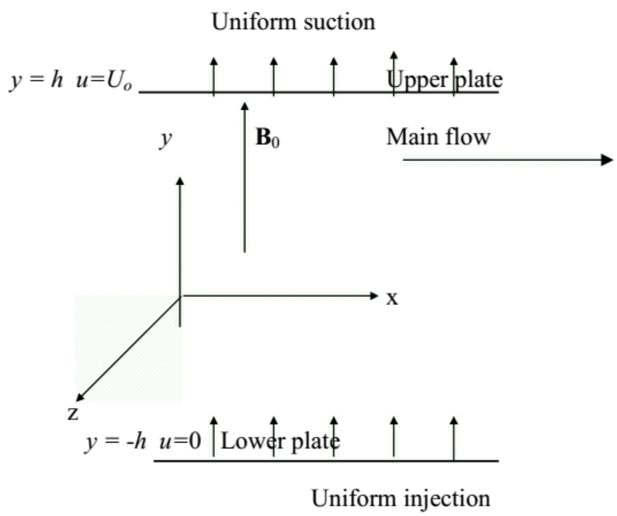


Fig. 1 The geometry of the problem

$$\vec{v}(y,t) = u(y,t)\vec{i} + v_o\vec{j}$$

with the initial and boundary conditions u=0 at t≤0, and u=0 at y=±h for t>0. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T=T_1$ at t≤0, $T=T_2$ at y=+h, and $T=T_1$ at y=-h for t>0. The fluid flow is governed by the momentum equation [16]

$$\rho\frac{\partial u}{\partial t} + \rho v_o\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K}u \tag{1}$$

where ρ and μ are, respectively, the density and the coefficient of viscosity and K is the Darcy permeability [13-15]. To find the temperature distribution inside the fluid we use the energy equation [17]

$$\rho c\frac{\partial T}{\partial t} + \rho c v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu(\frac{\partial u}{\partial y})^2, \tag{2}$$

where c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second term on the right side represents the viscous dissipation.

Introducing the following non-dimensional quantities

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{z} = \frac{z}{h}, \hat{u} = \frac{\rho h u}{\mu}, \hat{P} = \frac{P\rho h^2}{\mu^2}, t = \frac{t\mu}{\rho h^2}, \hat{T} = \frac{T - T_1}{T_2 - T_1}$$

$S = \rho v_o h/\mu,$ is the suction parameter,

$\Pr = \mu c/k$ is the Prandtl number,

$Ec = \mu^2/\rho^2 c h^2 (T_2 - T_1)$ is the Eckert number,

$M = h^2/K$ is the porosity parameter,

NuL = $(\partial\hat{T}/\partial\hat{y})\,\hat{y}$=-1 is the Nusselt number at the lower plate,

NuU = $(\partial\hat{T}/\partial\hat{y})\,\hat{y}$=1 is the Nusselt number at the upper plate,

Equations (1)-(2) are written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{3}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ec(\frac{\partial u}{\partial y})^2, \tag{4}$$

The initial and boundary conditions for the velocity become

$$u = 0, t \le 0, u = 0, y = \pm 1, t > 0 \tag{5}$$

and the initial and boundary conditions for the temperature are given by

$$t \le 0 : T = 0, t > 0 : T = 1, y = +1, T = 0, y = -1. \tag{6}$$

3. Analytical Solution of the Equations of Motion

Equation (3) is a linear inhomogeneous partial differential equation which is solved analytically using the Laplace transform (LT) method, under the initial and boundary conditions given by Eq. (5) to give the velocity field as functions of space and time. Taking the LT of Eq. (3) yields

$$\frac{d^2U(y,s)}{dy^2} - S\frac{dU(y,s)}{dy} - K(s)U(y,s) = -\frac{C}{s} \tag{7}$$

where $U(y,s)$=L($u(y,t)$), C is the constant value of –dP/dx and $K(s) = M + s$. The solution of Eq. (7) with y as an independent variable is given as

$$U(y,s)=\frac{C}{Ks}\left(1+\exp(Sy/2)\left(\frac{\sinh(S/2)\sinh(\alpha y)}{\sinh(\alpha)}-\frac{\cosh(S/2)\cosh(\alpha y)}{\cosh(\alpha)}\right)\right)$$

where $\alpha^2=S^2/4+K$. Using the complex inversion formula and the residue theorem [16], the inverse transform of $U(y,s)$ is determined as

$$u(y,t)=\frac{C}{M}\left(1+\exp(Sy/2)\left(\frac{\sinh(S/2)\sinh(\alpha_1 y)}{\sinh(\alpha_1)}-\frac{\cosh(S/2)\cosh(\alpha_1 y)}{\cosh(\alpha_1)}\right)\right)$$

$$+2\pi C\exp(Sy/2)\sum_{n=1}^{\infty}(-1)^n[(n-1)\frac{\exp(Dn_1 t)}{(Dn_1+M)Dn_1}\sinh(S/2)\sin(\pi(n-1)y)$$

$$+(n-0.5)\frac{\exp(Dn_2 t)}{(Dn_2+M)Dn_2}\cosh(S/2)\cosh(\pi(n-0.5)y)] \qquad (8)$$

where,

$$Dn_1=-[\pi^2(n-1)^2+S^2/4+M],$$
$$Dn_2=-[\pi^2(n-0.5)^2+S^2/4+M],\alpha_1=S/4+M.$$

Equation (8) shows that u is directly proportional to the pressure gradient, that is u/C is independent of C. The expression for the velocity u is to be evaluated for various values of the parameters M and S.

4. Numerical Solution of the Energy Equation

The analytical solution of Eq. (8) determines the velocity field for various values of the parameters M and S. The values of the velocity, when substituted in the right-side of the inhomogeneous energy equation (4), make it too difficult to solve analytically. Therefore, the energy equation is to be solved numerically using the Crank-Nicolson implicit method [18] with the initial and boundary conditions given by Eq. (6). The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The viscous dissipation term is evaluated using the velocity components and their derivatives in the y-direction which are obtained from the exact solution. We introduce the variables $v=\partial u/\partial y$ and $H=\partial T/\partial y$ to reduce Eq. (4) from the second order to the first order. The computational domain is divided into meshes each of dimension Δt and Δy in time and space, respectively. The finite difference representation for the energy equation is given by

$$\left(\frac{T_{i+1,j+1}-T_{i,j+1}+T_{i+1,j}-T_{i,j}}{2\Delta t}\right)+S\left(\frac{H_{i+1,j+1}+H_{i,j+1}+H_{i+1,j}+H_{i,j}}{4}\right)$$
$$=\frac{1}{\Pr}\left(\frac{(H_{i+1,j+1}+H_{i,j+1})-(H_{i+1,j}+H_{i,j})}{2\Delta y}\right)+Ec\left(\frac{v_{i+1,j+1}+v_{i,j+1}+v_{i+1,j}+v_{i,j}}{4}\right)^2, \qquad (9)$$

Finally, the block tri-diagonal system is solved using Thomas' algorithm [18]. Equation (9) is rewritten in the following forms

$$b_1T_l+b_2T_k+b_3H_l+b_4H_k=b_5 \qquad (10)$$

Where b_m (where m =1,2 , …,5) are the coefficients of the difference equations (10) that is corresponding to Eq. (9), l and k are countries equations to (i, j+1) and (i+1, j+1), respectively. The position y=-1 is designated by l=1. We write the generalized Thomas-algorithm as in the following steps [18].

The unknowns are written as

$$T_l=\overline{T}_l H_l+\hat{T}_l \qquad (11)$$

$$H_l=\overline{H}_k H_k+\hat{H}_k \qquad (12)$$

where, the variables $\overline{T}_l, \hat{T}_l$, $\overline{H}_k$ and $\hat{H}_k$ are representing Thomas' coefficients. The equations relating the temperature T to its derivatives H are given by

$$\frac{T_k-T_l}{\Delta y}=\frac{H_l+H_k}{2} \qquad (13)$$

Substituting from Eqs. (11) and (12) into (13) and after manipulations we get,

$$T_k=((\overline{T}_l+\Delta y/2)\overline{H}_k+\Delta y/2)H_k+(\overline{T}_l+\Delta y/2)\hat{H}_k+\hat{T}_l \qquad (14)$$

From which we can get

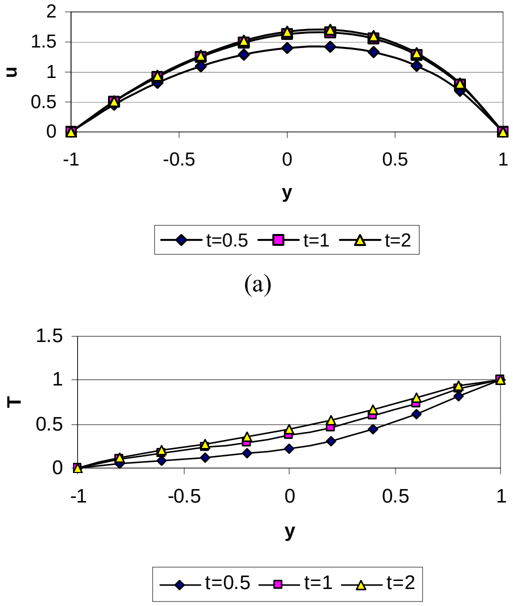
$$\overline{T}_k=(\overline{T}_l+\Delta y/2)\overline{H}_k+\Delta y/2, \qquad \hat{T}_k=(\overline{T}_l+\Delta y/2)\hat{H}_k+\hat{T}_l \qquad (15)$$

Substituting from Eqs. (13) and (15) into Eq. (10) we can obtain the two coefficients $\overline{H}_k$ and $\hat{H}_k$.

Unlike the velocity u, the temperature distribution depends on C. All calculations are carried out for Pr=1 and Ec=0.2.

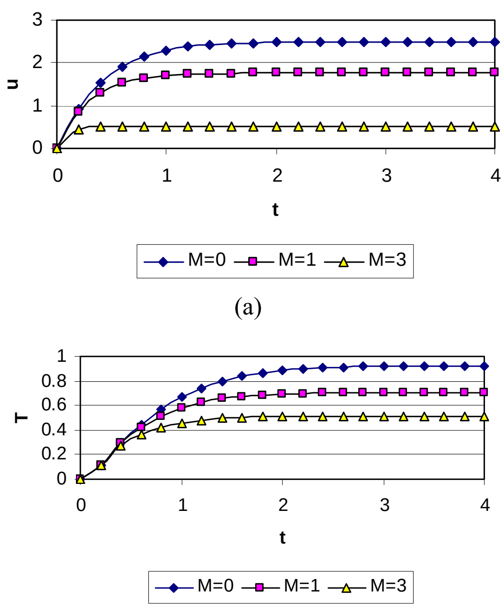
5. Results and Discussion

Fig. 2 shows the time progression of the velocity and temperature profiles up till the steady state and for M=1 and S=1. It is clear from Fig. 2a that the velocity charts are asymmetric about the y=0 plane because of the suction. The velocity component u reaches the steady state faster than T which is expected as u acts as the source of temperature.



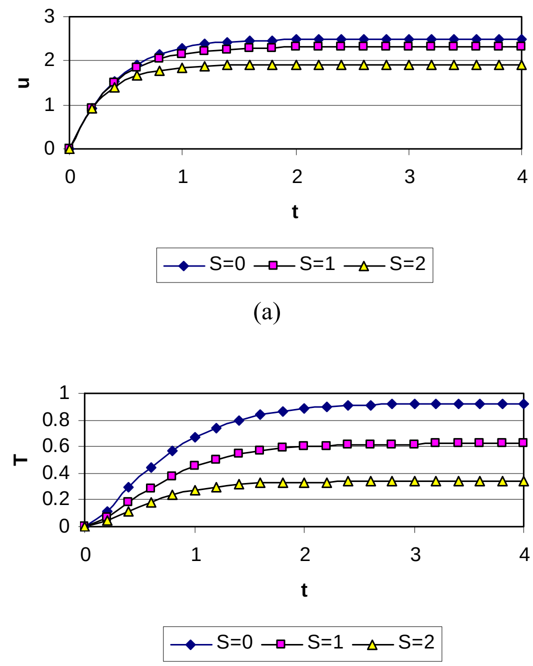
(b)
Fig. 2 Time development of the profile of: (a) u; and (b) T (M=1 and S=1)

Fig. 3 indicates the effect of the porosity parameter M on the time progression of the velocity u and temperature T at the centre of the channel and for S=0. It is clear from Fig. 3a that increasing the parameter M decreases u and its steady state time as a result of increasing the resistive porosity force on u. Fig. 3b shows that increasing M decreases T and its steady state time as increasing M decreases u which, in turn, decreases the viscous dissipation which decreases T.



(b)
Fig. 3 Effect of M on the time variation of: (a) u at y=0; (b) T at y=0. (S=0)

Fig. 4 indicates the effect of the suction parameter on the time progression of the velocity u and temperature T at the centre of the channel for M=0. In Fig. 4a, it is observed that increasing the suction decreases the velocity u at the center and its steady state time due to the convection of fluid from regions in the lower half to the center, which has higher fluid speed. In Fig. 4b, the temperature at the center is influenced more by the convection term, which pushes the fluid from the cold lower half towards the centre.



(b)
Fig. 4 Effect of S on the time variation of: (a) u at y=0; (b) T at y=0. (M=0)

Table 1 presents the variation of the steady state Nusselt number at the upper plate Nu_U and the lower plate Nu_L for various values of C and for $\text{Pr} = 1$, $\text{Ec} = 0.2$. It is clear that increasing the magnitude of the pressure gradient C decreases Nu_L while increases Nu_U. Table 2 presents the variation of the steady state Nusselt number at the upper plate Nu_U and the lower plate Nu_L for various values of Pr and for $\text{C} = -5$, $\text{Ec} = 0.2$. It is shown that increasing Pr decreases Nu_L and increases the magnitude of Nu_U. Table 3 presents the variation of the steady state Nusselt number at the upper plate Nu_U and the lower plate Nu_L for various values of Ec and for $\text{C} = -5$, $\text{Pr} = 1$. It is shown that increasing Ec increases Nu_L while decreases Nu_U.

Table 1 Variation of the steady state Nusselt number at the upper plate Nu_U and the lower plate Nu_L for various values of C ($Pr = 1, Ec = 0.2$)

C	Nu_L	Nu_U
-7	1.132888	-1.165262
-6	.8729877	-.5502089
-5	.6541663	-.02857843
-4	.4755399	.3986019
-3	.3353945	.7296044
-2	.2360443	.9668145
-1	.1761703	1.108868
0	.1561837	1.156197

Table 2 Variation of the steady state Nusselt number at the upper plate Nu_U and the lower plate Nu_L for various values of Pr ($C = -5, Ec = 0.2$)

Pr	Nu_L	Nu_U
1	.6541663	-.02857843
2	.8162214	-.5451652
3	.9549121	-1.079961
4	1.060888	-1.6484
5	1.138879	-2.247568
6	1.196886	-2.86929
10	1.327164	-5.471375

Table 3 Variation of the steady state Nusselt number at the upper plate Nu_U and the lower plate Nu_L for various values of $Ec(C = -5, Pr = 1)$

Ec	Nu_L	Nu_U
0	.1561835	1.156197
.1	.4052173	.5638403
.2	.6541663	-.02857843
.3	.90319	-.6209352
.4	1.152168	-1.213346
.5	1.401205	-1.805681
.6	1.650315	-2.397952
.7	1.899258	-2.990373
1	2.646212	-4.767562

6. Conclusions

The unsteady flow through a porous medium between parallel plates of a viscous incompressible fluid has been studied in the presence of uniform suction and injection. The effect of the porosity and the suction and injection velocity on the velocity and temperature distributions is investigated. It is found that both the porosity and suction or injection velocity has a marked effect on decreasing both the velocity and temperature distributions.

References

[1] Hartmann J, Lazarus F. Experimental investigations on the flow of mercury in a homogeneous magnetic field, K. Dan. Vidensk. Selsk. Mat. Fys. Medd, 1937, No. 7, Vol. 15, pp. 1–45.

[2] Orhan A, Mete A. Laminar forced convection with viscous dissipation in a couette–poiseuille flow between parallel plates, Applied Energy, 2006, Vol. 83, pp. 856-867.

[3] Tudor B, Ioana B. An unified numerical approach of steady convection between two parallel plates original, Applied Mathematics and Computation, 2009, Vol. 215, pp. 2673-2685.

[4] Alpher RA. Heat transfer in magnetohydrodynamic flow between parallel plates, International Journal of Heat and Mass Transfer, 1961, Vol. 3, pp. 108-115.

[5] Nigam SD, Singh SN. Heat transfer by laminar flow between parallel plates under the action of transverse magnetic field, The Quarterly Journal of Mechanics and Applied Mathematics, 1960, Vol. 13, pp. 85-97.

[6] Attia HA. Hall effect on Couette flow with heat transfer of a dusty conducting fluid in the presence of uniform suction and injection, African Journal of Mathematical Physics, 2005, No. 1, Vol. 2, pp. 97-110.

[7] Attia HA. Effect of porosity on unsteady Couette flow with heat transfer in the presence of uniform suction and injection, Kragujevac Journal of Science, 2009, Vol. 31, pp. 11-16.

[8] Erik S, Vajravelu K, Robert A, Van G, Pop I. Analytical solution for the unsteady MHD flow of a viscous fluid between moving parallel plates, Communications in Nonlinear Science and Numerical Simulation, 2011, Vol. 16, pp. 266-273.

[9] Attia HA, Abdeen MAM. Unsteady MHD flow and heat transfer between parallel porous plates with exponential decaying pressure gradient, Kragujevac Journal of Science, 2012, Vol. 34, pp. 15-22.

[10] Joseph DD, Nield DA, Papanicolaou G. Nonlinear equation governing flow in a saturated porous media, Water Resources Research, 1982, No. 4, Vol. 18, pp. 1049-1052.

[11] Kesavaiah D.Ch, et al. Effects of Radiation and free convection currents on unsteady couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium, International Journal of Engineering Research, 2013, No. 2, Vol. 2, pp 113-118.

[12] Das SS. Effect of constant suction and injection on MHD three dimensional couette flow and heat transfer through a porous medium, Journal of Naval Architecture and Marine Engineering, 2009, No. 1, Vol. 6.

[13] Narahari M. Effects of thermal radiation and free convection currents on the unsteady couette flow between two vertical parallel plates with constant heat flux at one boundary, Wseas Transactions on Heat and Mass Transfer, 2010, No. 1, Vol. 5.

[14] Ingham DB, Pop I. Transport phenomena in porous media, Pergamon, Oxford, 2002.

[15] Khaled ARA, Vafai K. The role of porous media in modeling flow and heat transfer in biological tissues, International Journal of Heat and Mass Transfer, 2003, Vol. 46, pp. 4989-5003.

[16] Schlichting H. Boundary Layer Theory, McGraw-Hill, New York, 1986.

[17] Kakac S, et al. Handbook of Single-Phase Convective Heat Transfer, John Wiley, New York, 1987.

[18] Mitchell AR, Griffiths DF. The Finite Difference Method in Partial Differential Equations, John Wiley, New York, 1980.