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Shakedown method versus pseudostaic method for seismic slope stability

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Abstract

Seismic stability of slopes is typically evaluated by conventional methods under the assumption that the slope is subjected to an earthquake just for one time. In general, time histories of loadings on slopes are unknown and loads are of variable repeated nature. Shakedown phenomenon can be considered as a safe state for slopes subjected to variable repeated loadings. In this study, lower bound dynamic shakedown theorem is employed for the seismic stability of slopes as a comprehensive verification. A numerical method applied previously to evaluate roads under the traffic loads was modified to make it appropriate for dynamic shakedown analysis in the present study. The numerical method is based on the combination of finite element and linear programming methods. Critical PGA is employed as a comparative parameter to compare shakedown and pseudostatic methods. Results show that, unlike pseudostaic method, shakedown approach is able to consider dynamic properties of load and slope. Also, it is indicated that contrary to pseudostaic approach, shakedown solutions are different for slopes and embankments. Shakedown and pseudostaic critical PGA versus dynamic properties of load and slope creates four distinct zones. It is shown that the forgoing zones can be used as appropriate tools for seismic zonation of slopes based on their short term and long term safety.

Keywords: Dynamic shakedown, Pseudostatic, Seismic slope stability, Earthquake

1. Introduction

For slopes in seismic regions, the top priority is to evaluate the safety against earthquake loads. Typically, well-known approaches such as pseudostatic method, Newmark displacement method and nonlinear force-displacement method are employed to assess the stability of slopes. Although the forgoing methods are different in some directions, they all consider the earthquake load as a one-time applied load. That is earthquake loading is supposed to be enforced on slope for just one time, and then one of the above mentioned methods is applied to evaluate the slope safety. However, the nature of possible loads on slopes is often unknown and in general, loads are of repeated variable type. In other words, direction, magnitude and time of loadings are changeable. Besides, loads can possibly be repeated for several times. Therefore to have a comprehensive evaluation of slope stability, variability and repetition of loading should be included in safety analysis.

Depending on load intensity, systems and in particular, slopes may show three different behaviors when subjected to variable repeated loads. If load magnitude is small enough, every point of system behaves elastically so that the whole system comes back to its previous position after load removal. In case of large load intensity, permanent displacements start to develop in the system and tend to increase after each load repetition so that after sufficient unloading and reloading, failure occurs due to excessive plastic strains. This kind of failure is called incremental plasticity. Another mode of failure that is also possible for systems under large magnitude loadings is alternating plasticity or low cycle fatigue. The two aforementioned type of failure are called inadaptation. For a particular load magnitude, systems first show permanent deformation, but the rate of plastic strain gradually decreases so that the system tends to behave elastically at the end. This kind of behavior is referred to as adaptation or shakedown. With regard to this definition, shakedown can be regarded as a safe state for structures under general manner of loading.

Shakedown load domain is a portion of the initial domain developed by all loads possibly can be applied on the system. In the literature, shakedown domain is obtained, multiplying a positive coefficient to the initial load domain. This coefficient

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is called shakedown load factor or shakedown factor of safety.

For the first time, Melan [1] introduced lower bound static shakedown theorem. Köiter [2], formulated upper bound static classical shakedown theorem.

Dynamic counterpart of Melan's lower bound theory was pioneered by Ceradini [3, 4] for systems subjected to dynamic loads. Maier used finite element method in combination with linear programming to solve the static [5] and then dynamic [6] shakedown problem in the form of an optimization process. Maier's method was able to be find the adaptation limit of complex structures.

Shakedown theories were being traditionally used to solve the discreet structure problems in early ages of their development. Just a few researches were devoted to application of those theories in practical geotechnical problems. Sharp and Booker [7] study can be considered as the first serious research in geotechnical area in which, they determined the adaptation limit of road pavement under traffic loads. Hossain &Yu [8] and Yu & Hossain [9] found the shakedown capacity of pavements subjected to traffic loads using a method based on the combination of triangular finite elements, stress discontinuity concept and linear programming approach. Their method was actually the extension of method of Boterro et al. [10] previously developed to obtain the bearing capacity of shallow foundations by the limit analysis method. Slopes were analyzed three dimensionally using Bottero's method by Askari et.al. [11].

Two and three dimensional pavements then solved by Shiau [12] using linear and nonlinear programming.

Shiau's formulations for 2D analysis are quite similar to that of Yu&Hossain [9].

Ohtsuka et al. [13] studied seismic stability of slopes by employing the shakedown method. They compared the static shakedown solutions of slopes under repeated static loads (no inertial) with that under an imaginary repeated earthquake time history. However, they did not provide details of their numerical approach.

In some ways, shakedown method is analogous to pseudostatic methods. For instance, both methods results in an individual factor of safety which shows the capacity of system against failure. However, type of loading and modes of failure are different in the two methods. Besides, as will be shown through the paper, unlike pseudostaic method, shakedown analysis is able to incorporate dynamic properties of load and material.

In this paper, shakedown is introduced as a complementary approach for a comprehensive evaluation of slope safety against seismic loads. An effective numerical method, previously utilized for shakedown of pavements under static traffic load was modified and used for dynamic shakedown analysis of slopes in the present research. Shakedown results are compared with corresponding pseudostatic solutions to illustrate their similarity and distinction. It will be shown that results of shakedown and pseudostatic method in the form of critical peak ground acceleration (PGA) can be used to perform seismic slope stability zonation regarding the short and long term behavior of slopes.

2. Shakedown theory and numerical approach

As a whole, structures are subjected to loads with unknown time history over their lifetime. Although time histories of possible loadings are anonymous for most situations, a bound on the amplitude of loads can be imagined or derived from structural codes. Therefore, an area in the load space, let's say the structure load domain, can be conceived, in which all loads that possibly might be applied on the structure exist. Under this load domain, an internal stress field is developed, which is the sum of the elastic and residual stress fields. Mathematically we can write.

$$\sigma_{ij}(x,t) = \sigma_{ij}^{E}(x,t) + \sigma_{ij}^{r}(x,t) \tag{1}$$

Superscripts E and r, indicates elastic and residual respectively.

A key question is: which portion of load domain cause system to shake down. For system under dynamic loadings a conservative solution can be found from the first dynamic shakedown theorem.

First dynamic shakedown theory states that if a fictitious response $u_i^*(x,t)$, $\varepsilon_{ij}^*(x,t)$, $\sigma_{ij}^*(x,t)$ and a time-independent residual stress field $\sigma^{x}_{ij}(x)$ can be found such that

$$f\left(\sigma_{ij}^*(x,t) + \sigma_{ij}^r(x)\right) \le 0 \tag{2}$$

Then shakedown will occur at the real response [4].

A fictitious response is any elastic solution of systems due to repeated external actions, including external forces and strains. It is called fictitious, first, because it is purely elastic and, second, because it is not necessarily obtained for the real initial conditions.

Shakedown theory can be stated mathematically in the form of an optimization process as below.

$$\lambda = \max\left\{ \alpha \;\middle|\; \begin{array}{l} f\left(\alpha\,\sigma_{ij}^*(x,t) + \sigma_{ij}^r(x)\right) \le 0 \\ \sigma_{ij,j}^r = non-repeated\ \ actions\ in\ domain \\ \sigma_{ij}^r.n_j = non-repeated\ actions\ on\ \ free\ boundaries \end{array} \right\} \tag{3}$$

The objective is λ, a coefficient by which the load domain is contracted or expanded to a sate under which shakedown occurs in the system. If constraints are presented in linear form, linear programming method may be utilized to find the optimization goal. The most significant advantage of linear programming is that the obtained solution is a global optimum.

3. Numerical approach

Numerical process to find the shakedown answer contains three fundamental stages.

1. A purely elastic-dynamic analysis of system under dynamic loadings to obtain fictitious response.
2. Developing equilibrium and yield constraints.
3. Optimization to find the best residual stress field and maximum load factor.

To achieve the second and third steps of the above schedule, method of Yu and Hossain [9] is employed with some modification to make it appropriate for dynamic shakedown analysis.

In the following all three steps mentioned above and the

numerical approach to accomplish them will be discussed in brief. Details can be found in Yu and Hossain [9].

3.1. Elastic-dynamic analysis

The equation governs the elastic-dynamic behavior of a system under seismic loads is of the following form.

$$M\ddot{u} + C\dot{u} + Ku = -M\ddot{X}_g \tag{4}$$

Where, M, C and K represents mass, damping and stiffness matrix respectively and $\ddot{X}_g$ is the ground acceleration. Ground acceleration is applied once on the slope and is supposed to be applied repeatedly. Finite element method and implicit time integration method of Newmark [13] were employed to solve the equation 4. To do this, the domain is discretized in to triangular elements. In order to have a linearly distributed elastic stress through elements, triangular elements are taken to have six node and displacement interpolation functions are supposed to be of quadratic form.

Damping is of classic type so that damping matrix is a linear combination of mass and stiffness matrix.

$$C = \eta M + \zeta K \tag{5}$$

The coefficients η and ζ are obtained considering first and second dominant periods of the structure.

At the end of the elastic-dynamic analysis, stresses in the corner nodes of the elements are determined.

3.2. Constraints development

In order to find a residual stress field, the same elements used to obtain the elastic-dynamic response of slopes are employed herein. The only modification is to lay discontinuity lines between adjacent elements (Fig. 1).

The bounding linear programming proposed by Yu&Hossain [9] involves equality constraints, comprising equilibrium of elements, discontinuity and boundary conditions and inequality constraints associated with the yield surface.

The linearization of equilibrium equations for a plane strain element with 3 corner nodes leads to the following equation.

$$A^e_{equil}\,\sigma^{re} = b^e_{equil} \tag{6}$$

Where A^e_{equil} is a 2*9 matrix with known components. For each element, Equation 6 creates two equality constraints.

To find a more accurate residual stress field, stress discontinuity lines are taken between adjacent elements. To have a statically admissible discontinuity, the stress components that are normal and tangential to discontinuity edges and common to adjacent elements must be in equilibrium. There is no restriction on the normal stress parallel to the discontinuity line (Fig. 2a).

The transformation of stresses onto the discontinuity lines will result in

$$A^d_{dis}\sigma^{rd} = b^d_{dis} \tag{7}$$

Where A^d_{dis} is a 4*9 known matrix. Thus, each discontinuity line forms four equality constraints.

Equilibrium must be satisfied on the free boundaries of system.

Because internal stresses are assumed to vary linearly along the edge of elements, they can be linked to external stresses according to

$$\sigma^r_{n1} = q_1\ ;\ \sigma^r_{n2} = q_2\ ;\ \tau^r_1 = t_1\ ;\ \tau^r_2 = t_2 \tag{8}$$

It should be noted that to derive the Eqn.8, external stresses on

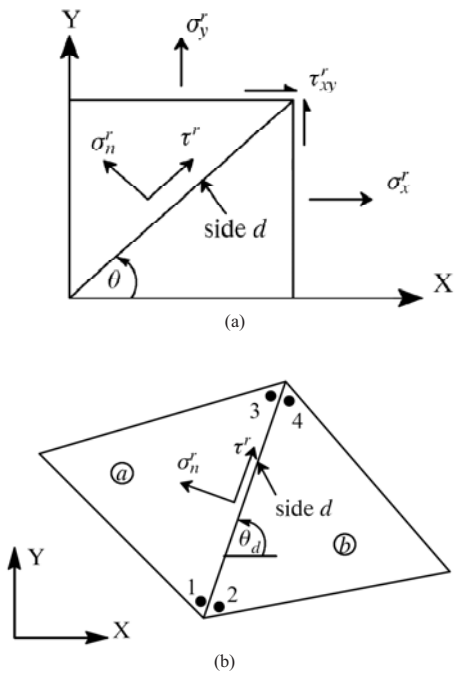

Fig. 2. (a) Normal and shear stresses acting on a plane. (b) Residual stress discontinuity between adjacent triangular elements a and b [11]

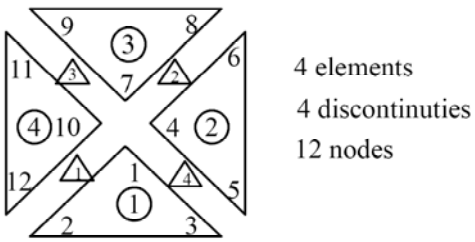

Fig. 1. Triangular element mesh for dynamic Shakedown analysis [11]

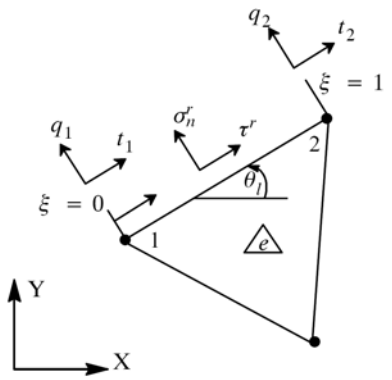

Fig. 3. Stress boundary conditions and external stresses acting on free boundary of element e [11]

the boundaries are assumed to be linearly distributed. Equilibrium of internal and external stresses can be simplified as

$$A^{l}_{bound}\,\sigma^{rl} = b^{l}_{bound} \tag{9}$$

Where A^{l}_{bound} is a 4*6 matrix with known arrays.

It is evident from Eq.9 that a boundary line produces four extra equality constrains.

The Mohr-Coulomb yield surface can be approximated by an interior polygon with p vertices and p sides as shown in Fig.4. The criterion of not violating the yield condition may be mathematically imposed on the linearized yield surface. The results according to Sloan [14] are

$$A_k\,\sigma_x + B_k\,\sigma_y + C_k\,\tau_{xy} \le D_k \tag{10}$$

Where A_k, B_k, C_k and D_k are known coefficients associated with kth failure mode (Fig.4).

Both the residual and elastic stresses are assumed to vary linearly through the elements. Therefore, the yield condition is satisfied for the whole body provided that it is satisfied at the corner nodes of each element.

If combined residual and elastic stresses at the shakedown state ($\sigma_{ij}=\sigma^{r}_{ij}+\lambda\sigma^{E}_{ij}$) are replaced with the general stress in Eq.10, then the nonequality constraint of yield will be as follows:

$$A_k\sigma^{r}_x + B_k\sigma^{r}_y + C_k\tau^{r}_{xy} + E_k\lambda \le D \quad ; \quad k = 1,2,3,.....,p \tag{11}$$

Where

$$E_k = A_k\sigma^{E}_x + B_k\sigma^{E}_y + C_k\tau^{E}_{xy} \tag{12}$$

For each corner node *i* of an arbitrary element, Eq.11 can be written in matrix form as

$$A^{i}_{yield}\,\sigma^{ri} \le b^{i}_{yield} \tag{13}$$

Where A^{i}_{yield} is a p*3 matrix with known arrays so that it forms p inequality constraints for each node.

Stability requires the yield condition to be satisfied at every point of the system after removal of repeated dynamic loads. That is, the residual stresses need to be inside the yield surface, or

$$A_k\sigma^{r}_x + B_k\sigma^{r}_y + C_k\tau^{r}_{xy} \le D \quad ; \quad k = 1,2,3,.....,p \tag{14}$$

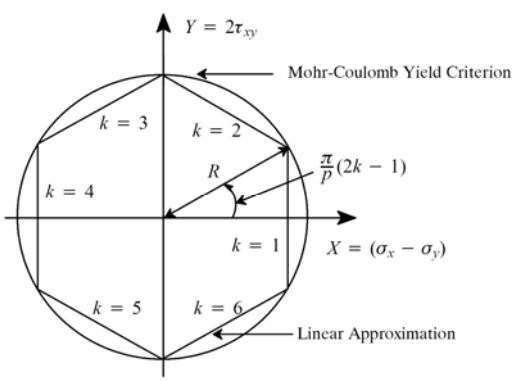

Fig. 4. Linearized Mohr-Coulomb yield surface [11]

Because dynamic loads are represented as load time histories (here, the acceleration time history), the corresponding stresses are also obtained as stress time histories. An evident solution might be satisfying equation 11 at every point and for the entire time history of stresses, which is clearly a cumbersome and time consuming task. According to equations 11 and 14, for each dynamic time step and at each element node, 2*p* yield constraints must be taken into account. For a system with *E* number of elements under a seismic load with *S* time steps, the yield criteria add 6*pES* inequality constraints to the optimization procedure.

According to Maier [5, 6], the time variable may be eliminated if the maximum components of stress along normals to the yield surface and over the whole time history are considered. In other words, the optimization procedure proposed in Eq.1 will be reduced to

$$M(x) = \max\left(N\,\sigma^{*}_{ij}(x,t)\right) \tag{15}$$

$$\lambda = \max\left\{\alpha \;\middle|\; \begin{array}{l} f\left(\alpha\,M + \sigma^{r}_{ij}(x)\right) \le 0 \\ \sigma^{r}_{ij,j} = non-repeated\ \ actions\ in\ domain \\ \sigma^{r}_{ij}.n_j = non-repeated\ actions\ on\ \ free\ \ boundaries \end{array}\right\}$$

In Eq.15, *N* is the normal vector to the yield surface. Based on the first part of Eq.15, the yield constraint in Eq.11 may be rewritten as

$$A_k\sigma^{r}_x + B_k\sigma^{r}_y + C_k\tau^{r}_{xy} + M_k(x)\lambda \le D \quad ; \quad k = 1,2,3,.....,p$$
$$M_k(x) = \max\left(A_k\sigma^{E}_x(x,t) + B_k\sigma^{E}_y(x,t) + C_k\tau^{E}_{xy}(x,t)\right) \tag{16}$$

Because the time variable was eliminated in Eq.16, the number of yield constraints is reduced from 6*pES* to 6*pE*.

Assembling the equality and nonequality constraints proposed in equations described above, the optimization procedure now can be summarized in the following compact form:

$$\begin{array}{ll} Minimise & -\lambda \\ subject\ to: & A_1\,X = b_1 \\ & A_2\,X \le b_2 \end{array} \tag{17}$$

In this study, the simplex linear programming approach was employed to find the best maximum value of the shakedown factor λ by the optimization procedure proposed in Eq.17.

4. Pseudo static versus shakedown method

Pseudstatic method is known as the simplest approach to estimate the seismic stability of slopes. In this method, earthquake load is simply replaced by a horizontal body load which is proportional to the weight of slope by factor K_y. The slope factor of safety is obtained subsequently, using the same way applied to estimate the static safety factor. The value of K_y corresponding to *F*S=1 is referred to as K_{cr}. Since all complex properties of earthquake record and slope material behavior have been summarized in an individual factor K_y, determination of a reasonable K_y and corresponding safety

factor is the most significant part of a pseudostatic analysis.

Varieties of technique have been innovated to find acceptable value of K_y and corresponding factor of safety. Strength reduction method has been found familiarity due to its accuracy to achieve earth and rock slope stability analysis and obtaining K_y [15]. Available methods, mostly judge slope safety by determination of safety factor when slope subjected to a predetermined psuodostaic load factor. In case the safety factor is larger than one, slope is considered to be safe. A variety of predetermined pseudostaic load factors are presented by different methods. For instance, FS>1.15 is acceptable when slope is under K_y=0.15 according to Seed [16] and based on Marcuson and Franklin [15] FS>1 is reasonable if K_y=1/3-1/2PGA/g. . Recently, Miraboutalebi et.al. [17], combined pseudostatic and Newmark method to investigate the seismic slope stability in the presence of inclined bedrock. Further details about available pseudo static methods can be observed in Abramson et al. [18]. Regarding the consequences of available methods, one may consider K_y=1/2PGA/g and correspondingly, FS=1 as a reasonable criteria to judge about slope safety by pseudostaic method. Therefore, in case the values of PGA and Kcr are known, we can make a judgment about the condition of slope in terms of stability according to table 1 [19].

Considering Table 1, PGA/g>2 K_{cr} implies slope instability and conversely, PGA/g<2 Kcr denotes stability of slope. Since PGA/g is a multiplier of PGA, for convenience, let's denote PGA/g as PGAC or peak ground acceleration coefficient. Therefore, it can be concluded that PGAC=2 K_{cr} is the threshold of slope instability and can be regarded as critical pseudostaic peak ground acceleration or $PGAC_{cr}^{PS}$.

A criterion, similar to $PGAC_{cr}^{PS}$, obtained for pseudostaic approach can be conceived for shakedown method. Consider a slope under external dynamic loading in the form of acceleration time history a(t). The corresponding elastic stress field is $\sigma_{ij}^{E}(x,t)$. If the resulting shakedown factor becomes λ, allowable external action will be $\lambda.a(t)$ and obviously, the maximum allowable load intensity is $\lambda.a_{max}$, where a_{max} is the peak acceleration magnitude along the time history of loading. For external action a(t), slope will collapse due to inadaptation beyond $\lambda.a(t)$. Therefore, $\lambda.a_{max}/g$ can be referred to as the critical shakedown peak ground acceleration or $PGAC_{cr}^{SD}$.

It is possible to compare pseudostatic and shakedown method by drawing an analogy between $PGAC_{cr}^{PS}$ and $PGAC_{cr}^{SD}$. It should be noted that critical PGAC for pseudostaic and shakedown methods are conceptually different in some ways. The dynamic properties of the soil and earthquake greatly affect the value of $PGAC_{cr}^{SD}$, while not having any effect on $PGAC_{cr}^{PS}$. Furthermore, loading is of monotonic type for pseudostaic analysis, whereas variable repeated loading is considered for shakedown approach. In this study, the pseudostatic and shakedown results are presented in the form of $PGAC_{cr}^{PS}$ and $PGAC_{cr}^{SD}$ respectively.

Table 1. Guidelines for pseudostatic analysis

Condition	Remarks
K_{cr} > PGA/**g**	Slope can be expected to survive the design earthquake
PGA/(2**g**) < K_{cr} < PGA/**g**	Minor damage is possible
K_{cr} < PGA/(2**g**)	Slope may be unstable

5.Numerical results

Behavior of slopes under repeated dynamic loading are investigated employing some illustrative examples. Results of shakedown method are compared subsequently with those of pseudostaic method to evaluate their differences.

Pseudostaic solutions for slope and embankment are equal if they are the same with respect to geometric and material

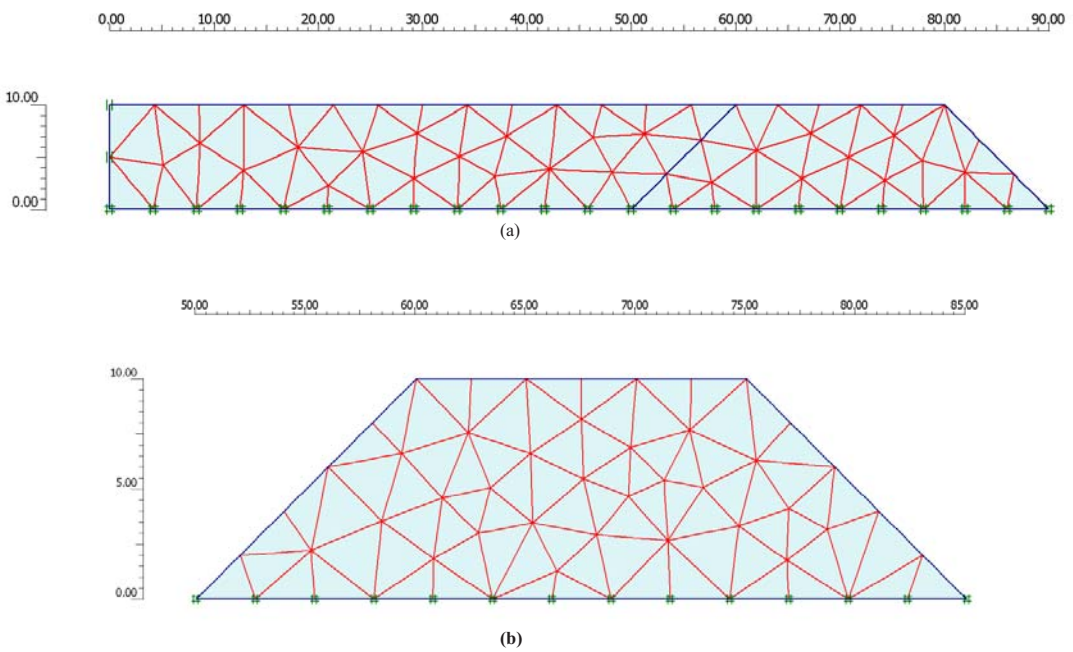

Fig. 5. Mesh and geometrical properties of examples considered in the present study. (a) Slope. (b) Embankment

Table 2. Characters of earthquakes considered in the present study

Earthquake	Deyhook (transverse)	Abbar (Longitudinal)	Zanjiran (Longitudinal)	Bam (transverse)	Zanjiran (transverse)	Avaj (transverse)
T_m (s)	0.496	0.313	0.159	0.396	0.185	0.257
a_{max} (m/s^2)	-2.49<a<3.75	-4.38<a<5.77	-10.44<a<8.8	-5.49<a<6.22	-8.35<a<10.3	-4.43<a<4.29

properties. In order to show the ability of shakedown approach to differentiate between slope and embankment, a slope and an embankment were selected as examples so that their only difference is that the embankment has one extra slope on the left (Fig 5). For convenience, mesh generation ability of Plaxis software [20] was employed to produce finite elements (Fig 5).

Six major earthquake records of Iran were chosen as dynamic loads. The earthquakes were selected so that varieties of mean period (T_m) are provided. The earthquake features have been illustrated in Table 2.

Furthermore, in order to assess the effect of dynamic properties of slope on the results, soils with different shear modulus (G) have been taken to produce a broad range of slope dominant period (T_s) (Table 3). Pisson ratio is taken 0.333 for all example resolved. To determine the effects of dynamic properties of both slope and load, Shakedown limit of slopes are presented as $PGAC^{SD}_{cr}$ against variation in T_s/T_m. Results of pseudostatic analysis were obtained using TRASS program developed by Farzaneh and Askari [21].

For slope and embankment shown in Fig 5, soil properties are assumed to be $\gamma H/c$=4, φ=30° and DR=5%. Shakedown analysis then performed to obtain shakedown critical acceleration of slope and embankment for variety of T_s/T_m as described in Table 2 and Table3. Results are presented in Fig 6.

Critical shakedown PGAC first decrease at lower values of T_s/T_m, then start to rise with increase in T_s/T_m. Critical PGAC reaches its minimum value at around T_s/T_m=1 where resonance occurs. It seems that shakedown behavior is better as slope rigidity decreases or for higher frequency loads.

Results for embankment and slope are approximated by exponential trend lines as depicted in Fig 5.

As Fig 5 shows, embankment can undergo higher PGAC compare to slope, especially at higher values of T_s/T_m.

Pseudostaic critical factor for both slope and embankment are the same and is equal to 0.75, consequently, critical pseudostaic PGAC is equal to 1.5 as illustrated in Fig 5.

6. Discussion about results

Allowable pseudostatic solutions are located under the $PGAC^{PS}_{all}$ curve and correspondingly points under the curve of $PGAC^{SD}_{all}$ suggest permissible shakedown peak ground acceleration. Accordingly, looking Fig 5, arrangement of shakedown and pseudostatic curves has divided the PGAC-T_s/T_m space into four distinct parts.

These four zones are illustrated in a more clear form in Fig 7 and described in Table.4 with respect to modes of failure associated with pseudostatic and shakedown methods.

Regarding Table 4, Region 2, is the most critical area, because slope is in danger of failure due to both plastic collapse and inadaptation. In region 3, slope shows stable behavior based on pseudostatic analysis while fails under load repetition. It shows that safety is provided under load application for just one time and further load enforcements are required for slope failure in zone 3. That is, in this region, problem may cause concerning stability of slopes in long time due to possibility of further load applications.

With contrast to zone 2, region 4 proposes the safest situation for slopes, because safety is provided with respect to both pseudostatic and shakedown methods.

However, behavior of slope in region 1 seems to be unusual. As Fig 7 shows, in region 1, slopes are stable under load repetition, while fail due to load application for only one time. This shows an apparent contradiction in slope behavior under seismic loads. With this regard it can be concluded that the result of pseudostaic analysis is not reliable in region 1. That is, just like regions 3 and 4, stability of slope is guaranteed in

Table 3. Dominant period (Ts) of slope and embankment examples Presented in this study

G (MPa)	2	15	30	100	300
T_s (s) Slope	1.195	0.437	0.309	0.169	0.098
T_s (s) Embankment	1.126	0.411	0.291	0.159	0.092

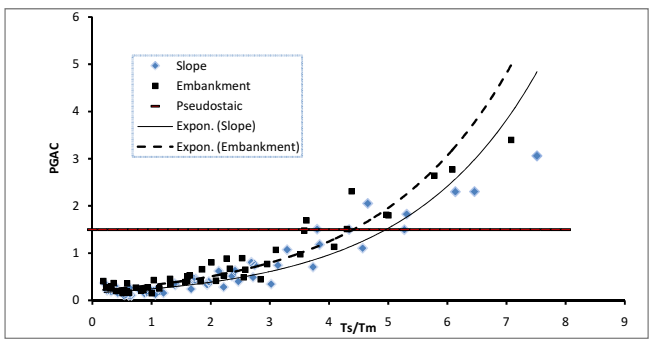

Fig. 6. Pseudostatic and shakedown critical PGAC against variation in Ts/Tm for slope and embankment examples presented in this study (γH/c=4, φ=30° and DR=5%)

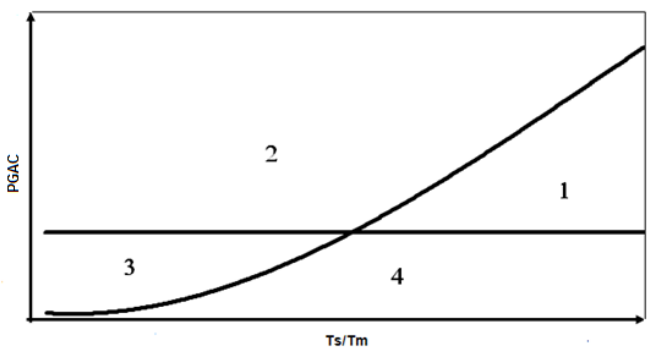

Fig. 7. Independent zones on shakedown-pseudostaic diagram

Table 4. Independent zones on shakedown-pseudostatic diagram and associated slope failure

Region \ Failure Mode	Plastic collapse	Inadaptation
1	Yes	No
2	Yes	Yes
3	No	Yes
4	No	No

region 1, regardless of the pseudostaic result.

Another conclusion that may be drawn from Fig 7 is that, although slopes in zones 2 and 3 are both unsafe when loads are repeated, it seems that number of load repetition to cause failure in zone 3 be less than that of region 2. The reason is that, in region 2, load intensity is higher than magnitude of critical collapse load, whereas load intensity is under the collapse limit load in region 3.

In shakedown analysis, loads are supposed to be repeated while for pseudostaic approach, loads are of monotonic type. Therefore Table 4 can be rearranged to indicate a description of slope behavior subjected to monotonic and repeated loads as illustrated in Table 5.

The concept described above can be used in seismic zonation of slopes. Slopes can be classified according to their geometrical and dynamical properties. For a given seismic region, dynamic characteristics of earthquake loads can be used to produce a diagram similar to Fig 7 for each of aforementioned slope groups. Let's call this kind of diagram slope zonation diagram or SZD. Now, having available PGA, slope characteristics in terms of geometry and material and mean period of loading, one can determine the safety of slope from corresponding SZD and Table 5.

7. Conclusions

Conventional methods to verify seismic stability of slopes assume that slope is under non-repeated, non-variable external loads while in practice such an assumption is not often the case. In this study, shakedown method was employed to evaluate the stability of slopes under the general form of loading, namely repeated variable loading. A numerical method used previously to find shakedown limit of pavements under traffic loads was modified to be appropriate for dynamic shakedown analysis of slopes. Employing some illustrative examples, results of dynamic shakedown of slopes then compared to those of pseudostaic analysis to find out their similarity and differences. The following conclusions were obtained from the present study.

1. Unlike conventional limit state methods, dynamic shakedown approach is able to involve dynamic properties of both load and slope into analysis.

2. Dynamic properties of load and subsoil greatly affect the shakedown response of slopes. Shakedown factor increases as Ts/Tm increases and minimum value of shakedown limit is obtained at around Ts/Tm=1 where resonance occurs.

3. It is shown that, shakedown peak ground acceleration, and its pseudostatic counterpart, , as identified in the text, can be used to compare shakedown and pseudostaic methods.

4. Results of pseudostatic method for a slope are the same regardless the geometry and dynamic properties of the slope, whereas shakedown limits is affected by variations in slope properties.

5. Drawing shakedown and pseudostatic curves in PGAC-Ts/Tm space, four distinct regions can be recognized, each one specifies slope stability condition with respect to failure modes of plastic collapse and inadaptation.

6. The four zones described above may be used for slope stability zonation. Having available PGA, slope dominant period (Ts) and load mean period (Tm), state of slope in PGAC-Ts/Tm is specified as a point which lies in one of the four zones. Then, it is possible to judge whether stability is provided under monotonic and repeated loads.

Table 5. Independent zones on shakedown-pseudostatic diagram and associated safety of slopes with respect to type of loading

Region \ Loading type	Monotonic loading	repeated loading
1	Safe	Safe
2	Unsafe	Unsafe
3	Safe	Unsafe
4	Safe	Safe

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