1. INTRODUCTION

It has been estimated that large amounts of the total manufacturing operating expenses can be attributed to material handling and that an effective facility layout can reduce these costs. Therefore, the layout problem in manufacturing systems is very important. A machine layout problem is different from a traditional layout problem in that there is an additional constraint on a machine’s shape. The pick-up/drop off points are assumed to be located at the machine centroid, this is typically justified given the size of the machine relative to the size of the entire floor space. Thus, the machine layout design problem described in this paper includes determining the location of the machine by specifying the spatial coordinates of the machine’s centroid and specifying the orientation of each machine in either a horizontal or a vertical position.

Flexible manufacturing systems are often used in dynamic environments to deal with uncertainties in production demands. A flexible machine layout that allows the system to efficiently respond to these dynamic and uncertain requirements is critical to achieving a cost-effective system design. Furthermore, a flexible layout is usually equipped...

Fig. 1. Mobile material handling shelf

Fig. 2. Cross over fixtures
with mobile machine tools as well as mobile material handling systems. An Example of mobile material handling system is presented in Fig.1.

Also some kind of mobile machine tools are demonstrated in Fig.2,3&4.

To achieve high productivity in a flexible manufacturing system, using of mobile machine tools can be useful. Usually mobile machines create higher investment costs. Hence, a good machine layout is needed to ensure good system performance and thereby create the benefits necessary to justify the capital investment. The flexible machine layout problem (FMLP) involves planning a machine layout design over a planning horizon with the possibility of rearranging machines if needed.

The desired layouts exhibit flexibility in two ways: through robustness to changes in production requirements and through adaptability of the layout to these new requirements. A robust layout is one that is “good” (or close to optimal) for a wide variety of demand scenarios even though it may not be optimal under any specific demand scenario. For a specific planning horizon, a robust layout design procedure attempts to minimize the total expected material handling costs over this horizon. A dynamic layout design procedure responds to changing production requirements that cause increased material handling costs by adapting the layout to these new requirements. It is hoped that the reduction in material handling costs will offset the layout rearrangement costs. Rosenblatt (1986) formulated this problem as a dynamic program under the assumption of equal size machines.

The flexible machine layout design procedure proposed in this paper generates a flexible layout for a set of unequal size machines over a planning horizon by optimizing the trade-offs between increased material handling costs and machine rearrangement costs as the production requirements change over time. It may choose robust layouts when machine rearrangement costs are high, adaptable or dynamic layouts when rearrangement is easy or the production requirements change drastically or a combination of the two strategies. A single robust layout provides an upper bound on the expected material handling cost while creating a new layout each period provides a lower bound on the expected material handling cost. The majority of practical problems will be neither one of the two extremes; therefore, a flexible layout design is necessary.

2. LITERATURE REVIEW

The existing robust and dynamic layout design procedures usually use a quadratic assignment problem (QAP) formulation as the core mechanism. In other words, the candidate machine location sites are known a priori and it is assumed that any machine can be assigned to any site. Since the QAP is an NP-complete problem (Sahni and Gonzalez 1976), most researchers use heuristic approaches, e.g., Golany and Rosenblatt (1989), Kouvelis et al. (1992), Raoot and Rakshit (1994), and Brian, Madhusudanan et al (2011) for creating robust layouts and Rosenblatt (1986), Kouvelis et al. (1992), Lacksonen and Enscore (1993), Urban, (1993) and Conway and Venkataramanan
(1994), Dunker et al (2005), Ming et al (2009), Junfeng et al (2011) for the dynamic layout problem. The major disadvantages of using these QAP based approaches are that shape constraints cannot be considered in the design stage and facility sites are known and have equal sizes. Hence, they are not well suited for the flexible machine layout problem discussed in this paper. However, some conceptual ideas from these approaches will be used to build a solution methodology for this problem. Montreuil and Laforge (1992) also solve a dynamic layout design problem given a scenario tree of probable future scenarios. Several layout design alternatives for the future are constructed based on this tree. They use a design skeleton approach to determine the machine layout. Lacksonen (1994) uses Montreuil’s formulation for a machine layout design problem to formulate a dynamic layout design problem. This formulation is solved with a two-step procedure similar to the idea of a design skeleton for the machine layout problem. Both Montreuil’s approach and Lacksonen’s approach allow the machines to have varying areas but not a fixed shape or geometry. Furthermore, the pickup and drop-off stations are allowed to move in order to further reduce the objective value.

3. PROPOSED PROCEDURE

In this paper the basic approach is to determine a robust machine layout over a set of planning time windows, which covers the total planning horizon. A planning time window is taken as a consecutive time span with a layout rearrangement occurring only at the beginning of the time span. Also the number and length of these time windows are determined based on the trade-off between material handling costs and machine rearrangement costs. Thus, the strategy is to modify the layout at the beginning of each time window, but not to change the layout within these windows. Hence, a pure robust strategy will have one time window equal to the total planning horizon, a dynamic or adaptive strategy will have several time windows each one period long, and a flexible strategy will choose the time windows to minimize the total cost and includes robust and dynamic strategies as special cases where they are appropriate. The robust machine layout subproblem is represented with an integer programming formulation. Its objective function includes both material handling and machine rearrangement costs. The length of the planning time window is determined using a heuristic that is motivated by the Silver-Meal lot-sizing heuristic (Silver and Peterson). Unlike the QAP-based formulations, the FMLP does not have prespecified layout sites but uses a continuous floor space; hence, there are an infinite number of candidate locations for each machine. Accordingly, the machine rearrangement alternatives cannot be evaluated by simple enumeration of all possible candidate locations for each machine. Furthermore, we assume a fixed cost for moving or rearranging a machine in the layout. Generally, this rearrangement cost does not depend on the distance the machine must be moved. Instead, it is the accumulation of fixed costs due to changing the utilities’ configuration, interrupting or disrupting production, using personnel and equipment to move the machine, etc. This approach differs from previous researches that assume the rearrangement costs are a linear function of the number of square-feet being rearranged or the distance moved.

4. MODEL FORMULATION

We propose to solve the FMLP using a heuristic procedure that is based on a construction type layout design algorithm. It assumes an open-field floor space and alternative production scenarios over multiple time periods. The FMLP is a multiple period design problem with uncertainties in production demands. The planning horizon is assumed to be discrete in time. Let \( s(t) \) represent a possible production scenario in period \( t \). Each production scenario, \( s(t) \), has an associated flow matrix, \( f[s(t)] \) and, a probability of occurrence, \( P[s(t)] \). The collection of all the production scenarios in planning period \( t \) is denoted as the set \( S(t) \). Since \( P[s(t)] \) is a probability function, we have \( \sum_{s(t) \in S(t)} P[s(t)] = 1 \) for all \( t \). Since the proposed approach is to develop a robust layout over a planning time window, an aggregate flow matrix will be used for this time window. We define \( F_{k,T}(i,j) \) as the expected flow density between machines \( i \) and \( j \) from time period \( k \) to period \( k + T \). Note that since we are dealing with a rolling planning horizon problem, no specific ending time period is necessary. Thus,

\[
F_{k,T}(i,j) = \sum_{t=k}^{k+T} \sum_{s(t) \in S(t)} P[s(t)] f[s(t)]
\]  

(1)
Additional notations required are as follows:
\( N = \) total number of machines in a layout,
\( A_i = \) fixed layout rearrangement cost for machine \( i \),
\( \{a_i, b_i\} = \) spatial coordinates of the centroid of machine \( i \) in the existing layout,
\( V_i = \begin{cases} 1, & \text{machine } i \text{ in the existing layout, (the shorter side at the bottom)} \\ 0, & \text{machine } i \text{ in the existing layout, (the longer side at the bottom)} \end{cases} \) for existing layout
\( Z_i = \begin{cases} 1, & \text{machine } i \text{ in the existing layout, (the shorter side at the bottom)} \\ 0, & \text{machine } i \text{ in the existing layout, (the longer side at the bottom)} \end{cases} \)
\( \omega_i = \) the width of machine \( i \) (shape constraint),
\( v_i = \) the length of machine \( i \) (shape constraint),
\( I_i = \begin{cases} 0, & \psi_i = 0 \text{ and } Z_i = 0 \\ 1, & \text{Otherwise} \end{cases} \)
\( e_{ij} = \) the unit flow distance weight between machine \( i \) and machine \( j \),
\( M = \) a big number (i.e., a surrogate for infinity),
\( W = \) the floor width,
\( H = \) the floor length,
\( (x_i, y_i) = \) spatial coordinates of the centroid of machine \( i \),
\( \alpha_i = \begin{cases} 1, & \text{if } x_i \geq x_j \text{ and } 0 \text{ otherwise}, \\ 0, & \text{Otherwise} \end{cases} \)
\( \beta_i = \begin{cases} 1, & \text{if } y_i \geq y_j \text{ and } 0 \text{ otherwise}, \\ 0, & \text{Otherwise} \end{cases} \)
\( \sigma_i = \begin{cases} 1, & \text{if } x_i \geq a_i \text{ and } 0 \text{ otherwise}, \\ 0, & \text{Otherwise} \end{cases} \)
\( \tau_i = \begin{cases} 1, & \text{if } y_i \geq b_i \text{ and } 0 \text{ otherwise}, \\ 0, & \text{Otherwise} \end{cases} \)
\( \varphi_i = \begin{cases} 1, & \text{if } \tau_i = 0 \text{ and } 1 \text{ otherwise}, \\ 0, & \text{Otherwise} \end{cases} \)
\( \theta_{ij} = \) binary variable indicating machine interference,
\( E_{ij}(F_{ij}) = \) the magnitude of the positive (negative)
component of \( x_i - x_j \),
\( G_{ij}(H_{ij}) = \) the magnitude of the positive (negative)
component of \( y_i - y_j \),
\( P_i(Q_i) = \) the magnitude of the positive (negative)
component of \( x_i - a_i \),
\( R_i(S_i) = \) the magnitude of the positive (negative)
component of \( y_i - b_i \),
\( \varnothing = \{(i,j) | i = 1,\ldots,N; j = i+1,\ldots,N; i \neq j\} \),
\( \Delta = \{(i) | i = 1,\ldots,N\} \)

The formulation for the robust machine layout problem over the time window from period \( k-1 \) to period \( k + T \) is shown below. This formulation calculates the material handling costs over the time window. In addition, it considers potential layout changes and includes the cost of rearranging machines to get from the current layout (which was used in period \( k - 1 \) and is denoted by \((a_i, b_i)\) and \( V_i \) to the new layout.

\[
\begin{align*}
\text{minimize} & \quad \sum_{(i,j) \in \varnothing} e_{ij} F_{ij}(i,j) [E_{ij} + F_{ij} + G_{ij} + H_{ij}] + \sum_{i \in \Delta} A_i I_i \\
\text{subject to} & \quad \left| E_{ij} + F_{ij} - \left(\frac{1 - Z_i}{2} w_i - \frac{Z_j}{2} v_j - \frac{1 - Z_j}{2} w_j \right) \right| - \frac{Z_j}{2} v_j \geq -\theta_{ij} M \quad (i,j) \in \varnothing \\
& \quad \left| G_{ij} + H_{ij} - \left(\frac{1 - Z_i}{2} w_i - \frac{1}{2} w_i - \frac{1 - Z_j}{2} v_j \right) \right| - \frac{Z_j}{2} v_j \geq -(1 - \theta_{ij}) M \quad (i,j) \in \varnothing \\
& \quad x_i - a_i = P_i - Q_i \quad i \in \Delta \\
& \quad P_i \leq a_i M \quad i \in \varnothing \\
& \quad Q_i \leq (1 - a_i) M \quad i \in \varnothing \\
& \quad P_i + Q_i - \tau_i M \leq 0 \quad i \in \Delta \\
& \quad x_i - a_i = P_i - Q_i \quad (i,j) \in \varnothing \\
& \quad F_{ij} \leq (1 - a_i) M \quad (i,j) \in \varnothing \\
& \quad G_{ij} \leq \beta_i M \quad (i,j) \in \varnothing \\
& \quad H_{ij} \leq (1 - \beta_i) M \quad (i,j) \in \varnothing \\
& \quad y_i + b = R_i - S_i \quad i \in \Delta \\
& \quad R_i \leq \rho_i M \quad i \in \Delta \\
& \quad S_i \leq (1 - \rho_i) M \quad i \in \Delta \\
& \quad R_i + S_i - \gamma_i M \leq 0 \quad i \in \Delta \\
& \quad \tau_i + \gamma_i - 2 \varphi_i \leq 0 \quad i \in \Delta \\
& \quad \varphi_i + Z_i - 2I_i \leq 0 \quad \text{when } V_i = 0 \quad i \in \Delta \\
& \quad \varphi_i + (1 - Z_i) - 2I_i \leq 0 \quad \text{when } V_i = 1 \quad i \in \Delta \\
& \quad \left\{ Z_{ij}, a_{ij}, \beta_{ij}, \sigma_i, \tau_i, \rho_i, \varphi_i, \varphi_i, i \in \{0,1\} \right\} \quad i \in \Delta \text{ and } (i,j) \in \varnothing
\end{align*}
\]

The objective function (2) minimizes the sum of the weighted flow distance and machine rearrangement costs. Eqs. (3) and (4) ensure that there is no overlap between any pair of machines. When Eq. (3) is active, oii is equal to 0, which means the distance between the centroids of cells i and j is too far to overlap in the
horizontal direction. When $\theta_{ij}$ equals 1, Eq. (4) is active, and the distance between the centroids of cells $i$ and $i$ is too far to overlap in the vertical direction. One or the other of these constraints must be active for the layout to be feasible.

The formulation explicitly considers the current layout and tracks all machine rearrangements required to obtain the new layout. Eqs. (5) – (8) determine whether a machine’s location in the new layout was moved in the $x$ direction from its location in the old layout. The status of whether a machine has moved in the $x$ direction is assigned to the indicator variable, $x_i$ and is used to determine the rearrangement cost. Eqs. (9)-(14) are used to eliminate the absolute value sign from the rectilinear distance measure between the centroid of machine $i$ and the centroid of machine $j$. Similarly, Eqs. (15) – (18) detect machine location changes in the $y$ direction and assign the status to the indicator variable, $y_i$.

Eq. (19) determines whether a machine has been moved in either the $x$ or $y$ direction in the new layout and assigns the status to the indicator variable, $\varphi_i$. If the machine’s centroid is located at the same position as the existing one, then $\varphi_i$ is equal to zero. The machine orientation also impacts the machine rearrangement cost. When $\varphi_i$ is equal to zero and the orientation of a machine in the new layout is the same as in the existing layout, then no rearrangement cost is incurred for that machine. Otherwise, if $\varphi_i$ is equal to one or the orientation has changed, then there is a fixed machine rearrangement cost added to the objective function. Eqs. (20) and (21) regulate these machine rearrangement costs, where $I_i$ is equal to zero when there is no rearrangement associated with machine $i$. Finally, Eq. (22) specifies the bounds for each variable.

5. COMPUTATIONAL EXPERIMENTS

An experimental problem is used to demonstrate the proposed flexible machine layout design procedure. The problem is a 7-machine, 7-time period problem, P7. The floor space is assumed to be 30 units by 30 units. The machine specifications and the existing layouts are shown in following table.

Table 1. Machine specifications and the existing layouts

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i$</td>
<td>13</td>
<td>4</td>
<td>6.5</td>
<td>10</td>
<td>2</td>
<td>9.5</td>
<td>14.5</td>
</tr>
<tr>
<td>$b_i$</td>
<td>8.5</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>8.5</td>
<td>3</td>
</tr>
<tr>
<td>$V_i$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$w_i$</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$h_i$</td>
<td>4</td>
<td>8</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2. The costs per period, $C(k,T)$

<table>
<thead>
<tr>
<th>$C(k,T)$</th>
<th>$C(1,0)$</th>
<th>$C(1,1)$</th>
<th>$C(1,2)$</th>
<th>$C(1,3)$</th>
<th>$C(1,4)$</th>
<th>$C(5,0)$</th>
<th>$C(5,1)$</th>
<th>$C(5,2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>1600</td>
<td>1385</td>
<td>1350</td>
<td>1275</td>
<td>1390</td>
<td>1450</td>
<td>1350</td>
<td>1280</td>
</tr>
</tbody>
</table>

Fig. 5. Costs per period

Planning time window
rearrangement cost, \( A_i \), is 100 for all \( i \).

The costs per period, \( C(k,T) \), for the problem are shown in Table 2, and are further illustrated in Figs. 5 and 6.

The cost per period for P7 consistently decreases over the first four planning periods, then increases in period 5. Hence, the layout is rearranged at the beginning of period 1 and is used for the additional three periods. Then, a new layout is determined for periods 5-7 by using the layout from periods 1-4 as the existing layout.

The resulting flexible machine layout design has a cost per period of 1275 and 1280 for periods 1-4 and periods 5-7, respectively. The final proposed design for P7 is shown in Fig. 6.

6. CONCLUSION

In this paper we proposed a flexible machine layout design procedure that represents a significant step toward solving an important problem in an automobile manufacturing system setting. The model incorporates the machine rearrangement costs, as well as the material handling costs, into the layout design procedure to create a cost-effective machine layout over the planning horizon. It is different from the existing flexible layout methods because machines are of varying sizes and have fixed geometries and load/unload point positions; and the layout rearrangement cost is a fixed cost based on changing the position of a machine in the layout.

REFERENCES