Evaluating the Effect of Dissipated Viscous Energy of a Rolling Tire on Stress, Strain and Deformation Fields Using an Efficient 2D FE Analysis

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Abstract

The dissipated energy from periodic deformation is regarded as the main reason for heat generation and temperature rise inside the tire domain. However, the mechanical behavior of rubber parts is highly temperature dependent. In most performed investigations, the influence of thermal effects on stress/deformation fields of pneumatic tires is ignored and just temperature distribution is considered. Hence in this study, using a series of 2D and 3D finite element models, a robust and efficient numerical study is presented for thermo-mechanical analysis of pneumatic tires specially 115/60R13 radial tire. Finally, the effects of loading conditions and ambient temperature on the thermo-mechanical properties of tire are investigated in detail. Comparing the obtained results with the available results in literature, shows a good agreement of the presented studies with related published works.

Keywords: Pneumatic tire; Thermal analysis; FEM analysis; Stress-deformation field.

1. Introduction

Design and developing a new product is an expensive costing process for tire manufacturers. Virtual simulation of real conditions during tire operation can dramatically reduce the cost and time [1]. The finite element method is a very useful numerical tool for predicting the tire behavior in various conditions. The thermal effects are key factors that should be considered for long service life and optimal operation of vehicles tire. A portion of periodic deformation energy of the tire is converted to heat energy due to hysteretic property of rubber. Because of low diffusivity of rubber, temperature locally rises in the tire structure and as a result, the rolling resistance of tire in contact with the pavement and the energy efficiency of the vehicle is strongly affected [2]. The fatigue life of rubber drops also with an increase in temperature especially at the temperature of 90°C-120°C [3]. So, it is necessary to evaluate the tire temperature distribution at different operating conditions.

Assuming that the inelastic energy of deformation process is completely converted into volumetric heat, Mc Allen et al. investigated the temperature rise in aircraft tires [4]. Later, Lin and Hwang obtained the temperature distribution of rolling tires in a steady state thermal analysis [5]. Sokolov calculated the temperature in material body of self-heating pneumatic tires using a 3D FE analysis for stress strains [3]. By fitting Fourier series on the strain and stress of nodes, also using a 3D finite element model, Zepeng evaluated the heat generation for thermal analysis of a rolling tire [6]. Li et al. employed the Algor software for assessment the influence of the ambient temperature on temperature gradient and stress fields of a dump truck tire [7]. In most of the mentioned studies, just temperature distribution is considered as the major objective of the analysis and effect of created heat in mechanical behavior of rolling tire is ignored. In the present study, using set of numerical investigations based on finite element model, the effect of ambient temperature and different loading conditions on thermal/structural behavior of a radial tire 115/60R13 have been evaluated. The available experimental and numerical data are used for verifications the obtained results.
2. Tire Construction

For investigating the thermo-mechanical properties of tire, a radial pneumatic tire is considered where the schematic view of radial tire is illustrated in figure 1 [8]. Compared with the traditional bias tires, the radial-ply tires have become more popular because of less rolling resistance and heat generation. Several layers of steel belts are laid under the tread rubber at the low crown angle; i.e. the angle between the ply and circumferential line of the tire. Besides strengthen the composite structure of tire, these steel belts also help to distribute the created heat and improving fuel consuming. The tread layer of the tire is usually patterned with longitudinal or transverse grooves and serves as a wear-resistance layer that provides sufficient frictional contact with the pavement [9]. Flexibility of the sidewalls makes it act like a moving hinge and the tread blocks remain flat against the road surface with maximum grip.

3. Theoretical Background

3.1. Rubber Material Model

Since rubber is a highly extensible material, small-strain elasticity theory is not suitable for describing the responses of tires to large deformations. Therefore, the incompressible hyper-elastic Mooney-Rivlin model is commonly used for tire studies [10-13]. According to this model, the stresses are obtained from the partial derivatives of the strain energy functions as [14].

\[
\sigma = \frac{\partial U}{\partial \lambda} = \frac{\partial U}{\partial I_1} \frac{\partial I_1}{\partial \lambda} + \frac{\partial U}{\partial I_2} \frac{\partial I_2}{\partial \lambda} + 2 \left(1 - \lambda^3\right) \left( \frac{\partial U}{\partial \varepsilon_{11}} + \frac{\partial U}{\partial \varepsilon_{22}} \right)
\]

(1)

Where \( \sigma \) is the stress, \( U \) is the strain energy function, \( \lambda \) is the stretch ratio and \( \varepsilon_{11}, \varepsilon_{22} \) are the two first deviator of strain invariants. Therefore, the strain energy used in the model is

\[
U = C_{01} (I_1 - 3) + C_{10} (I_2 - 3)
\]

(2)

In equation (1), coefficients \( C_{01} \) and \( C_{10} \) are the Mooney-Rivlin constants which can be experimentally determined for each rubber [15].

3.2. Heat generation rate

A rolling pneumatic tire exhibits both elastic and viscous characteristics due to embedded rubber compounds. As result of imposing time dependent loads to such structures, hysteretic loss occurs and some of restoring strain energy is dissipated in each cycle of periodic loading.

At each operating temperature, the ratio of loss strain energy to the total strain energy of the rubber, which is called hysteresis, can be estimated by experimental methods. According to the results obtained by Lin and Hwang [5], it is reasonable to take this ratio 0.1 for vast variety of operating temperatures.

The total strain energy that is calculated from deformation module is then multiplied with hysteresis to find the lost strain energy which is assumed to completely convert to internal heat generation in this study

\[
U_{\text{loss}} = H U_{\text{total}}
\]

(3)

Knowing the velocity of rolling tire, heat generation rate \( Q_v \) for each element of tire can be easily obtained as:

\[
Q = \frac{U_{\text{loss}} \cdot V_I}{2\pi R}
\]

(4)

Where \( V_I \) is the velocity and \( R \) is the radius of the rolling tire.

3.3. Heat convection

Determining the heat generation rate, proper boundary conditions are needed for evaluating the temperature distribution of rolling tire.

The flow of air with velocity of \( v \) around the outer surfaces of the tire causes forced heat transfer convection with the heat transfer coefficient \( h_c \) where \( h_c \) is determined through the following empirical relation [16]

\[
h_c = 5.9 + 3.7v \left( \frac{W}{m^2 \times ^\circ C} \right)
\]

(5)

The airflow over the treaded surface has a single value that can be easily obtained, but the relative velocity around the sidewall of rolling tire is a function of linear distance measured from the hub center. So, the average velocity on this surface is substituted in Equation (5). Compared with the exposed surfaces, the inner surface is taken to be
insulated due to negligible heat transfer to the inside air [4].

4. Simulation Details

4.1. 3D FEM for Pneumatic Tire

Evaluating the total stored strain energy and amount of its dissipation in each revolution is an essential prerequisite for any thermal/mechanical analysis. So as first step, a 3D FE model of radial tire 115/60R13 is prepared for evaluating the hysteretic loss of the tire [17]. For simplicity, the tire is assumed to be composed of carcass-ply, tread, and bead. A 3D model axisymmetric model is constructed by revolving the cross-section of the tire in circumferential direction. The thickness of tread, sidewall and ring are 10, 7.5 and 5 mm, respectively.

The tread as an isotropic and homogenous rubber is built by solid element with Mooney-Rivlin hyper-elastic property. The carcass-ply and bead are also created as solid element with elastic properties shown in table 1. As preprocessing stage, the tire is meshed using 10369 linear 8-node hexahedral elements with dimensions and aspect ratio of 1 and 1.5 mm, respectively. For improving the accuracy of the analysis, a dense mesh is used in the vicinity of tire footprint.

![Fig1. Components of a radial-ply tire](image)

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>Tread</th>
<th>Belt</th>
<th>Carcass</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-10</td>
<td>15</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Mooney-Rivlin constants (Mpa)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{01}$</td>
<td>8.061</td>
<td>2.0477</td>
<td>0.550966</td>
<td>-</td>
</tr>
<tr>
<td>$C_{10}$</td>
<td>1.806</td>
<td>1.1859</td>
<td>0.00373778</td>
<td>-</td>
</tr>
<tr>
<td>Density, $\rho \left( \text{kg/m}^3 \right)$</td>
<td>1400</td>
<td>1140</td>
<td>1100</td>
<td>7644</td>
</tr>
<tr>
<td>Modulus of elasticity, $E \left( \text{Gpa} \right)$</td>
<td>-</td>
<td>55</td>
<td>0.794</td>
<td>-</td>
</tr>
<tr>
<td>Poisson ratio, $\nu$</td>
<td>-</td>
<td>0.3</td>
<td>0.45</td>
<td>-</td>
</tr>
<tr>
<td>Thermal conductivity, $\mu \left( \text{W/m°C} \right)$</td>
<td>0.293</td>
<td>60.5</td>
<td>0.293</td>
<td>0.055</td>
</tr>
<tr>
<td>Hysteresis constant, $H$</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1. Material properties used in tire FE model [7].
For imposing boundary conditions, pressure, static load and contact conditions are applied to the tire. Thorough finite element analysis, the stress, strain, deformation and strain energy of deformed elements are also calculated for entire tire domain (Figure 2). As shown in figure 2, a 3D FE tire model is presented where inflation pressure, load, and contact condition are considered for calculating stored strain energy in deformed tire elements.

Using determined hysteresis, the lost strain energy can be evaluated in one cycle of rotation of a rolling tire. Therefore, results are used in two successive 2D finite element analysis for evaluating the temperature distribution and thermal effects on stress gradient of a pneumatic tire. The whole procedures of modeling and simulating applied in this study are illustrated as a flowchart shown in figure 3.

4.2. The 2D Thermo- Mechanical Analysis for Pneumatic Tire
2D finite element thermal and mechanical models are developed for 115/60R13 tire. The appropriate restraints and boundary conditions are imposed to both models (Figure 4).
Utilizing a steady-state thermal analysis, the temperature distribution for different cases is obtained. On a non-linear loading analysis, these results are then used for evaluating the thermal effects on mechanical behavior of the tire. Stress distribution, displacement and strain results under different axle loads and inflation pressure are obtained in the following.

5. Results and Discussions

5.1. Loading Analysis

For a variety of inflation pressure, vertical displacements of tire hub center, loaded by axle force is evaluated using a 3D finite element model. Data of experimental investigation are used to verify the obtained results. As depicted in Figure 5, it is clear that the simulation results have a good consistency with the empirical observations. Moreover, the obtained results have a good agreement with the outputs of results presented by Behroozi et al. [18] for analysis of aircraft tire. Thus, the calculated values for strain energy of tire, which is used as major criteria for prediction of thermal and mechanical behavior, have been found with an acceptable accuracy.

5.2. Thermal analysis

Variation of temperature across the tire cross section is shown in figure 6. For the given ambient temperature, the exposed zones to the airflow; i.e. the tread blocks and inner/outer surfaces of the tire, are cooler than belt regions due to low thermal conductivity of the rubber.

Since the highest temperatures occur in the belt edge, therefore it is regarded as the main cause of structural failure. On the other hand, it is seen that the temperature distribution radially symmetric, which is similar to that of presented by many researchers [2-4, 7, 16].

For the considered tire with inflation pressure and ambient temperature 25°C, variation of maximum temperature created in belt layer of rolling tire versus different values of speeds is depicted in figure 7. For the same axle load, it is obvious that any increase in tire velocity leads to smaller period of one revolution and consequently increase in amount of heat generation rate. So, the maximum temperature found to be raised by increasing in tire speed which shows a good consistency with the experimental measures [2, 5, 19].

On the other hand, for higher values of axle loads, the maximum temperature has sharper variations with speed; an approximate 45°C increase can be seen for axle load 2.5 kN and 65°C increase at the axle load 7.5 kN.

![Fig7. The effect of speed and axle load on maximum tire temperature.](image-url)
Temperature and deformation fields of tire are plotted in figures 8. In figures 8a, temperature fields of the tire, rolling on three different ambient temperatures are shown. A cold operating temperature of -10 °C, a moderate temperature of 15 °C and 40 °C as hot ambient condition are considered in the analysis. The inflation air pressure of 200kPa is imposed on internal surface of tire model with 2.5kN weight load on its axle. The maximum tire temperatures of 29.2 °C, 54.2 °C and 79.2 °C are obtained respectively for three ambient temperatures. It is inferred that, warmer conditions of working will cause more intense temperature gradient which is proved by results of experimental analysis [2, 7, 20].

Based on the results of thermal analysis for three ambient temperatures, the counters of deformation for the tire at the end of 1 second time step are depicted in figures 8b. For the same axle load and inflation pressure, any increase in ambient temperature leads to large tire hub displacement. For three different ambient temperatures, the variations of hub displacements within time of 0 to 1 second are depicted in figure 9. It is clear that for the given time step, the calculated values of displacement increase from ambient temperature -10 °C to 40 °C. These observations closely match the published results reported by Yanjin et al. [7] and Li et al. [21].

The von Mises stress contour in tire subjected to an 200kPa inflation pressure, 2.5kN axle load and ambient temperature 15 °C is shown in figure 10a. According to the results, the maximum stress occurs in belt layer and the stress decreases from the tire bottom to top along the whole circumference. For different ambient conditions, the variations in maximum values of von Mises stress during 1 second are illustrated in figure 10b. It is found that any change in ambient temperature has a considerable effect on created stresses inside the tire domain.
Conclusions

In this work using a successive numerical based finite element method, the effects of dissipated energy on deformation and stress fields of radial tire 115/60R13 is investigated for different axle loads and inflation pressures. It is concluded that the maximum values for temperature and von Mises stress occur while, the bottom regions of tire endure larger deformations in comparison with the other parts. The results show that for 25 °C difference in ambient temperature, the maximum values for hub displacement and created stress is increased by 115% and 40%, respectively. Therefore, sensitivity of tire deformation to the thermal effects is more than the stress.
References


[9]. Erşahin MA. Finite element analysis of cornering characteristics of rotating tires: MIDDLE EAST TECHNICAL UNIVERSITY; 2003.


