

## Control of Two-Wheels Inverted Pendulum Using Parallel Distributed Compensation and Fuzzy Linear Quadratic Regulator

Rasool Kahani

M.Sc. Student of Control Engineering  
Islamic Azad University, Sci & Res Branch  
Tehran, Iran  
E-mail: rasool.kahani65@gmail.com

Bijan Moaveni

Department of Railway Engineering  
Iran University of Science Technology  
Tehran, Iran  
E-mail: b\_moaveni@iust.ac.ir

**Abstract**--This paper, in the first step, deals with the modeling of two-wheels inverted pendulum using Lagrange method. The final model is represented in state-space form. Then Parallel Distributed Compensation (PDC) controller is designed for it. The stability condition of whole system is shown by employing the Linear Matrix Inequality (LMI) approach. At the end to access the minimum error between the set-point and the output and to obtain the best performance in the control signal, Fuzzy Linear Quadratic Regulator (FLQR) controller is introduced and designed for this system. Two computer simulations are presented to show the effectiveness of the proposed control methods.

**Keywords**- Wheeled inverted pendulum, Parallel Distributed Compensation, Linear Matrix Inequality, Fuzzy Linear Quadratic Regulator

### I. INTRODUCTION

A two-wheels inverted pendulum robot has some useful characteristics as a human-assistant robot. SEGWAY, the personal transporting vehicle is the representative example for it. In 2002, Grasser [3] used Newtonian approach to derive the dynamical equations. Later, in [4], similar work has been reported using LQR controller. In this paper, we focus on the inverted pendulum type robot, which employs two wheels for its mobility, that wheels are coupled to the same geared DC motor. On the other hand the motion of two wheels is depended on each other, so system can move just in one direction as a Single Input Single Output (SISO) system. The matter of wheeled inverted pendulum is moving wheels to the desired position with keeping vertical stability of inverted pendulum, so the output is the angle of the pendulum which is originally unstable via rotation of wheels. In next section to obtain the dynamic equations of this system, Lagrange method is used and the nonlinear state space form is given sequentially. As the inverted pendulum system is nonlinear it is well-suited to be controlled by fuzzy logic. So, PDC (Parallel Distributed Compensation) approach is chosen that it employs the corresponding T-S (Takagi-Sugeno) fuzzy model. There are two ways to obtain T-S fuzzy model, "sector nonlinearity" and "local approximation" [1]. In this paper the second method is used since it reduces the number of fuzzy rules, and as a result, the designing procedure is more convenient.

There are few results so far that can provide an effective way of designing optimal controllers for general nonlinear systems. So we propose the FLQR as an optimal controller which can provide an effective control performance.

### II. MODELING

Fig. 1 shows the block diagram of system. The mass of chassis is  $m_p$  that has rotation inertia  $I_p$ , and the centre of chassis is located as far as the distance  $L$ . The chassis would be unstable in the wheeled inverted pendulum system and if the controller doesn't work properly it'll fall down by gravity. At this moment, system moves toward the direction which the chassis is falling down so an inertial force is produced that prevent falling down and keeping balance.

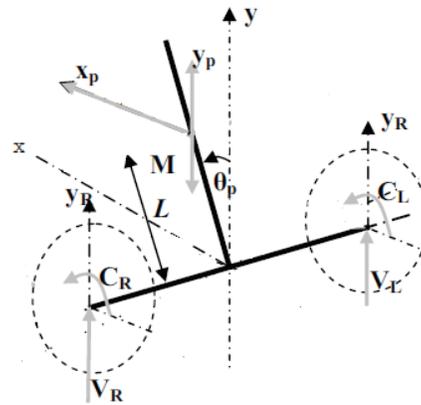


figure 1. Block Diagram of chassis and wheels

#### A. Dynamic Equations

Dynamic model is useful to predict the system behavior after applying controller to the system. In this research, dynamic equation of the system is obtained by Lagrange method. To use this method, at first the kinetic and potential energy are determined.

The kinetic energy of pendulum is:

$$T_1 = \frac{1}{2} I_p \dot{\theta}_p^2 + \frac{1}{2} m_p v^2 \quad (1)$$