Bifurcation Control in Hodgkin–Huxley Model based on the State Feedback Theory

M. Asgharpour , PhD Student of Control System Design, Department of technical and engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran.
e-mail: mahnaz_asgharpour@yahoo.com

B. Moaveni, Assistant Professor of Control Systems Design, School of Railway Engineering, Iran University of Science and Technology, Tehran, Iran.
e-mail: b_moaveni@yahoo.com

Abstract- Since the Hodgkin–Huxley (HH) equations depend on various parameters, changing each of parameters will cause the HH model to represent qualitatively different behaviors such as bifurcation. In this paper We designed a closed loop system to control the bifurcation by introducing a digital state feedback which causes the bifurcation point to move to an unreachable point at the same time and this will lead to a complete bifurcation control. Our simulation results validate the applied theoretical analysis and control method. This digital control strategy could be useful to design a new closed loop electrical stimulation system to cure some nervous system diseases specially ion channel dysfunctions.

Keywords- Bifurcation Control, State Feedback Theory, Modeling, Hodgkin-Huxley Model

I. Introduction

In the last decades, due to the electrical nature of the mechanisms of travelling and processing the neural messages through the neural cells in central nervous system (CNS), the idea of using extracellular stimulation to affect the activity and the responses of the human nervous system has been widely investigated. The excitable membrane of a neuron contains several ion channels such as sodium Na, potassium K, and extra leakage l channels. The opening of sodium channels changes the membrane potential in approximately 10^{-3} second and ,this rapid change of voltage will produce a signal which is called action potential. The neurons of the nervous system would communicate together by these action potentials. In order to model the excitable membrane of the neural cells, different models have been proposed such as: Hodgkin–Huxley (HH)[10] , Frankenhaeuser (FH)[13], McNeal[12], Izhikevich, CRRSS[8], etc. .Most of these models are designed based on the two basic models: HH and FH . HH model has been presented based on the electrical conductance of the ion channels of the excitable membrane while the FH model is described based on the cell's ionic permeability. Furthermore, The FH model was achieved by electrophysiological experiments on the myelinated fibers of frog but HH model was extracted by an elegant series of experiments on the giant nerve fiber of a squid. The characteristic feature of the squid giant axon is its extraordinary large diameter which is about 0.5 mm [9].This large size lets the nerve fiber to conduct the action potentials rapidly. Because of this large diameter which is 100 times thicker in comparison with the other nerve fibers of squid's nervous system and with the other nervous systems, Hodgkin and Huxley were able to manipulate their model more feasibly than smaller fiber models. Hodgkin and Huxley proposed their model of nerve conduction in 1952 and they developed the core mathematical framework for modern biophysically neural modeling [9][16]. After that, the HH model has been widely studied experimentally and mathematically by other researchers. The nerve conduction HH model consists of a set of four equations as follows:

$$\begin{cases} \dfrac{dV}{dt} = \dfrac{1}{C_m}[I_{ext} - g_{Na} m^3 h\left(V - V_{Na}\right) \\ \qquad - g_K n^4 \left(V - V_K\right) - g_l \left(V - V_l\right)], \\ \dfrac{dy}{dx} = \alpha_y \left(1 - y\right) - \beta_y \left(V\right) y \end{cases} \qquad (1)$$

$y = m, h, n$

Where V represents the trans-membrane voltage of the muscle cell that is produced due to the charge accumulation and transportation across the membrane [16]. m, h and n are the gating variables representing the sodium current activation, sodium current inactivation and the potassium current activation functions , respectively which their values are between 0 and 1. α_m ,β_m ,α_h ,β_h , α_n and β_n are nonlinear functions of V as follows :