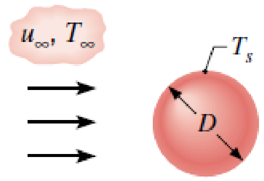
2. Flow across Cylinders and Spheres



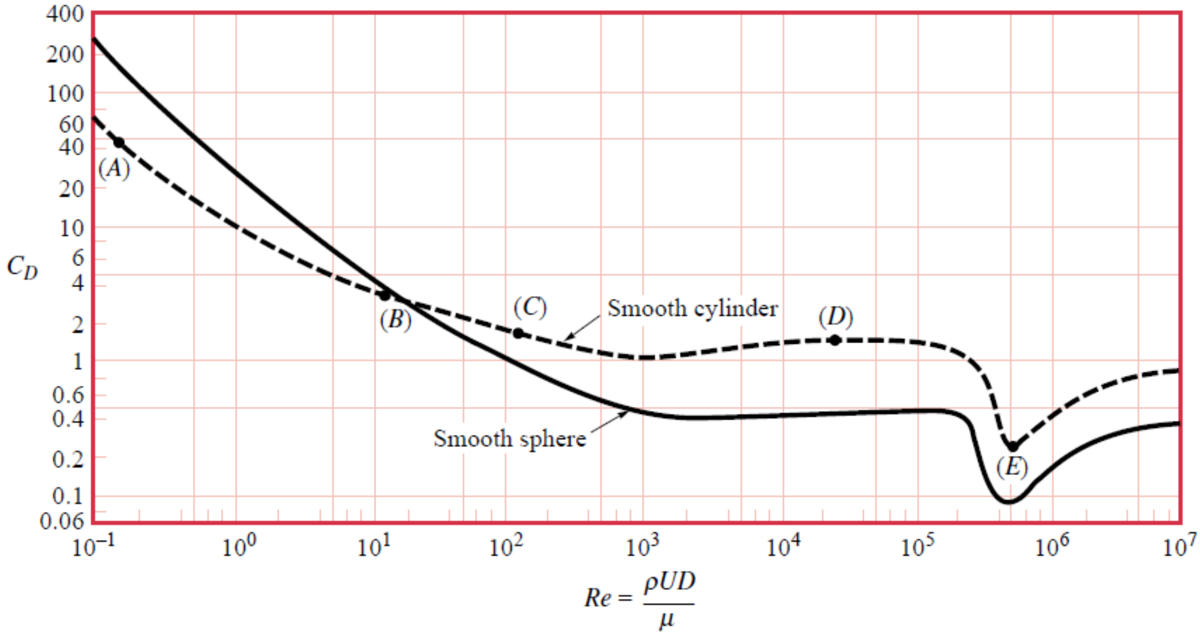


Figure 1 Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere.

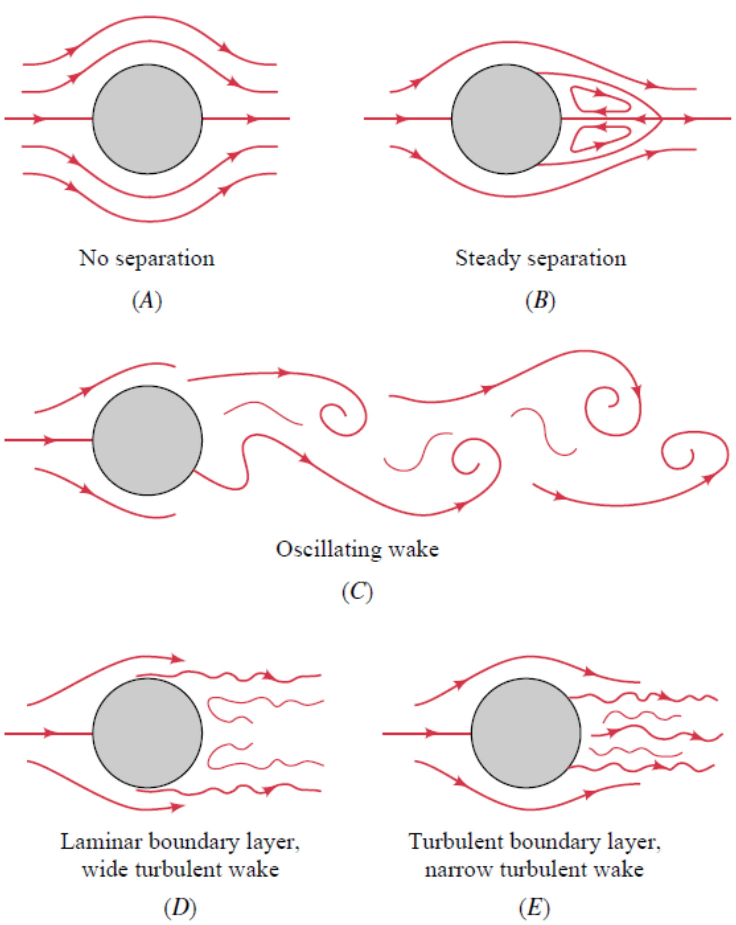


Figure 2 Typical flow patterns for flow past a circular cylinder.

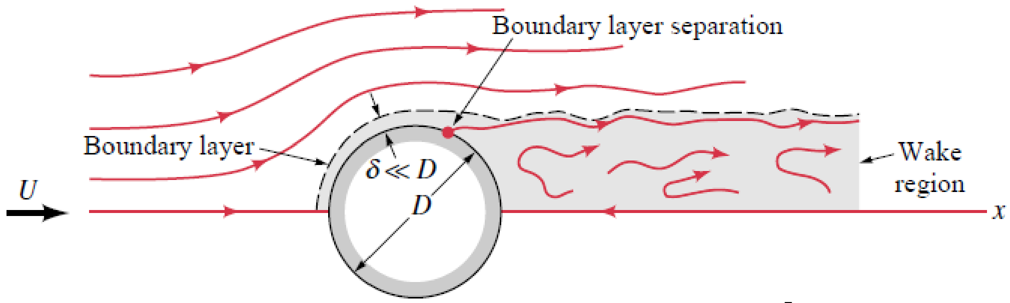


Figure 3 Flow past a circular cylinder, Re=10^5.

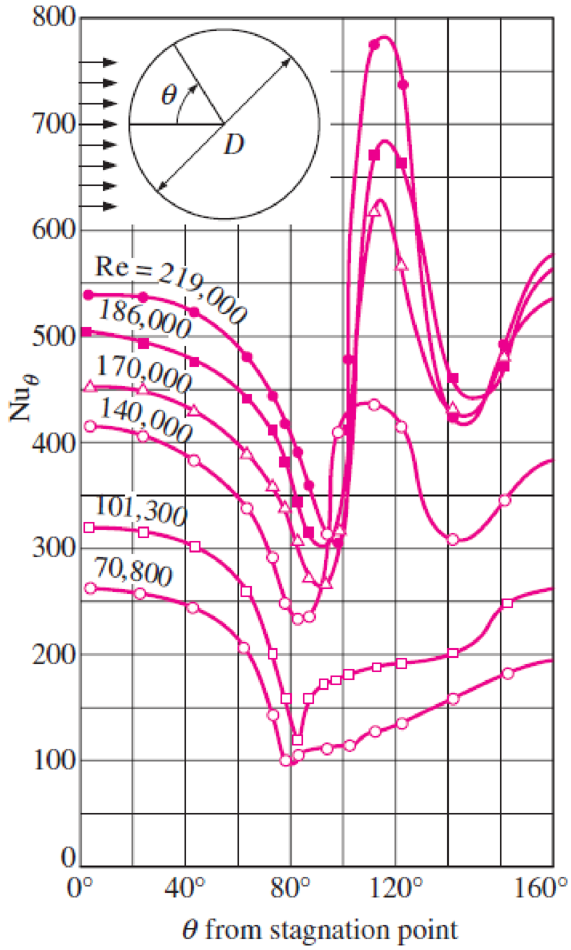


Figure 4 Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross-flow of air (from Giedt).

-Empirical Relations:

1. Churchill and Bernstein Relation for Cylinder:

$$\overline{Nu}_{cyl} = \frac{\bar{h}\,D}{k} = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4\,\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$\mathrm{Re}_D\,\mathrm{Pr} > 0.2$, smooth surface

2. Whitaker Correlation for Sphere:

$$\overline{Nu}_{sph} = \frac{\bar{h}\,D}{k} = 2 + \left[0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

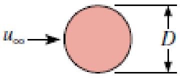
$3.5 \le \mathrm{Re}_D \le 80{,}000 \quad 0.7 \le \mathrm{Pr} \le 380$, smooth surface

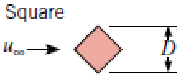
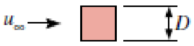
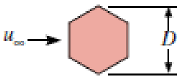
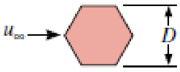
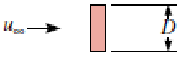
3. Hilpert Correlation (general case):

$$\overline{Nu}_D = \frac{\bar{h}\,D}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Pr} \geq 0.7$$

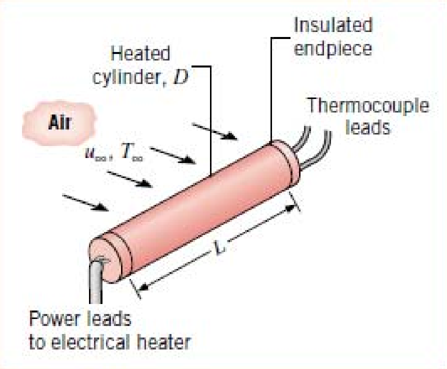
Table 1 Constants for the Hilpert Correlation, for Circular (Liquids and Gases) and Noncircular (Gases only) in Cross Flow

Geometry	Re_D	C	m
Circular 	0.4–4	0.989	0.330
	4–40	0.911	0.385
	40–4000	0.683	0.466
	4000–40,000	0.193	0.618
	40,000–400,000	0.027	0.805

Geometry	Re_D	C	m
Square 	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon 	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate 	4×10^3–1.5×10^4	0.228	0.731

Example 1:

Experiments have been conducted to measure the convection coefficient on a polished metallic cylinder 12.7 mm in diameter and 94 mm long (Fig. E17.4*a*). The cylinder is heated internally by an electrical resistance heater and is subjected to a cross flow of air in a low-speed wind tunnel. Under a specific set of operating conditions for which the free stream air velocity and temperature were maintained at u_∞ = 10 m/s and 26.2°C, respectively, the heater power dissipation was measured to be P_e = 46 W, while the average cylinder surface temperature was determined to be T_s = 128.4°C. It is estimated that 15% of the power dissipation is lost by conduction through the endpieces.



(a) Determine the convection heat transfer coefficient from the experimental observations.
(b) Compare the experimental result with the convection coefficient computed from an appropriate correlation.

Figure E17.4a

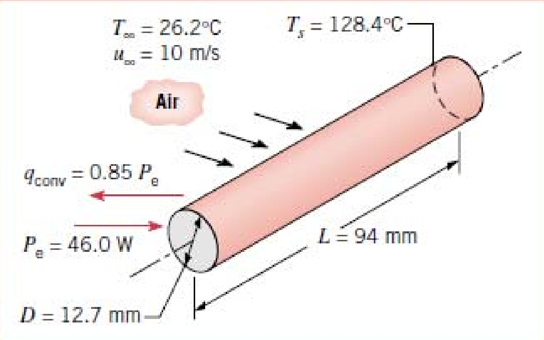
Solution

Known: Operating conditions for a heated cylinder.

Find:

(a) Convection coefficient associated with the operating conditions.
(b) Convection coefficient from an appropriate correlation.

Schematic and Given Data:



Assumptions:
1. Steady-state conditions.
2. Uniform cylinder surface temperature.
3. Negligible radiation exchange with surroundings.

Figure E17.4b

Properties: Table HT-3, air ($T_f \approx 350$ K): $\nu = 20.92 \times 10^{-6}$ m²/s, $k = 30 \times 10^{-3}$ W/m · K, Pr = 0.700.

Analysis:

(a) The convection heat transfer coefficient may be determined from the *experimental observations* by using Newton's law of cooling. That is

$$\bar{h} = \frac{q_{\text{conv}}}{A(T_s - T_\infty)}$$

Since 15% of the electrical power is transfered by conduction from the test section, it follows that $q_{\text{conv}} = 0.85P_e$, and with $A = \pi DL$

$$\bar{h} = \frac{0.85 \times 46 \text{ W}}{\pi \times 0.0127 \text{ m} \times 0.094 \text{ m} (128.4 - 26.2)°\text{C}} = 102 \text{ W/m}^2 \cdot \text{K} \ \triangleleft$$

(b) Using the *Churchill-Bernstein correlation,* Eq. 17.35

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

With all properties evaluated at T_f, $Pr = 0.70$ and

$$Re_D = \frac{u_\infty D}{\nu} = \frac{10\text{ m/s} \times 0.0127\text{ m}}{20.92 \times 10^{-6}\text{ m}^2/\text{s}} = 6071$$

Note that $Re_D\,Pr = 6071 \times 0.700 = 4250 > 0.2$, so that the correlation is within the recommended range. Hence, the Nusselt number and the convection coefficient are

$$\overline{Nu}_D = 0.3 + \frac{0.62(6071)^{1/2}(0.70)^{1/3}}{[1 + (0.4/0.70)^{2/3}]^{1/4}}\left[1 + \left(\frac{6071}{282{,}000}\right)^{5/8}\right]^{4/5} = 40.6$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 40.6 \frac{0.30\text{ W/m}\cdot\text{K}}{0.0127\text{ m}} = 96\text{ W/m}^2\cdot\text{K} \quad \triangleleft$$

Comments:

1. The Hilpert correlation, Eq. 17.34, is also appropriate for estimating the convection coefficient

$$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$$

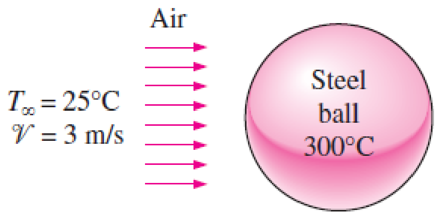
With all properties evaluated at the film temperature, $Re_D = 6071$ and $Pr = 0.70$. Hence, from Table 17.2, find for the given Reynolds number that $C = 0.193$ and $m = 0.618$. The Nusselt number and the convection coefficient are then

$$\overline{Nu}_D = 0.193(6071)^{0.618}(0.700)^{0.333} = 37.3$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 37.3 \frac{0.030\text{ W/m}\cdot\text{K}}{0.0127\text{ m}} = 88\text{ W/m}^2\cdot\text{K}$$

2. Uncertainties associated with measuring the air velocity, estimating the heat transfer from cylinder ends, and averaging the cylinder surface temperature, which varies axially and circumferentially, render the experimental result accurate to no better than 15%. Accordingly, calculations based on the two correlations used here are within the experimental uncertainty of the measured result.

Example 2:



A 25-cm-diameter stainless steel ball ($\rho = 8055$ kg/m^3, $C_p = 480$ J/kg · °C) is removed from the oven at a uniform temperature of 300°C (Fig. 19–18). The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.

SOLUTION A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\,250°C} = 2.76 \times 10^{-5}$ kg/m · s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A–22)

$$k = 0.02551 \text{ W/m} \cdot °\text{C} \qquad \upsilon = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr} = 0.7296$$

Analysis The Reynolds number is determined from

$$\text{Re} = \frac{\mathcal{V} D}{\upsilon} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\text{Nu} = \frac{hD}{k} = 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + [0.4(4.802 \times 10^4)^{1/2} + 0.06(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4}$$
$$= 135$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\mathrm{Nu} = \frac{0.02551 \text{ W/m}\cdot{}^\circ\text{C}}{0.25 \text{ m}}(135) = \mathbf{13.8 \text{ W/m}^2\cdot{}^\circ\text{C}}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25 \text{ m})^2 = 0.1963 \text{ m}^2$$

$$\dot{Q}_{\text{ave}} = hA_s(T_{s,\text{ ave}} - T_\infty) = (13.8 \text{ W/m}^2\cdot{}^\circ\text{C})(0.1963 \text{ m}^2)(250 - 25)^\circ\text{C} = 610 \text{ W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho\tfrac{1}{6}\pi D^3 = (8055 \text{ kg/m}^3)\,\tfrac{1}{6}\pi(0.25 \text{ m})^3 = 65.9 \text{ kg}$$

$$Q_{\text{total}} = mC_p(T_2 - T_1) = (65.9 \text{ kg})(480 \text{ J/kg}\cdot{}^\circ\text{C})(300 - 200)^\circ\text{C} = 3{,}163{,}000 \text{ J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{\text{ave}}} = \frac{3{,}163{,}000 \text{ J}}{610 \text{ J/s}} = 5185 \text{ s} = \mathbf{1 \text{ h } 26 \text{ min}}$$

Discussion The time of cooling could also be determined more accurately using the transient temperature charts or relations introduced in Chap. 18. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between 1 and 2 h.